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CHEAT SHEET # 011 January 2025

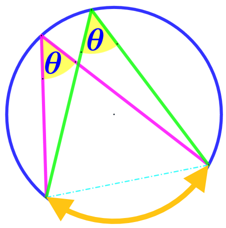
Australian Curriculum Cheat Sheet Year 9 Mathematics

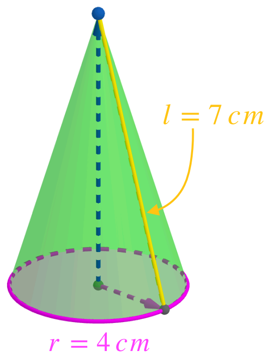
Free and always will be!

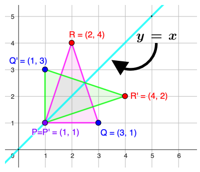
Aim: A cheat sheet for Year 9 Mathematics under the Australian Curriculum, focusing on the key concepts, examples, formulas, and tips:

Year 9 Units Overview:

Term 1 **Unit 1** - Real Numbers, Indices, and Surds **Unit 2** - Algebraic Techniques, and Scientific Notation
Term 2 **Unit 3** - Equations, Inequalities, Quadratics, and Finance. **Unit 4** - Simultaneous Equations, and Circles.
Term 3 **Unit 5** - 3D Pythagoras, Spheres, Cones, and Geometric Proofs **Unit 6** - Trigonometry, and Transformations.
Term 4 **Unit 7** - Data Representation and Interpretation. **Unit 8** - Statistics, and Probability of Combined Events.







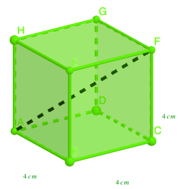
Cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

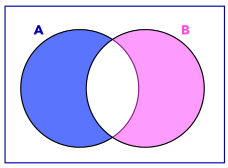
Sine Rule : $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Area Rule: $Area = \frac{1}{2}ab\sin(C)$

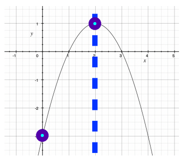
$\mathbb{R}$ = *Set of Real Numbers* = $\{\mathbb{N}^*, \mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{I}\}$

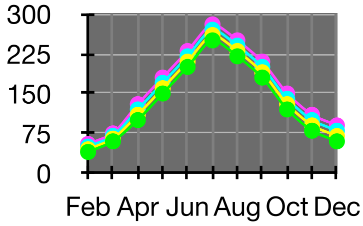
$\mathbb{N}^*$ = Natural numbers,	$\{1, 2, 3 \ldots\}$
$\mathbb{N}$ = Natural numbers including zero,	$\{0, 1, 2, 3 \ldots\}$
$\mathbb{Z}$ = Integers,	$\{\ldots -2, -1, 0, 1, 2 \ldots\}$
$\mathbb{Q}$ = Rational numbers,	$\{\ldots \frac{-3}{5}, \frac{-1}{2}, \frac{-2}{5}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5} \ldots\}$
$\mathbb{I}$ = Irrational numbers,	$\{\ldots -\pi, -\sqrt{3}, -\psi, -\sqrt{2}, \sqrt{2}, \psi, \sqrt{3}, \pi \ldots\}$

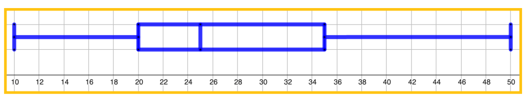




Multiplication:	$a^m \times a^n = a^{m+n}$
Division:	$a^m \div a^n = a^{m-n}$
Power of a Power:	$(a^m)^n = a^{m\times n}$
Negative Power:	$a^{-m} = \frac{1}{a^m}$
Rational Power:	$a^{\frac{m}{n}} = \sqrt[n]{a^m} = a^{\frac{1}{n}\times m} = \left(\sqrt[n]{a}\right)^m = a^{m\times\frac{1}{n}} = \sqrt{(a^m)}$
Zero Power:	$a^0 = 1$







Term 1

Unit 1

1. Number and Algebra

Real Numbers, Indices, and Surds::

Understanding irrational numbers, square roots, and cube roots.

Example: Simplify $\sqrt{48}$.

Solution:

$\sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$.

Laws of indices, simplifying surds.

