F_WORKSHEET # 013 Australian Curriculum Year 8 Mathematics FWM8T3U5

Volume, Surface Area, and Transformations 8

ACACIA TUTORING

Free and always will be!

Focus: A set of questions and solutions for Year 8 students on Volume, Surface Area, and Transformations, tailored to the Australian Curriculum under the strand 'Measurement and Geometry':

1. Volume of Prisms:

a) Calculate the volume of a rectangular prism with dimensions $5\,cm$, $3\,cm$, and $8\,cm$.

b) Find the volume of a triangular prism where the triangle's base is $6\,cm$, height is $4\,cm$, and the length of the prism is $10\,cm$.

2. Surface Area of Prisms:

a) Calculate the surface area of a cube with side length $4\,cm$.

b) Find the surface area of the rectangular prism from question 1a.

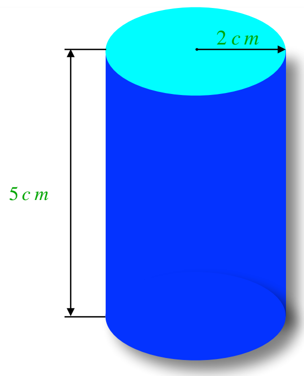
3. Volume of Cylinders:

a) Determine the volume of a cylinder with radius $3\,cm$ and height $10\,cm$.

b) If the height of the cylinder is doubled, what is the new volume?

4. Surface Area of Cylinders:

Find the surface area of a cylinder where the radius is $2\ cm$ and the height is $5\ cm$.



5. Real-World Applications:

a) How much sand would be needed to fill a sandbox with the shape of a rectangular prism that is $2\,metres$ long, $1.5\,metres$ wide, and $0.5\,metres$ deep?

b) A cylindrical water tank has a diameter of $1.4\,metres$ and a height of $2\,metres$. How much water can it hold in litres? ($1m^3 = 1000L$)

6. Composite Shapes *(tricky)*:

A shape consists of a cube with side length $3\,cm$ and a cylinder on top with the same base area as the cube's top face and a height of $4\,cm$. Calculate the volume of this composite shape.

7. Reflection.

A point $P(3, 4)$ is reflected over the $x - axis$ to produce point P'. What are the coordinates of P'? Describe the effect of this reflection on the $y - coordinate$ of the point.

8. Rotational Symmetry.

A square has four lines of symmetry. How many degrees is the angle of rotational symmetry for a square? Explain how this relates to the number of times the square maps onto itself in a full 360° rotation.

9. Combining Transformations.

A triangle with vertices $A(1, 2)$, $B(3, 2)$, and $C(1, 5)$ is first translated 4 units to the right and 2 units down, then reflected over the $y-axis$. Determine the coordinates of the vertices of the final image triangle $A''B''C''$. Use a digital tool or graph paper to verify your answer (optional for students).

10. Real World Application.

A logo designer is creating a pattern for a company's branding. The logo is a rectangle with vertices at $D(0, 0)$, $E(4, 0)$, $F(4, 2)$, and $G(0, 2)$ on a coordinate grid. The designer wants to create a repeating pattern by rotating the rectangle $90°$ clockwise about the origin and then translating the rotated rectangle 6 units to the right. Find the coordinates of the vertices of the final rectangle $D''E''F''G''$. Describe one way this transformation could be used in a real-world branding context.

Solutions

1a.

$$\text{Volume } V \text{ of a Prism} = \text{Area of Base} \times \text{Height:}$$
$$\text{Volume } V \text{ of a rectangular prism} = \text{length} \times \text{width} \times \text{height:}$$
$$V = 5\,cm \times 3\,cm \times 8\,cm$$
$$= 120\,cm^3.$$

b.

$$\text{Area of the triangular base} = \frac{1}{2} \times \text{base} \times \text{height:}$$
$$= \frac{1}{2} \times 6\,cm \times 4\,cm$$
$$= 12\,cm^2.$$

$$\text{Volume of the prism} = \text{Area of base} \times \text{length (height of prism):}$$
$$= 12\,cm^2 \times 10\,cm$$
$$= 120\,cm^3.$$

2a.

