F_WORKSHEET # 003 Australian Curriculum Year 7 Mathematics FWM7T2U3

ACACIA TUTORING

Formulas, and Finance 7

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Focus: A set of questions and solutions for Year 7 students focused on the topic of Formulas, and Finance, tailored to the Australian Curriculum:

1. Financial planning with percentages.

A student is saving for a new portable speaker as part of a budgeting project. A speaker costs $\$99$. A store offers a $10\,\%$ discount during a sale.

a) Calculate the discount amount and the sale price of the speaker.

b) The student has saved 80 . How much more do they need to save to buy the bicycle at the sale price?

c) Justify whether waiting for the sale is a good financial decision compared to buying the speaker now at full price.

2. Using Formulas in a Financial Context

A student is organising banners to advertise this year's musical and needs to calculate costs using a formula. The cost of producing custom banners for the musical is given by the formula $C = 20 + 99n$, where C is the total cost in dollars and n is the number of banners.

a) Calculate the cost of producing half a dozen banners.

b) If the fundraiser has a budget of $\$1{,}000$, how many banners can they afford to produce?

Extra space for 2b...

c) Explain why the formula $C = 20 + 99n$ is appropriate for this context.

3. Financial Decision with Algebraic Modelling

A student is comparing two options for earning pocket money to save for a concert tickets. They can earn pocket money by designing digital artwork for a local printing company. They are given two options for receiving payment. Option A pays $\$50$ per piece of art. Option B pays a $\$100$ base fee plus $\$25$ per piece of art.

a) Write algebraic expressions for the total earnings from each option, where n is the number of pieces of art sold.

b) If the student were to sell 6 pieces of art, which option pays more, and by how much?

c) Determine the number of pieces of art where both options pay the same amount, and justify which option is better to choose for more than this number.

4. Financial Modelling with Percentages and Formulas

A student is planning a fundraiser event and needs to manage ticket sales. They charges $\$12$ per ticket, and groups of 5 or more get a $10\,\%$ discount per ticket.

a) Write a formula for the total cost T of n tickets, considering the discount for groups of 5 or more, use a piecewise formula to display the final two equations.

b) Calculate the cost for a group of 12 students and verify the discount was applied correctly.

c) If a class has $\$120$, how many tickets can they buy, and justify whether they should buy as a group or individually.

5. Simple Interest

a) A savings account earns simple interest using the formula $I = PRT$, where I is the interest, P is the principal, R is the annual interest rate (as a decimal), and T is the time in years. Calculate the interest earned on $\$1{,}200$ at $10\,\%$ per year for $2\,years$.

b) A savings account earns simple interest using the formula $I = PRT$. Calculate the interest earned on $\$1{,}500$ at $1\,\%\,per\ quarter$ for $5\,years$, then calculate the total amount in the account using the formula $A = P + I$.

c) You wish to open a savings account and you have two options:

Option A: Earns simple interest at rate of $0.40\,\%$ **per month for** $2\,years$ **.**
Option B: Earns simple interest at rate of $5\,\%$ **per year for** $2\,years$ **.**

Calculate the interest earned on a $\$100$ **deposit and decide which account you will invest your money in, ensuring you justify your choice.**

Solutions

1a.

Discount Amount and Sale Price

$$\begin{aligned}\text{Discount} &= 10\,\% \text{ of } \$99\\ 10\,\% &= 0.10\\ \text{Discount} &= 0.10 \times \$99\\ &= \$9.90\,,\end{aligned}$$

$$\begin{aligned}\text{Sale price} &= \text{Original price} - \text{Discount}\\ \text{Sale price} &= \$99 - \$9.90\\ &= \$89.10\end{aligned}$$

b.

Additional Savings Needed

$$\begin{aligned}\text{Student's savings} &= \$80\\ \text{Sale price} &= \$89.10\\ \text{Amount needed} &= \$89.10 - \$80\\ &= \$9.10\end{aligned}$$

The student needs to save an additional $\$9.10$.

c.

Justification of Waiting for the Sale

Buying now at full price costs $\$99$, while the sale price is $\$89.10$, saving $\$9.10$. Waiting for the sale is a good financial decision because it reduces the cost by $10\,\%$, allowing the student to afford the speaker with less money, and this additional saving could go towards the students next savings goal.

Since the student has $\$80$, they only need $\$9.10$ more for the sale price, compared to $\$19$ more for the full price. This smaller savings goal is more achievable and reduces financial strain. However, the student should ensure the sale is happening soon to avoid missing out or needing the speaker urgently.

2a.