Example: Simplify $(2^3)^2$.

Solution:

$2^{3 \cdot 2} = 2^6 = 64$.

Example: Simplify $\sqrt{75}$.

Solution:

$\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$.

Unit 2

Algebraic Techniques, and Scientific Notation:

Expanding and factorising quadratic expressions.

Example: Expand $(x+3)^2$.

Solution:

$x^2 + 6x + 9$.

Example: Factorise $x^2 - 5x + 6$.

Solution:

$(x-2)(x-3)$.

Writing large and decimal numbers in scientific notation.

Example: Write in scientific notation $314{,}000$.

Solution:

3.14×10^5

Term 2

Unit 3

Equations, Inequalities, Quadratics, and Simple Interest:

Solving linear equations algebraically.

Example: Solve $4x + 6 = 2$.

Solution:
$x = -1$.

Solving pairs of linear equations algebraically.

Example: Solve the system of equations using substitution: $\begin{cases} y = 2x + 1 \\ 3x - y = 4 \end{cases}$.

Solution:
$x = 5$, $y = 11$.

Solving inequalities involving negative multiplication.

Example: Solve $-2x > 10$.

Solution:
$x < -5$.

Solving quadratics by factorisation, completing the square, and using the quadratic formula.

Example: Solve $x^2 - 7x + 10 = 0$.

Solution:
$x = 2$ **or** $x = 5$.

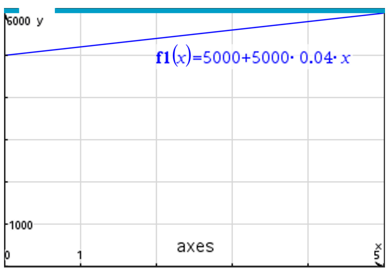
Simple interest and linear equations.

Example: Calculate the simple interest earned on an investment of $\$5{,}000$ at an interest rate of $4\,\%$ $per\,annum$ for 3 $years$. Show your working using the formula $I = PRT$.

Solution:
$I = \$600$.

Example: Calculate the amount owing on a loan of $\$5{,}000$ at an interest rate of $4\,\%$ $per\,annum$ for 3 $years$. Show your working using the formula $I = PRT$ and $A = P + I$.

Solution:
$A = \$5{,}600$.



Unit 4

Simultaneous Equations, and Circles.

Solving simultaneous equations algebraically through substitution, and elimination.

Example: Solve $\begin{cases} x + y = 5 \\ 2x - y = 4 \end{cases}$

Solution:
$x = 3,\ y = 2.$

Example: Solve by substitution $\begin{cases} x + y = 5 \\ y = 4 + 2x \end{cases}$

Solution:
$x = \frac{1}{3},\ y = \frac{14}{3}.$

Example: Solve by elimination $\begin{cases} x + 2y = 6 \\ 9x - 2y = 4 \end{cases}$

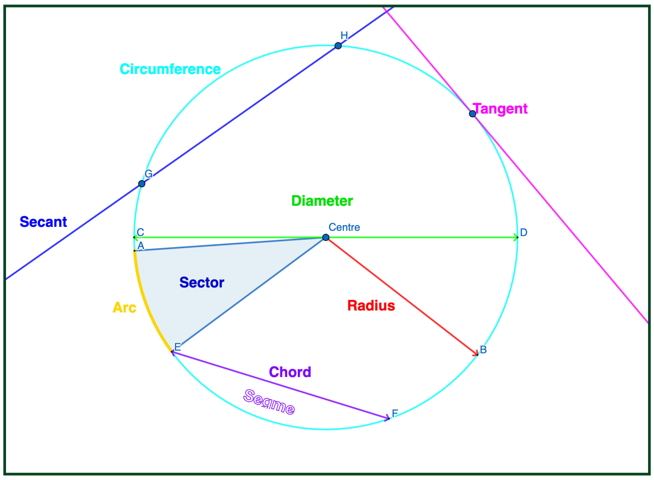
Solution:
$x = 1,\ y = 2.5.$

2. Measurement and Space.

Identify parts of a circle.

Example: Label the different parts of a circle.

Solution:



Circumference, area, arc length, sector area.