A cube has 6 faces, each with area $4^2 = 16\,cm^2$:

$$\text{Surface Area} = 6 \times 16\,cm^2$$
$$= 96\,cm^2.$$

b.

There are 3 pairs of identical faces:

2 faces with area $5 \times 3 = 15\,cm^2$

2 faces with area $5 \times 8 = 40\,cm^2$

2 faces with area $3 \times 8 = 24\,cm^2$

$$\text{Surface Area} = 15 + 40 + 24$$
$$= 79\,cm^2.$$

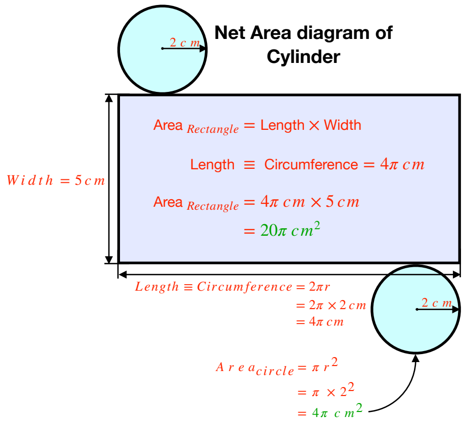
3a.

$$\text{Volume } V \text{ of a prism} = \text{Area of Base} \times \text{Length (Height)}$$
$$\text{Volume } V \text{ of a cylinder} = \pi r^2 h :$$
$$V = \pi \times (3\,cm)^2 \times 10\,cm$$
$$= 90\pi\,cm^3$$
$$\approx 282.74\,cm^3.$$

b.

$$\text{New height} = 20\,cm :$$
$$\text{New Volume} = \pi \times (3)^2 \times 20$$
$$= 180\pi\,cm^3$$
$$\approx 565.49\,cm^3.$$

4.



Surface Area includes the top, bottom, and lateral side:

$$
\begin{aligned}
\text{Area of the } two \text{ circular bases } &= 2 \times \pi r^2 \\
&= 2 \times \pi \times (2\,cm)^2 \\
&= 2 \times \pi \times 2^2\,cm^2 \\
&= 2 \times \pi \times 4\,cm^2 \\
&= 8\pi\,cm^2
\end{aligned}
$$

Area of the lateral surface (rectangle):

$$
\begin{aligned}
&= 2\pi rh \\
&= 2\pi \times 2\,cm \times 5\,cm \\
&= 2 \times 2 \times 5\pi\,cm^2 \\
&= 20\pi\,cm^2
\end{aligned}
$$

Total Surface Area:

$$
\begin{aligned}
&= 8\pi + 20\pi \\
&= 28\pi\,cm^2 \\
&\approx 87.96\,cm^2\,.
\end{aligned}
$$

5a.

Volume of the sandbox:

$$
\begin{aligned}
V &= 2\,m \times 1.5\,m \times 0.5\,m \\
&= 1.5\,m^3\,.
\end{aligned}
$$

b.

First, find the radius $\left(r = \dfrac{\text{diameter}}{2} = 0.7\,m\right)$:

Volume of Prism $=$ Area of Base $\times$ Height

$$
\begin{aligned}
\text{Volume } &= \pi r^2 \times h \\
&= \pi \times (0.7\,m)^2 \times 2\,m \\
&= 0.98\pi\,m^3 \\
&\approx 3.077\,m^3
\end{aligned}
$$

Convert to litres:

$$
\begin{aligned}
&3.077\,\cancel{m^3} \times 1000\,L/\cancel{m^3} \\
&\approx 3077\,L\,.
\end{aligned}
$$

