ACACIA TUTORING

Ratio, Rates, Proportions, and Pythagoras

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Focus: A set of questions and solutions for Year 8 students on Ratio, Rates, and Proportions, tailored to the Australian Curriculum under the strand 'Number and Algebra', and Pythagoras under the strand 'Measurement and Geometry' :

1. Ratios:

a) Simplify the ratio $15 : 20$**.**

b) If the ratio of red to blue marbles is $3 : 5$ **and there are** 24 **red marbles, how many blue marbles are there?**

2. Rates:

a) If a car travels $180\,km$ **in** $3\,hours$ **, what is its average speed in** km/h **?**

b) A machine produces 300 **widgets in** $5\,minutes$ **. What is the production rate in widgets per** $minute$ **?**

3. Proportions:

a) Solve the proportion $\frac{2}{3} = \frac{x}{12}$.

b) If 4 apples cost $\$3$**, how much would** 10 **apples cost?**

4. Unit Rates:

a) A shop sells $5\,kg$ of sugar for $\$6$. What is the price per $kilogram$?

b) If you can buy 3 pens for $\$2.70$, what is the cost of one pen?

5. Real-World Applications:

a) A recipe for cookies calls for a ratio of 2 cups of flour to 1 cup of sugar. If you have 6 cups of flour, how much sugar do you need?

b) A map has a scale of $1\ cm$ to $5\ km$. If the distance between two cities on the map is $4\ cm$, what is the actual distance?

6. Comparing Rates:

a) Which is the better deal: $4\,litres$ **of paint for** $\$24$ **or** $7\,litres$ **for** $\$42$ **?**

b) If one printer prints 10 **pages in** $2\,minutes$ **and another prints** 20 **pages in** $3\,minutes$**, which printer is faster?**

7. Basic Application of the Pythagorean Theorem:

a) In a right-angled triangle, one leg is $6\,cm$ and the other leg is $8\,cm$. Find the length of the hypotenuse.

b) If the hypotenuse of a right triangle is $13\,cm$ and one leg is $5\,cm$, find the length of the other leg.

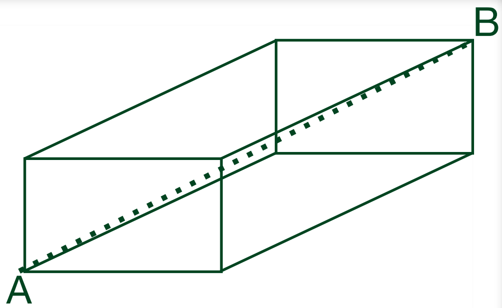
8. Using Pythagorean Theorem with Real-World Scenarios:

a) A ladder is leaning against a wall. The base of the ladder is $3\, metres$ from the wall, and the ladder reaches $4\, metres$ up the wall. How long is the ladder?

b) A baseball diamond is a square with sides of 90 feet . How far does a player run from home plate to second base directly?

9. Finding Unknown Lengths in 3D:

A box has dimensions of $3\,cm$ by $4\,cm$ by $5\,cm$. What is the longest straight line you can draw inside the box, connecting opposite corners?



10. Verification of Right Triangles:

a) Check if a triangle with sides 5, 12, and 13 is a right triangle.

b) Determine if a triangle with sides $7, 24,$ and 25 is a right triangle.

11. Problem Solving:

A kite string makes a $30 - degree$ angle with the ground, and the string is $100\,metres$ long. How high is the kite if you assume the string is straight and taut?

Solutions

1a.

Find the greatest common divisor (GCD) of 15 and 20, which is 5 :

$$15 \div 5 : 20 \div 5$$
$$= 3 : 4 .$$

b.

Let x be the number of blue marbles. The ratio gives us:

$$\frac{\text{Number of Red Marbles}}{\text{Number of Blue Marbles}} = \frac{\text{Total Number of Red Marbles}}{\text{Total Number of Blue Marbles}}$$
$$\frac{3}{5} = \frac{24}{x}$$

Cross multiply:

$$3x = 5 \times 24$$
$$3x = 120$$
$$x = \frac{120}{3}$$
$$x = 40$$

There are 40 blue marbles.

2a.

$$\text{Average speed } = \frac{\text{Distance}}{\text{Time}}$$
$$= \frac{180\,km}{3\,hours}$$
$$= 60\,km/h .$$

b.

$$\text{Production rate } = \frac{300 \text{ widgets}}{5\,minutes}$$
$$= 60 \text{ widgets} / minute .$$

3a.

$$\frac{2}{3} = \frac{x}{12}$$

Cross multiply:

$$2 \times 12 = 3 \times x$$
$$24 = 3x$$
$$x = \frac{24}{3}$$
$$x = 8 .$$

b.

Set up the proportion:

$$\frac{4}{3} = \frac{10}{x}$$

Cross multiply:

$$4x = 3 \times 10$$
$$4x = 30$$
$$x = \frac{30}{4}$$
$$x = 7.5\,.$$

Therefore, 10 apples would cost $\$7.50$.

4a.

Unit rate $= \dfrac{\$6}{5\,kg}$

$= \$1.20$ per kg .

b.

Unit rate $= \dfrac{\$2.70}{3 \text{ pens}}$

$= \$0.90$ per pen .

5a.

The ratio is $2 : 1$, so:

$$2 \times x = \frac{6}{\cancel{x}} \cancel{\times x}$$
$$\frac{\cancel{2}x}{\cancel{2}} = \frac{6}{2}$$
$$x = 3\,.$$

You need 3 cups of sugar.

b.

Using the scale:

$$1\,cm : 5\,km$$
$$1\,cm \times 4 : 5 \times 4$$
$$\rightarrow 4\,cm : 20\,km\,.$$

So $4\,cm$ on the map $= 20\,km$ in real life.