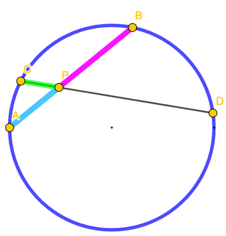
Example: Find the area of a sector with angle $60°$ in a circle of radius $5\,cm$.

Solution:

$$\rightarrow \frac{60}{360} \times \pi \times 5^2$$
$$= \frac{25\pi}{6}\ cm^2$$

Circle Theorems.

Example: In a circle, two chords AB and CD intersect at point P . If $AP = 16\,cm$, $PB = 24\,cm$, and $CP = 12\,cm$, what is the length of PD ?



Solution:
$PD = 32\,cm$.

Term 3

Unit 5

3D Pythagoras, Spheres, Cones, and Geometric Proofs

Applying Pythagoras in three dimensions.

Example: Find the length of the diagonal of a cube with side length 3 cm.

Solution:

$\sqrt{3^2+3^2+3^2} = \sqrt{27} = 3\sqrt{3}\ cm$.

Volume and surface area of spheres and cones.

Example: Find the surface area of a sphere with a diameter of $6\,cm$. Use $\pi \approx 3.14$.

Solution:

Radius, $r = 3cm$,

$$A_{sphere} = 4\pi r^2$$
$$= 113.04\,cm^2.$$

Example: Calculate the volume of a cone with a radius of $3\,cm$ and a height of $10\,cm$. Use $\pi \approx 3.14$.

Solution:

$$V_{cone} = \frac{1}{3}\pi r^2 h$$
$$r = 3\,cm.$$
$$h = 10\,cm.$$
$$= 94.2\,cm^3.$$

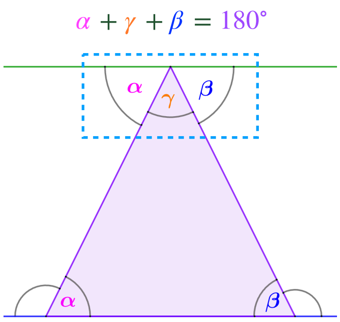
Geometric Proofs and congruence theorems.

Example: What is a geometric proof?

Solution:

A geometric proof is a logical argument that uses previously established facts, definitions, postulates, and theorems to show that a particular statement is true in geometry.

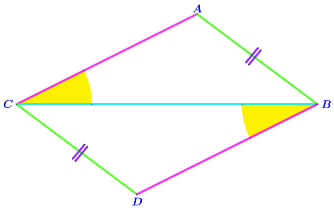
Example: Prove, that the sum of the interior angles in a triangle, add up to $180\,degrees$.



Example: Prove that if two angles and the included side of one triangle (blue triangle) are equal to two angles and the included side of another triangle (red triangle), then the triangles are congruent by the ASA criterion.

Solution:

Given triangles CAB and CDB where :



$AB \parallel CD$,

$\rightarrow \angle ACB \cong \angle DCB$ (Alternate interior angles)

And, $CB \cong BC$.

$\therefore \triangle ABC \cong \triangle DCB$.

Unit 6

Trigonometry, and Transformations.

Trigonometric rules and ratios for right-angled triangles.

$$SOH \rightarrow \sin(\theta) = \frac{opposite}{hypotenuse} = \frac{O}{H}$$

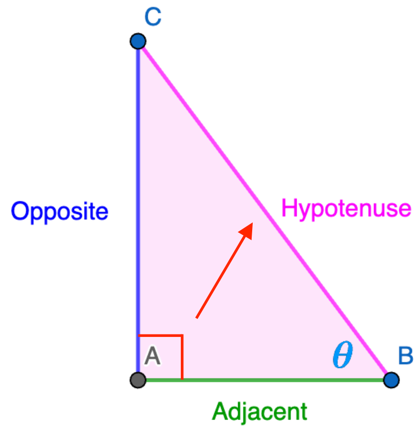
$$CAH \rightarrow \cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{A}{H}$$

$$TOA \rightarrow \tan(\theta) = \frac{opposite}{adjacent} = \frac{O}{A}$$

$SOH\ CAH\ TOA$

Some Old Hags,
Can't Always Hide,
Their Old Age.

SOH
CAH
TOA



Example: If $\sin(\theta) = \frac{3}{5}$, find $\cos(\theta)$ in a right triangle.

