ACACIA TUTORING

Expanding and Factorising 8

Free and always will be!

Focus: A set of questions and solutions for Year 8 students on Expanding and Factorising, tailored to the Australian Curriculum under the strand 'Number and Algebra':

1. Expanding Brackets:

a) Expand: $3(x + 4)$.

b) Expand and simplify: $2(a - 3) + 5(a + 1)$.

2. Expanding Binomial Products:

a) Expand: $(x+3)(x-2)$.

b) Expand: $(2y-1)(y+4)$.

3. Factorising:

a) Factorise: $x^2 + 5x + 6$.

b) Factorise: $a^2 + 2a - 24$.

c) *(Advanced Extension)* **Factorise:** $6a^2 - 13a + 6$ **.**

4. Factorisation using a Common Factor:

a) Factorise: $15x + 25$.

b) Factorise: $4a^3 + 12a^2$.

5. Difference of Two Squares:

a) Factorise: $x^2 - 9$ **.**

b) Factorise: $16y^2 - 25$ **.**

6. Mixed Practise:

a) Expand and simplify: $(2x+3)(x-1)+(x+2)^2$.

b) Factorise fully: $2x^2 + 8x + 8$ **.**

Solutions

1a.

Use the distributive property:

$$3(x+4) \quad\quad a(b+c) = ab + ac$$
$$= 3 \times x + 3 \times 4$$
$$= 3x + 12\,.$$

b.

Apply the distributive property to each bracket:

$$2(a-3) = 2a - 6$$
$$5(a+1) = 5a + 5$$

Combine:

$$2a - 6 + 5a + 5$$
$$= 7a - 1\,.$$

2a.

Use the FOIL method (First, Outer, Inner, Last):

$$(x+3)(x-2)$$
$$= x \cdot x + x \cdot (-2) + 3 \cdot x + 3 \cdot (-2)$$
$$= x^2 - 2x + 3x - 6$$
$$= x^2 + x - 6\,.$$

b.

FOIL again:

$$(2y-1)(y+4)$$
$$= 2y \cdot y + 2y \cdot 4 + (-1) \cdot y + (-1) \cdot 4$$
$$= 2y^2 + 8y - y - 4$$
$$= 2y^2 + 7y - 4\,.$$

Remember, mathematicians sometimes use the symbol ($\cdot$) instead of ($\times$) so we don't get ($\times$) confused with (x)
E.g. $2 \times x \equiv 2 \cdot x$

3a.

Look for two numbers that multiply to 6 (the constant term) and add up to 5 (the coefficient of x) :

These numbers are 2 and 3 .

So, $x^2 + 5x + 6$
$$= (x+2)(x+3)\,.$$

b.

$$a^2 + 2a - 24$$

We need numbers that multiply to give (-24) and add to give $(+2)$:

$$\rightarrow \_ \times \_ = -24 \ \& \ \_ + \_ = 2 \ ?$$
$$\rightarrow 6 \times -4 = -24 \ \& \ 6 + -4 = 2$$

So numbers are $(+6)$ and (-4)

$$\rightarrow a^2 + 2a - 24$$
$$= (a+6)(a-4) .$$

c. *(Advanced Extension)*

$$6a^2 - 13a + 6$$

We need numbers that multiply to give $(6 \times 6) = 36$ and add to -13 :

These numbers are -9 and -4 ,

so we break $-13a$ into $-9a - 4a$,

Thus, $6a^2 - 13a + 6$

$$= 6a^2 - 9a - 4a + 6$$
$$= 6a \times a - 9a - 4a + 6$$
$$= 2 \times 3a \times a - 3 \times 3a - 2 \times 2a + 2 \times 3$$
$$= 2 \times 3a \times a - 3 \times 3a - 2 \times 2a - 2 \times -3$$
$$= 2 \times a \times 3a - 3 \times 3a - 2 \times 2a - 2 \times -3$$
$$= 2a \times 3a - 3 \times 3a - 2 \times 2a - 2 \times -3$$
$$= 3a(2a-3) - 2(2a-3)$$
$$= 3a(2a-3) - 2(2a-3)$$
$$= (3a-2)(2a-3)$$

4a.

The greatest common factor of 15 and 25 is 5 :

$$\rightarrow 15x + 25$$
$$= 5(3x+5) .$$

b.

The GCF here is $4a^2$:

$$4a^3 + 12a^2$$
$$\rightarrow 4a^2 \times a + 4a^2 \times 3$$
$$= 4a^2(a+3) .$$
$$= 4a^2(a+3) .$$

5a.

Recognise it as a difference of squares:

$$x^2 - 9 \qquad x^2 - a^2 = (x - a)(x + a)$$
$$x^2 - 3^2$$
$$= (x - 3)(x + 3)\,.$$

b.

This is also a difference of squares:

$$16y^2 - 25$$
$$= (4y)^2 - 5^2 \qquad x^2 - a^2 = (x - a)(x + a)$$
$$= (4y + 5)(4y - 5)\,.$$

6a.

Expand each separately:

$$\rightarrow (2x + 3)(x - 1) \qquad (a + b)(x + y) = ax^2 + ay + bx + by$$
$$= 2x^2 - 2x + 3x - 3$$
$$= 2x^2 + x - 3$$

$$\rightarrow (x + 2)^2$$
$$= (x + 2)(x + 2) \qquad (a + b)(x + y) = ax^2 + ay + bx + by$$
$$= x^2 + 2x + 2x + 4$$
$$= x^2 + 4x + 4$$

Then combine:

$$\rightarrow (2x^2 + x - 3) + (x^2 + 4x + 4)$$
$$= 3x^2 + 5x + 1\,.$$

b.

Factor out the GCF, which is 2 :

$$\to 2(x^2 + 4x + 4)$$

The expression inside the brackets is a perfect square trinomial:

$$\to x^2 + 4x + 4$$
$$= x^2 + 2^2x + 2^2$$
$$= (x + 2)^2$$

So, fully factored, it is:

$$= 2(x + 2)^2 .$$

Additional Notes for Teachers:

Learning Outcomes:
Students should be able to expand expressions with brackets and factorise quadratic expressions, recognising patterns like difference of squares and perfect square trinomials.

Teaching Strategies:
Use visual aids like algebra tiles for expanding and factorising. Encourage students to create their own examples and challenge each other with different expressions.

Assessment:
Assess through a mix of direct computation problems and more abstract problems where students must decide the best method of factorisation or expansion.

Resources:
Use algebra software for interactive practice or traditional methods like factoring games or puzzles to engage students with these algebraic concepts.

This set of questions aligns with the Australian Curriculum for Year 8, aiming to develop fluency in algebraic manipulation through expanding and factorising.