6. (tricky)

$$\begin{aligned}\text{Volume of the cube} &= s^3 \\ &= 3^3 \\ &= 27\,cm^3\end{aligned}$$

$$\begin{aligned}\text{Area of one face of cube} &= s^2 \\ &= 3^2 \\ &= 9\,cm^2\end{aligned}$$

$$\begin{aligned}\text{Area of one face} &= \text{Area of Cylinder Base} \\ 9 &= \pi r^2 \\ \frac{9}{\pi} &= \frac{\cancel{\pi} r^2}{\cancel{\pi}} \\ \frac{9}{\pi} &= r^2 \\ \sqrt{\frac{9}{\pi}} &= \cancel{\sqrt{r^2}} \\ r &= \sqrt{\frac{9}{\pi}} \\ &\approx 1.693\,cm\end{aligned}$$

$$\begin{aligned}\text{Volume of the cylinder} &= \text{Area of Base} \times \text{Height} \\ &= \pi r^2 \times h \\ &\approx \pi \times (1.693)^2 \times 4 \\ &\approx 11.459\pi\,cm^3 \\ &= 36\,cm^3\end{aligned}$$

$$\begin{aligned}\text{Total Volume} &= 27\,cm^3 + 36\,cm^3 \\ &= 63\,cm^3.\end{aligned}$$

7.

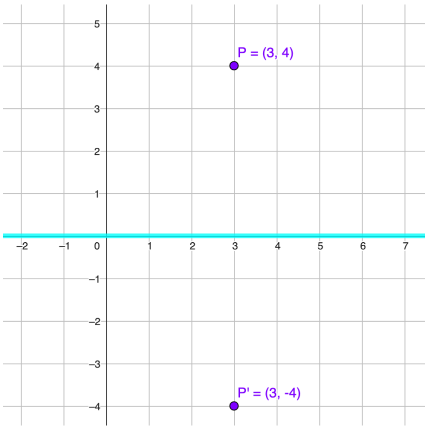
To reflect a point over the $x - axis$, the $x - coordinate$ remains unchanged, and the $y - coordinate$ is multiplied by -1, $(x, y) \rightarrow (x, -y)$

For $P(3, 4)$, the $x - coordinate$ is 3, and the $y - coordinate$ is 4.

After reflection :
$x' = 3, y' = -4$.

Thus, the coordinates of P' are $(3, -4)$. The effect on the $y - coordinate$ is that it changes from positive to negative (or vice versa), so 4 becomes -4.

Answer: The coordinates of P' are $(3, -4)$. The $y - coordinate$ is multiplied by -1, changing its sign.



8.

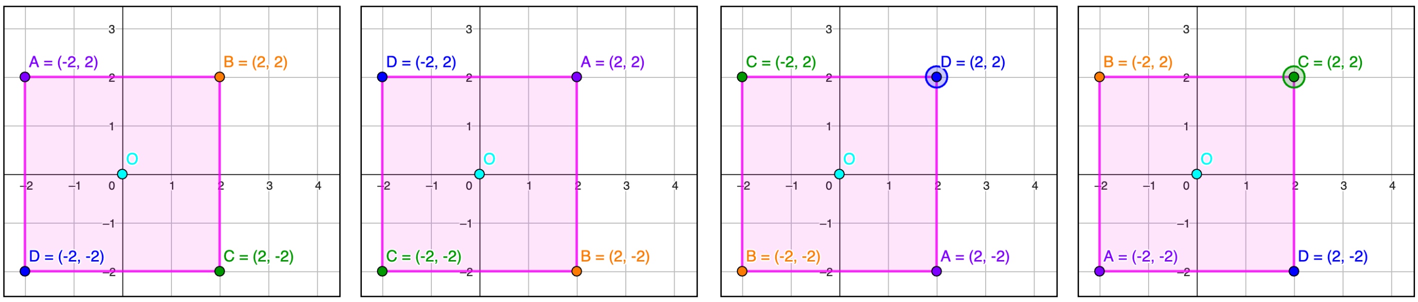
A square has rotational symmetry when it can be rotated about its centre and still look identical.