Solution:

Using Pythagorean identity, $\cos(\theta) = \frac{4}{5}$.

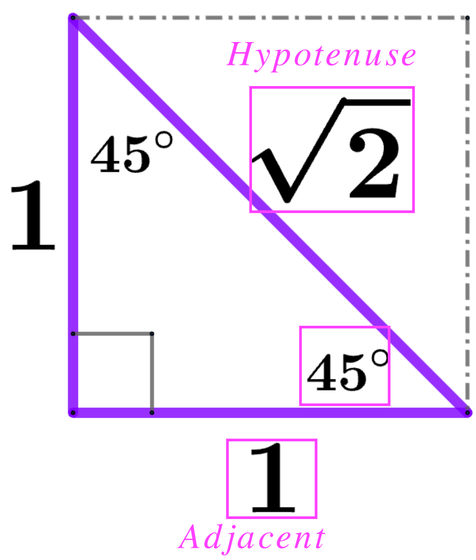
Example: Find the exact value of $\cos(45^\circ)$ in a right triangle.

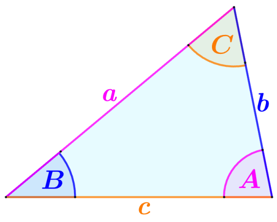
Solution:

Using the triangle to the right, use the identity:

$$\cos(\theta) = \frac{Adjacent}{Hypotenuse}$$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$





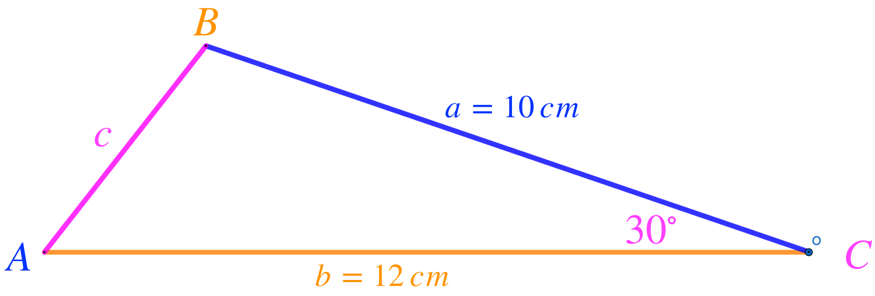
Cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Sine Rule : $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Area Rule: $Area = \dfrac{1}{2}ab\sin(C)$

Example: In triangle ABC, $AC = 12\,cm$, $BC = 10\,cm$, and $\angle BCA = 30^\circ$. Find the length of AB using the cosine rule.

Solution:



Here, $c = c$,

$b = 12$,

$a = 10$,

and $\cos 30^\circ = \dfrac{\sqrt{3}}{2}$

Cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

$c \approx 6.013\,cm$.

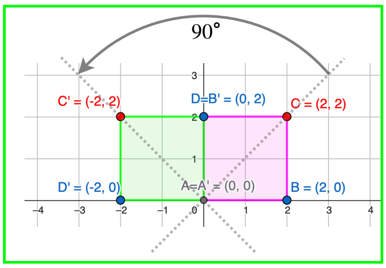
Example: Find $\angle B$ in the same triangle ABC using the sine rule.

Solution:

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

We know $b = 12,\ c = BA \approx 6.013,\ \angle C = 30^\circ$, so:

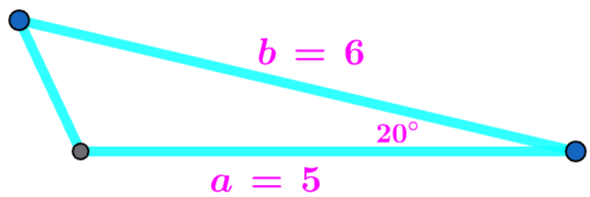
$B \approx 86.23^\circ$.



Example: Find the area of a triangle with side lengths $a = 5$, $b = 6$, and the angle between a and b is 20°.

Solution:

$$Area = \frac{1}{2}ab\sin(C)$$
$$= \frac{1}{2}\cdot 5\cdot 6\cdot\sin(20)$$
$$= 15\sin(20)$$
$$\approx 5.13\ units.$$



Example: A square with vertices $A(0,\ 0)$, $B(2,\ 0)$, $C(2,\ 2)$, and $D(0,\ 2)$ is rotated 90° counterclockwise about the origin. Determine the coordinates of the final image of vertex C.

