Linear Equations, Inequalities, and Linear Relationships

ACACIA TUTORING

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Focus: A set of questions and solutions for Year 8 students on Linear Equations and Inequalities, tailored to the Australian Curriculum under the strand 'Number and Algebra':

1. Solving Linear Equations:

a) Solve the equation to find the value of ' x **':** $2x + 5 = 13$ **.**

b) Solve for ' y **' in:** $3y - 7 = 2y + 3$ **.**

2. Linear Equations with Fractions:

a) Solve: $\dfrac{x}{3} + 2 = 5$.

b) Solve: $\dfrac{2y}{5} - 1 = \dfrac{y}{5} + 2$.

3. Solving Inequalities:

a) Solve the inequality: $3x - 1 > 8$.

b) Solve: $2(y + 3) \leq 4y - 2$.

4. Graphing Linear Inequalities:

a) Graph the solution to: $x + 2 < 5$ **on a number line .**

b) Sketch the region that satisfies: $2x + y \geq 4$ **.**

5. Word Problems:

a) If twice a number plus 3 equals 11 , what is the number?

b) A student has $50 and wants to buy books that cost $15 each. How many books can he buy if he wants to have at least $10 left for other expenses?

6. Linear Relationships:

a) Graph the following two points on a cartesian plane and find the slope of a line passing through the points $(1, 2)$ and $(5, 6)$:

b) Construct a table of values for the equation $y = 2x + 3$ **for** $x = (-1, 0, 1)$ **.**

c) A linear relationship is given by the equation $y = 2x + 3$ **. Find the** $x - intercept$ **and** $y - intercept$ **of the line. Draw the graph of this equation on the coordinate plane.**

d) A taxi company charges $\$7$ **for starting the trip and** $\$1.50\ per\ kilometre$ **traveled. Write the linear equation that represents the cost (** C **) of a trip based on the distance (** d **) in** $kilometres$ **. How much would a** $10\ km$ **trip cost?**

Solutions

1a.

Subtract 5 from both sides:
$$\rightarrow 2x \cancel{+5} \; \cancel{-5} = 13 \; -5$$
$$2x = 8$$

Divide by 2 :
$$\frac{\cancel{2}x}{\cancel{2}} = \frac{8}{2}$$
$$x = 4 \, .$$

b.

Subtract $2y$ from both sides to get all y terms on one side:
$$3y - 2y - 7 = \cancel{2y} \; \cancel{-2y} \; +3$$
$$y - 7 = 3$$

Add 7 to both sides:
$$y \cancel{-7} \; \cancel{+7} = 3 \; +7$$
$$y = 10 \, .$$

2a.

Subtract 2 from both sides:
$$\frac{x}{3} \cancel{+2} \; \cancel{-2} = 5 \; -2$$
$$\frac{x}{3} = 3$$

Multiply both sides by 3 to clear the fraction:
$$\frac{x}{\cancel{3}} \cdot \cancel{3} = 3 \cdot 3$$
$$x = 9 \, .$$

b.

Get rid of the fractions by multiplying every term by 5 :
$$\frac{2y}{\cancel{5}} \cdot \cancel{5} - 1 \cdot 5 = \frac{y}{\cancel{5}} \cdot \cancel{5} + 2 \cdot 5$$
$$2y - 5 = y + 10$$

Subtract y from both sides:
$$2y - y - 5 = y \; \cancel{-y} \; +10$$
$$y - 5 = 10$$

Add 5 to both sides:
$$y \cancel{-5} \; \cancel{+5} = 10 \; +5$$
$$y = 15 \, .$$

3a.

Add 1 to both sides:

$$3x - 1 + 1 > 8 + 1$$
$$3x > 9$$

Divide both sides by 3 :

$$\frac{3x}{3} > \frac{9}{3}$$
$$x > 3.$$

b.

Expand and simplify:

$$2y + 6 \leq 4y - 2$$

Subtract $2y$ from both sides:

$$2y - 2y + 6 \leq 4y - 2$$
$$6 \leq 2y - 2$$

Add 2 to both sides:

$$6 + 2 \leq 2y - 2 + 2$$
$$8 \leq 2y$$

Divide by 2 :

$$\frac{8}{2} \leq \frac{2y}{2}$$
$$4 \leq y \ or \ y \geq 4.$$

4a.

Solve the inequality:

$$x + 2 - 2 < 5 - 2$$
$$x < 3.$$

On a number line, draw an open circle at 3

(since x cannot equal 3)

and shade all numbers to the left of 3 .

b.

Convert to slope-intercept form to graph:

$$2x - 2x + y \geq 4 - 2x$$
$$y \geq 4 - 2x$$

Draw the line $y = -2x + 4$ (solid line since it's $\geq$).

Shade the region above this line because y needs to be greater than or equal to the line's values.

5a.

Let the number be x :

$$2x \cancel{+3-3} = 11-3$$
$$\frac{\cancel{2}x}{\cancel{2}} = \frac{8}{2}$$
$$x = 4$$

b.