A square maps onto itself after a rotation of 90°, as it has four identical sides and angles.
The angle of rotational symmetry is the smallest angle of rotation that maps the shape onto itself, which is 90°.

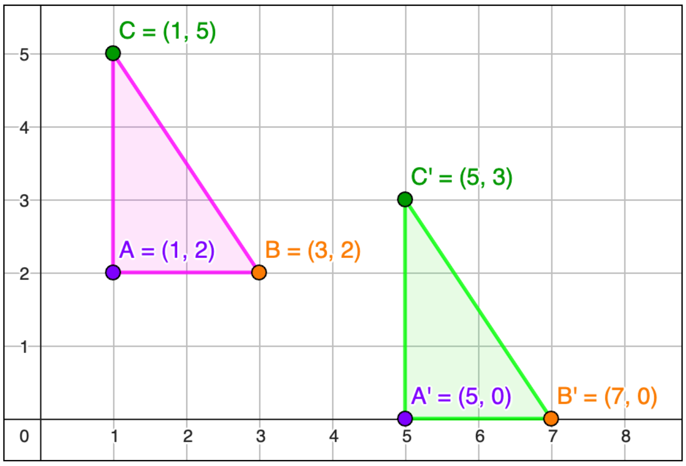
In a full 360° rotation, the square maps onto itself $\dfrac{360^\circ}{90^\circ} = 4$ times, (at $90^\circ, 180^\circ, 270^\circ, 360^\circ$).

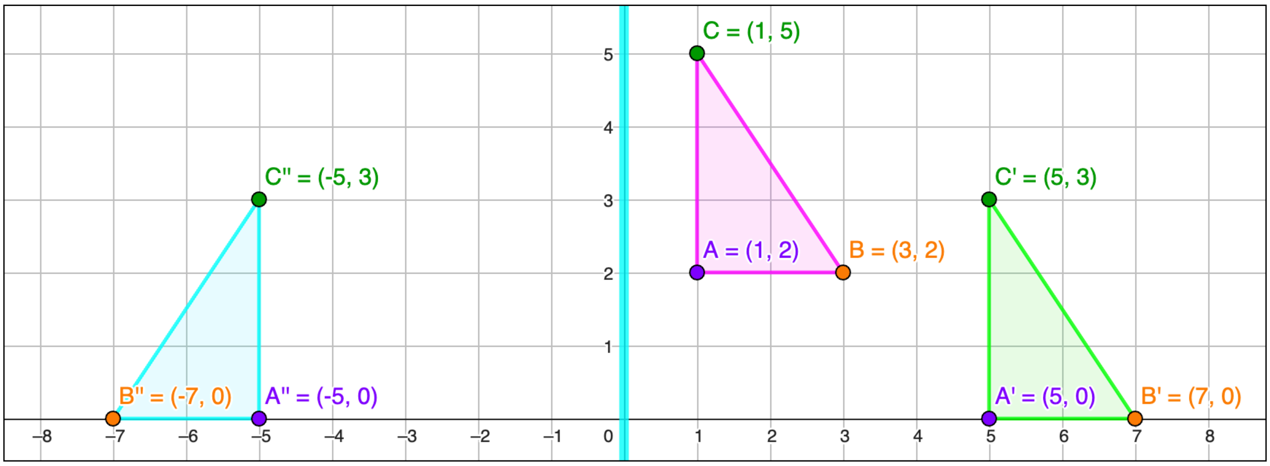
Answer: The angle of rotational symmetry is 90°. This means the square maps onto itself four times in a full 360° rotation, as $360 \div 90 = 4$.

Initial Image / 360° — 90° — 180° — 270°



9.





Step 1: Perform the translation.

A translation of 4 units right and 2 units down is represented by $(x, y) \rightarrow (x+4, y-2)$.