Solution:

For a 90° counterclockwise rotation about the origin, the rule is $(x,\ y) \to (-y,\ x)$:

The image of $C(2,\ 2)$ after rotation is $C'(-2,\ 2)$.

Term 4

Unit 7

3. Statistics and Probability

Data Representation and Interpretation:

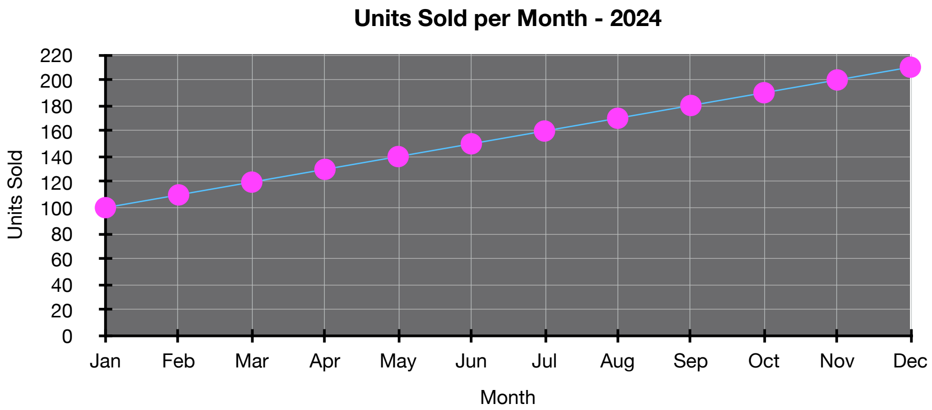
Trends, seasonal variations, smoothing data.

Example: Identify a trend in monthly sales data over a year.

Solution:

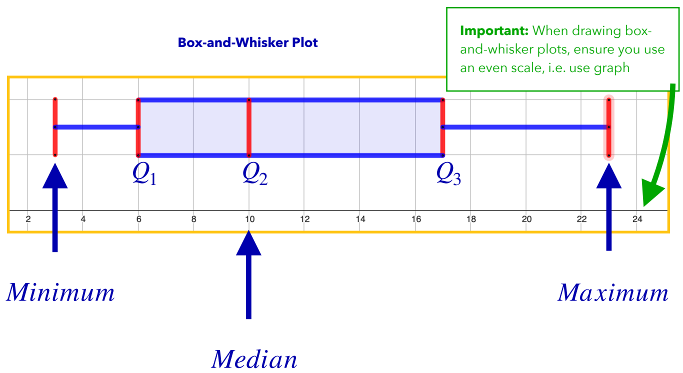
Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Units Sold	100	110	120	130	140	150	160	170	180	190	200	210

Yes, there is an upward trend as the sales increase each month, suggesting growth over time.
Graph data to visually observe trend. *(See below)*



Constructing and interpreting box plots to show data distribution.
Example: Identify the five-number summary used in a box plot.

Solution:
Minimum Value: The smallest data point excluding outliers.
First Quartile (Q_1) : The median of the lower half of the data.
Median (Q_2) : The middle value when data is ordered (median of whole data set).
Third Quartile (Q_3) : The median of the upper half of the data.
Maximum Value: The largest data point excluding outliers.



Example: Explain the difference between independent and dependent events.

Solution:
Independent Events: The occurrence of one event does not affect the probability of the other. E.g., flipping a coin twice; the result of the first flip does not change the probability of the second flip.
Dependent Events: The outcome of one event affects the probability of the other. E.g., drawing two cards without replacement from a deck; the probability of the second card depends on what was drawn first.

Unit 8

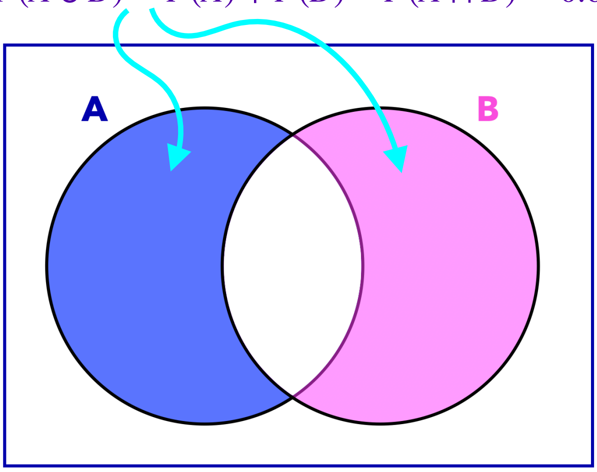
Statistics, and Probability of Combined Events.