6a.

Calculate the cost per $litre$ for each:

$4\,litres$ for $\$24$:

$$\rightarrow \frac{\$24}{4L} = \$6\,per\,litre\,.$$

$7\,litres$ for $\$42$:

$$\frac{\$42}{7L} = \$6\,per\,litre\,.$$

Both deals have the same unit price, so they are equally as good.

b.

Convert to pages per $minute$:

Printer 1 : $\dfrac{10 \text{ pages}}{2\,minutes} = 5$ pages per $minute$.

Printer 2 : $\dfrac{20 \text{ pages}}{3\,minutes} \approx 6.67$ pages per $minute$.

Therefore, Printer 2 is faster.

7a.

Using the Pythagorean theorem, $a^2 + b^2 = c^2$:

$$\begin{aligned} 6^2 + 8^2 &= c^2 \\ 36 + 64 &= c^2 \\ 100 &= c^2 \\ c^2 &= 100 \\ \cancel{\sqrt{c^2}} &= \sqrt{100} \\ c &= \sqrt{100} \\ &= 10\,cm\,. \end{aligned}$$

b.

Here, $a^2 + 5^2 = 13^2$:

$$\begin{aligned} a^2 \cancel{+25} \cancel{-25} &= 169 \ -25 \\ a^2 &= 169 - 25 \\ a^2 &= 144 \\ \cancel{\sqrt{a^2}} &= \sqrt{144} \\ a &= \sqrt{144} \\ &= 12\,cm\,. \end{aligned}$$

8a.

The ladder forms the hypotenuse of a right triangle,
where the wall and ground are the legs:

$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$
$$25 = c^2$$
$$\cancel{\sqrt{c^2}} = \sqrt{25}$$
$$c = \sqrt{25}$$
$$= 5\,metres\,.$$

b.

From home to second base forms the diagonal of the square,
which is the hypotenuse of a right triangle where each side of the square is a leg:

$$90^2 + 90^2 = c^2$$
$$8100 + 8100 = c^2$$
$$16200 = c^2$$
$$\cancel{\sqrt{c^2}} = \sqrt{16200}$$
$$c = \sqrt{16200}$$
$$\approx 127.28\,feet\,.$$

9.

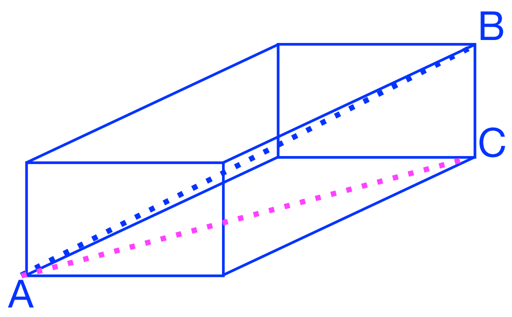
The longest line inside a rectangular prism is the space diagonal,
which can be found by considering the box as two right triangles stacked.

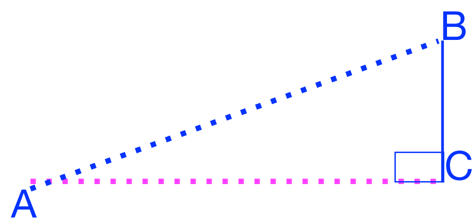
First, find the diagonal of the base A to C $(3\,cm$ by $4\,cm)$:

$$3^2 + 4^2 = d^2$$
$$9 + 16 = d^2$$
$$25 = d^2$$
$$d = 5\,cm\,.$$

Now, use this diagonal with the height $(5\,cm)$ to find the space diagonal A to B :

$$5^2 + 5^2 = l^2$$
$$25 + 25 = l^2$$
$$50 = l^2$$
$$l = \sqrt{50}$$
$$\approx 7.07\,cm\,.$$





10a.

If $5^2 + 12^2 = 13^2$, then it's a right triangle:

$25 + 144 \equiv 169$

$169 \equiv 169$, so it's a right triangle.

$\equiv$ means, 'the same as' or 'equivalent'.

b.

Check with $7^2 + 24^2 = 25^2$:

$49 + 576 \equiv 625$

$625 \equiv 625$, so it's a right triangle.

11.

Here, we're dealing with a right triangle where the angle with the ground is known, and we need to find the opposite side (height of kite):

Using *sine* :

$$\sin(30^\circ) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin(30^\circ) = \frac{h}{100}$$

$$0.5 \times 100 = \frac{h}{\cancel{100}} \times \cancel{100}$$

$$0.5 \times 100 = h$$

$$h = 0.5 \times 100$$

$$= 50\,metres.$$

Additional Notes for Teachers:

Learning Outcomes:

Students should understand how to work with ratios, rates, and proportions, converting between them, solving problems, and applying these concepts in real-world scenarios. Students should understand and apply the Pythagorean Theorem in various contexts, including verifying right triangles and solving problems in 2D and 3D.

Teaching Strategies:

Use practical examples like recipes, travel distances, or pricing to teach these concepts. Encourage students to find and compare rates in everyday life. Use practical applications like building projects or sports to illustrate the theorem's use. Encourage students to measure things around them to apply the theorem in a real-world context.

Assessment:

Test through problems where students must calculate, compare, or apply ratios, rates, and proportions in context. Test through problems that involve finding lengths, verifying right angles, and using trigonometry for more complex scenarios.

Resources:

Use digital tools or apps for interactive scaling and proportion exercises. Classroom activities could include shopping scenarios or map-reading exercises. Use geometry software or physical models for visualisation. Encourage students to explore Pythagorean triples or use protractors and rulers for hands-on learning.

This set of questions aligns with the Australian Curriculum for Year 8, focusing on developing students' skills in understanding and manipulating ratios, rates, and proportions, and the practical application of the Pythagorean Theorem in geometry.