Cost for half a dozen (12) banners

Formula :

$$C = 20 + 99n$$

Substitute $n = 12$:

$$\begin{aligned}C &= 20 + 99 \times 12\\ C &= 20 + 1{,}188\\ &= 1{,}208\end{aligned}$$

$$\text{Cost} = \$1{,}208\,.$$

b.

Number of banners for $\$1{,}000$ Budget

Budget: $C = 1{,}000$

Formula: $1{,}000 = 20 + 99n$

Solve for n :

Subtract 20 :

$$1{,}000 - 20 = \cancel{20} + 99n - \cancel{20}$$

$$980 = 99n$$

Divide by 99 :

$$\frac{980}{99} = \frac{\cancel{99}n}{\cancel{99}}$$

$$9.898989 \approx n$$

$$n \approx 9.898989$$

Since they can't buy 0.898989 of a banner, they round down to 9 ,

They can afford 9 banners.

Verification:

Substitute $n = 9$ into the formula :

$$C = 20 + 99 \times 9$$

$$= 20 + 891$$

$= \$911$, which is $< \$1{,}000$ and thus comes under budget.

c.

Explanation of the Formula

The formula $C = 20 + 99n$ is appropriate because it models the total cost of producing banners. The fixed cost of $\$20$ likely represents a setup fee (e.g., design setup), which is incurred regardless of the number of banners. The variable cost, $99n$, represents the cost per banner of $\$99$ each, which depends on the number of banners n . This linear relationship accurately reflects the cost of printing banners where there's an initial cost plus a per-item cost, making it suitable for budgeting and planning.

3a.

Algebraic Expressions

Option A :

$\$50$ per artwork, so earnings $= 50n$.

Option B :

$\$100$ base fee $+ \$25$ per artwork, so earnings $= 100 + 25n$.

b.

Earnings for 6 pieces of art

Option A :

$$\begin{aligned} 50n &= 50 \times 6 \\ &= 300 \\ \text{Earnings} &= \$300 \end{aligned}$$

Option B :

$$\begin{aligned} 100 + 25n &= 100 + 25 \times 6 \\ &= 100 + 150 \\ &= 250 \\ \text{Earnings} &= \$250 \end{aligned}$$

Comparison :

$$\$60 - \$50 = \$10$$

Option A pays $\$50$ more for 6 pieces of art.

c.

Number of Pieces of Art for Equal Pay and Justification

Set the expressions equal :

$$50n = 100 + 25n$$

Solve for n ,

Subtract $25n$:

$$\begin{aligned} 50n \ -25n &= 100 + \cancel{25n} \ \ \cancel{-25n} \\ 25n &= 100 \end{aligned}$$

Divide by 25 :

$$\begin{aligned} \frac{\cancel{25}n}{\cancel{25}} &= \frac{100}{25} \\ n &= 100 \div 25 \\ &= 4 \end{aligned}$$

Verification , for $n = 4$

Option A :

$$50 \times 4 = 200$$

Option B :

$$\begin{aligned} 100 + 25 \times 4 &= 100 + 100 \\ &= 200 \end{aligned}$$

Both pay $\$200$ for 4 pieces of art.

Justification:

For more than 4 pieces of art ($n > 4$), test $n = 5$,

Option A :

$$50 \times 5 = 250$$

Option B :

$$\begin{aligned} 100 + 25 \times 5 &= 100 + 125 \\ &= 225 . \end{aligned}$$

Option A pays more $\$250$ vs. $\$225$.

Option A is better for more than 4 pieces of art because its rate $\$50$ per piece, is higher than Option B's $\$25$ per piece, and the $\$100$ base fee in Option B becomes less significant as n increases. For fewer than 4 pieces of art, Option B pays more due to the base fee (e.g., for $n = 1$, Option A: $\$50$, Option B: $\$125$).

4a.

Formula for Total Cost

For $n < 5$:

Cost $= \$12$ per ticket, so $T = 12n$,

$n =$ Number of Tickets Sold

$T =$ Total Sales in $\$'s$.

For $n \geq 5$:

$$\begin{aligned} \text{Discount} &= 10\,\% \text{ of } \$12 \\ &= \frac{10}{100} \times 12 \\ &= \frac{1\cancel{0}}{10\cancel{0}} \times 12 \\ &= \frac{1}{10} \times 12 \\ &= 0.1 \times 12 \\ &= \$1.20 \text{ per ticket.} \end{aligned}$$

$$\begin{aligned} \text{Discounted price per ticket} &= \$12 - \$1.20 \\ &= \$10.80 \end{aligned}$$

So, total cost :

$$T = 10.8n$$

Piecewise formula :

$$T = \begin{cases} 12n & \text{if } n < 5 \\ 10.8n & \text{if } n \geq 5 \end{cases}$$

b.