Using Venn diagrams, tree diagrams for two-stage events.

Example: $If\ P(A) = 0.3,\ P(B) = 0.5,$ and $P(A\ and\ B) = 0.2,$ find $P(A\ or\ B)$.

Solution:
$P(A \cup B) = 0.3 + 0.5 - 0.2 = 0.6\,.$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6\,.$$



→ Blue Section + Pink Section
= Blue Circle + Pink Circle − White Intersection

Example: From a deck of cards, what's the probability of drawing two aces in succession without replacement?

Solution:

$$P(A) = \frac{n(A)}{n(S)}$$

$$\begin{aligned} P(A\,and\,B) &= P(A \cap B) \\ &= P(\mathrm{A}) \times P(B\,|\,A)\,. \\ &= \text{Probability of A} \times \text{Probability of } B \text{ given } A\,. \\ &= \frac{1}{221} \approx 0.0045 \approx 0.45\,\%\,. \end{aligned}$$

Using Probability Trees, and two-way tables.

Example: Construct a probability tree for the scenario of flipping two coins and determine the probability of getting at least one tail.

Solution:

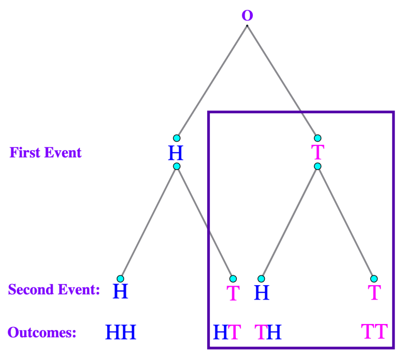
$$P(A)=\frac{n(A)}{n(S)}$$

For each coin flip, there are two outcomes:
$Heads\ (H)$ or $Tails\ (T)$.

The scenarios for at least one tail are:

$$\cancel{HH},\ HT,\ TH,\ TT$$

$$\text{Total probability } = \frac{3}{4} = 0.75 = 75\,\%\,.$$



Example: A survey results in a two-way table showing that out of 300 people, 180 like tea (T), 150 like coffee (C), and 80 like both (T and C). How many like tea but not coffee? What is the probability that a randomly selected person likes only coffee? Use a two-way table to solve the problem, or a Venn diagram.

Solution:
Two-way Table:

$$P(A)=\frac{n(A)}{n(S)}$$

$$=\frac{7}{30}=0.23\dot{3}=23.\dot{3}\,\%\,.$$

	Like Tea	Dislike Tea	**Total**
Like Coffee	80	70	150
Dislike Coffee	100	50	150
Total	180	120	300

Example: Given the dataset: $\{\ 10,\ 14,\ 18,\ 22,\ 26\ \}$, calculate the standard deviation.

Solution:

$$\{\ 10,\ 14,\ 18,\ 22,\ 26\ \}$$

$$\text{Standard Deviation } = \sigma = \sqrt{\text{Variance}} = \sqrt{32}$$

$$\approx 5.66\,.$$

General Tips.

Problem-Solving: Use strategies like drawing, listing, or working backwards.
Mental Math: Enhance skills in quick calculations, especially with fractions and percentages.
Understanding vs. Memorising: Focus on understanding concepts to apply them in varied contexts.
Units: Always check and convert units appropriately, especially in geometry.

Additional Tips

Technology: Use graphing calculators or software for complex calculations, but ensure you understand the steps.
Real-Life Applications: Connect math to real-world scenarios like financial planning, architecture, or data analysis.
Common Mistakes: Watch out for errors in simplification, especially with indices and surds, or misapplying trigonometric functions.

This cheat sheet provides a snapshot, but mastering Year 9 Mathematics involves extensive practice, deep understanding of principles, and applying them across different problem types. For more resources, consider textbooks aligned with the Australian Curriculum, online educational tools, or interactive math platforms.