Let b be the number of books:

$$15b \leq 50 - 10$$
$$15b \leq 40$$

Divide by 15 :

$$\frac{\cancel{15}b}{\cancel{15}} \leq \frac{40}{15}$$
$$b \leq \frac{40}{15}$$
$$b \leq \frac{40 \div 5}{15 \div 5}$$
$$b \leq \frac{8}{3}.$$
$$\approx b \leq 2.67$$

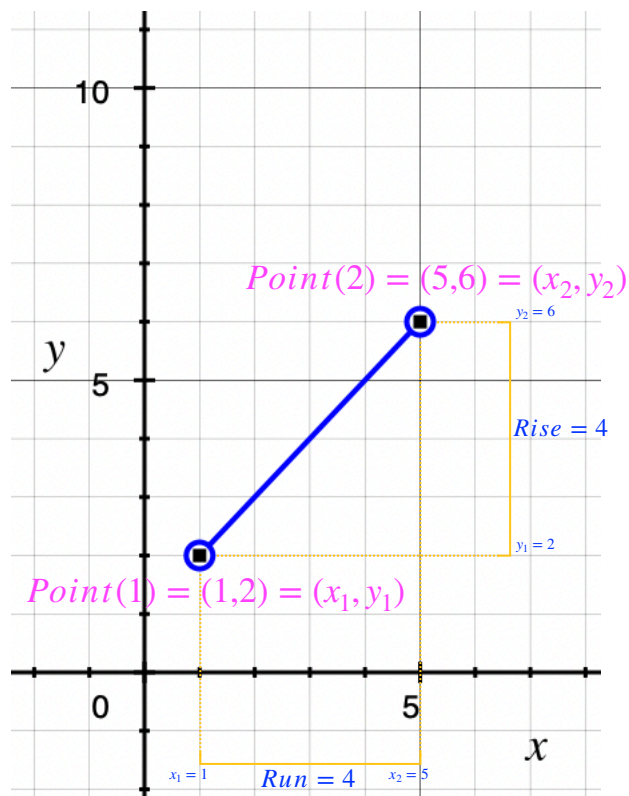Since he can only buy whole books, he can buy at most 2 books.

6a.

B . 2 - Slope is calculated as:

$$Point(2) = (5,6) = (x_2, y_2)$$
$$Point(1) = (1,2) = (x_1, y_1)$$

$$m = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{5 - 1}$$

$$= \frac{4}{4}$$
$$= 1 \, .$$
$$= \frac{1}{1} = \frac{\text{Rise}}{\text{Run}}$$

$$Point(2) = (5,6) = (x_2, y_2)$$
$$y_2 = 6$$
$$Rise = 4$$
$$y_1 = 2$$
$$Point(1) = (1,2) = (x_1, y_1)$$
$$x_1 = 1 \qquad Run = 4 \qquad x_2 = 5$$

b.

Substitute the values for $x = (-1, 0, 1)$, into the equation $y = 2x + 3$:

$$x = -1 \to 2 \times (-1) + 3$$
$$= 1 \, ,$$
$$x = 0 \to 2 \times (0) + 3$$
$$= 3 \, ,$$
$$x = 1 \to 2 \times (1) + 3$$
$$= 5 \, .$$

x	y
−1	1
0	3
1	5

c.

To find the y-intercept, set x to 0 :

$$y = 2 \times (0) + 3,$$
$$y = 0 + 3,$$
$$y = 3.$$

The $y - intercept$ is at $(0, 3)$.

To find the $x - intercept$, set y to 0 : $0 = 2x + 3$,

$$0 - 2x = 2x - 2x + 3,$$
$$-2x = 3,$$
$$\frac{-2x}{-2} = \frac{3}{-2}, \; (\text{Remember}, \; + \; \text{and} \; - = - \;)$$
$$x = \frac{-3}{2} = -1.5.$$

The x-intercept is at $\left(\dfrac{-3}{2}, 0 \right)$.

d.

The equation is $C = 7d + 1.5$.

For a $10\,km$ trip: $C = 7\,\$/km \times (10\,km) + 1.5$

$$= 70 + 1.5$$
$$= \$71.5.$$

The cost would be $\$71.50$.

Additional Notes for Teachers:

Learning Outcomes:
Students should be adept at solving linear equations and inequalities, understanding how to manipulate them and interpret solutions in context.

Teaching Strategies:
Use physical or digital tools for graphing inequalities. Engage students with real-life scenarios where they must set up and solve equations or inequalities.

Assessment:
Evaluate through varied problems requiring students to solve, graph, and apply linear equations and inequalities.

Resources:
Graph paper for manual graphing, interactive algebra software, or apps for visual learning. Classroom activities might include creating scenarios where students must use algebra to solve problems.

This set of questions aligns with the Australian Curriculum for Year 8, focusing on deepening understanding and proficiency in linear equations and inequalities.

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