Cost for 12 Students

Since , $n = 12 \geq 5$, use $T = 10.8n$:

$$T = 10.8 \times 12$$
$$= 129.6$$
$$\text{Cost} = \$129.60$$

Verification ,

Without discount :

$$12 \times 12 = \$144$$

Discount amount :

$$10\,\%\ of\ \$144 = 0.1 \times 144$$
$$= \$14.40$$

Discounted cost :

$\$144 - \$14.40 = \$129.60$, which matches.

c.

Number of Tickets for $\$120$ and Justification

Since they're a class, assume they buy as a group $n \geq 5$, so use $T = 10.8n$.

Budget $\$120$.

Solve :

$$10.8n \leq 120$$

Divide by 10.8 :

$$\frac{\cancel{10.8}n}{\cancel{10.8}} \leq \frac{120}{10.8}$$
$$n \leq 120 \div 10.8$$
$$= 11.1\dot{1} .$$

Since n must be a whole number, use $n = 11$:

Cost :

$10.8 \times 11 = 118.8$, which is $\leq \$120$.

Try $n = 12$

$10.8 \times 12 = 129.6$, which exceeds $\$120$.

Maximum tickets $= 11$.

Justification: Buying as a group is better because the discounted price of $\$10.80$ per ticket, allows them to buy more tickets than the full price, $\$12$.

For 11 tickets,
Group cost: $\$118.80$
Individual cost: $12 \times 11 = \$132$, which exceeds $\$120$.

With $\$120$, they can only buy $120 \div 12 = 10$, or 10 tickets individually. The group discount maximises the number of tickets (11 vs. 10) and saves money.

5a.

Simple interest :

$$I = PRT$$

Given :

$$P = 1{,}200\,,$$

$$R = 10\,\% = \frac{10}{100} = 0.10 = \ per\ year$$

$$T = 2\,years\,.$$

$$I = 1{,}200 \times 0.10 \times 2$$
$$= \$240\,.$$

b.

Simple interest :

$$I = PRT$$

Given :

$$P = \$1{,}500$$

$$R = 1\,\%\ per\ quarter$$
$$= 1\% \times 4 \text{ quarters per year}$$
$$= 4\,\%\,per\ year \div 100$$
$$= 0.04\,per\ year$$

$T = 5\,years\,.$ Ensure Time and Rate are in the same units i.e. years.

$$I = 1{,}500 \times 0.04 \times 5$$
$$= \$300\,.$$

$$A = P + I$$
$$= \$1{,}500 + \$300$$
$$= \$1{,}800\,.$$

c.

$$I = PRT$$

Option A :

Given :

$P = 100$,

$R = 0.40\% \div 100 = 0.0040$, $per\ month$

$T = 2\,years = 24\,months$. Ensure Time and Rate are in the same units i.e. months.

$$I = 100 \times 0.0040 \times 24$$
$$= \$9.60.$$

Option B :

Given :

$P = 100$,

$R = 5\% \div 100 = 0.05$, $per\ year$

$T = 2\,years$. Ensure Time and Rate are in the same units i.e. years.

$$I = 100 \times 0.05 \times 2$$
$$= \$10.00.$$

Option B is the best choice as it earns more interest , $\$10.00$ compared to $\$9.60$.

Additional Notes for Teachers:

Learning Outcomes: Students should be able to solve simple financial equations, understand and solve formulas in financial situations, and apply these concepts to real-world problems.
Teaching Strategies: Use digital tools to visualise formulas and financial concepts. Encourage students to compare and estimate costs to check the reasonableness of their calculations. Encourage students to check their solutions by substituting back into the original equation or inequality.
Assessment:
Assess students on accurate calculations, correct use of formulas, and clear justifications, aligning with QCAA's five-point scale (A–E). Emphasise reasoning in justifications to meet the curriculum's focus on interpreting solutions in context.
Resources:
Refer to QCAA's "Sequence of content descriptions: Years 7–10 – Mathematics" and ACARA's Australian Curriculum V9.0 for content and standards. Use QCAA's P–10 Planning app for aligned assessment tasks.

These questions and solutions provide a comprehensive approach to teaching and assessing Year 7 Number and Algebra topics related to finance, formulas, and justification, ensuring alignment with the Australian Curriculum V9.0 and QCAA guidelines.