For $A(1, 2)$: $(1+4, 2-2) = (5, 0)$, so $A' = (5, 0)$.
For $B(3, 2)$: $(3+4, 2-2) = (7, 0)$, so $B' = (7, 0)$.
For $C(1, 5)$: $(1+4, 5-2) = (5, 3)$, so $C' = (5, 3)$.
The intermediate triangle has vertices $A'(5, 0)$, $B'(7, 0)$, $C'(5, 3)$.

Step 2: Reflect over the $y-axis$.

A reflection over the $y-axis$ is represented by $(x, y) \rightarrow (-x, y)$.
For $A'(5, 0)$: $(-5, 0)$, so $A'' = (-5, 0)$.
For $B'(7, 0)$: $(-7, 0)$, so $B'' = (-7, 0)$.
For $C'(5, 3)$: $(-5, 3)$, so $C'' = (-5, 3)$.

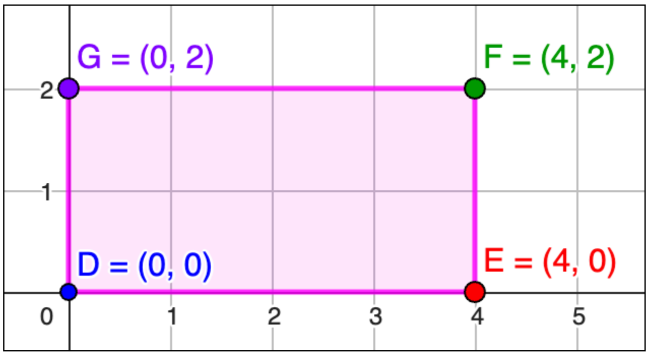
Step 3: Verify (optional for students).

Plotting the original triangle ABC, the intermediate triangle $A'B'C'$ and the final triangle $A''B''C''$ on a Cartesian plane (manually or using a digital tool like GeoGebra) confirms the coordinates.
The final triangle should be a mirror image of the translated triangle across the $y-axis$.

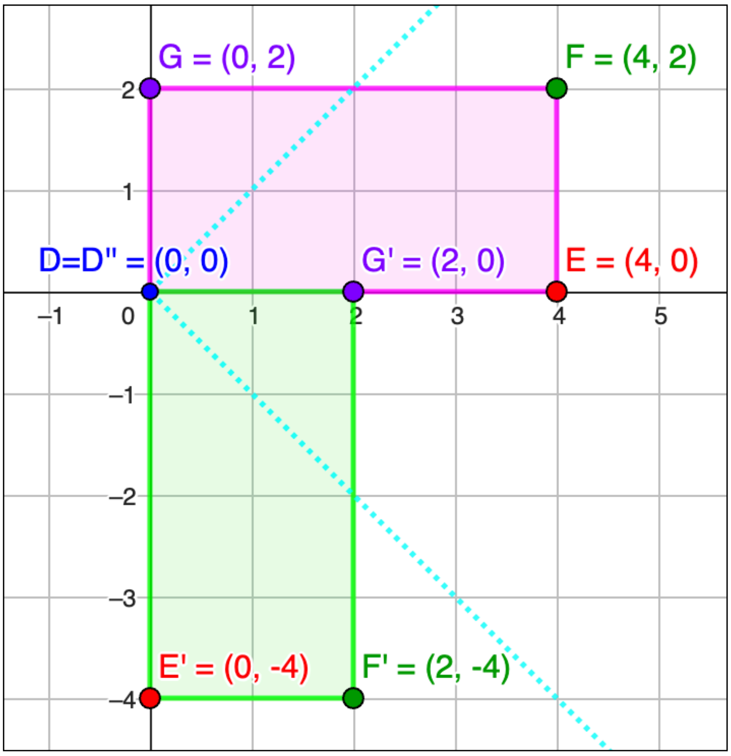
Answer: The coordinates of the final image triangle are $A''(-5, 0)$, $B''(-7, 0)$, $C''(-5, 3)$.

10.

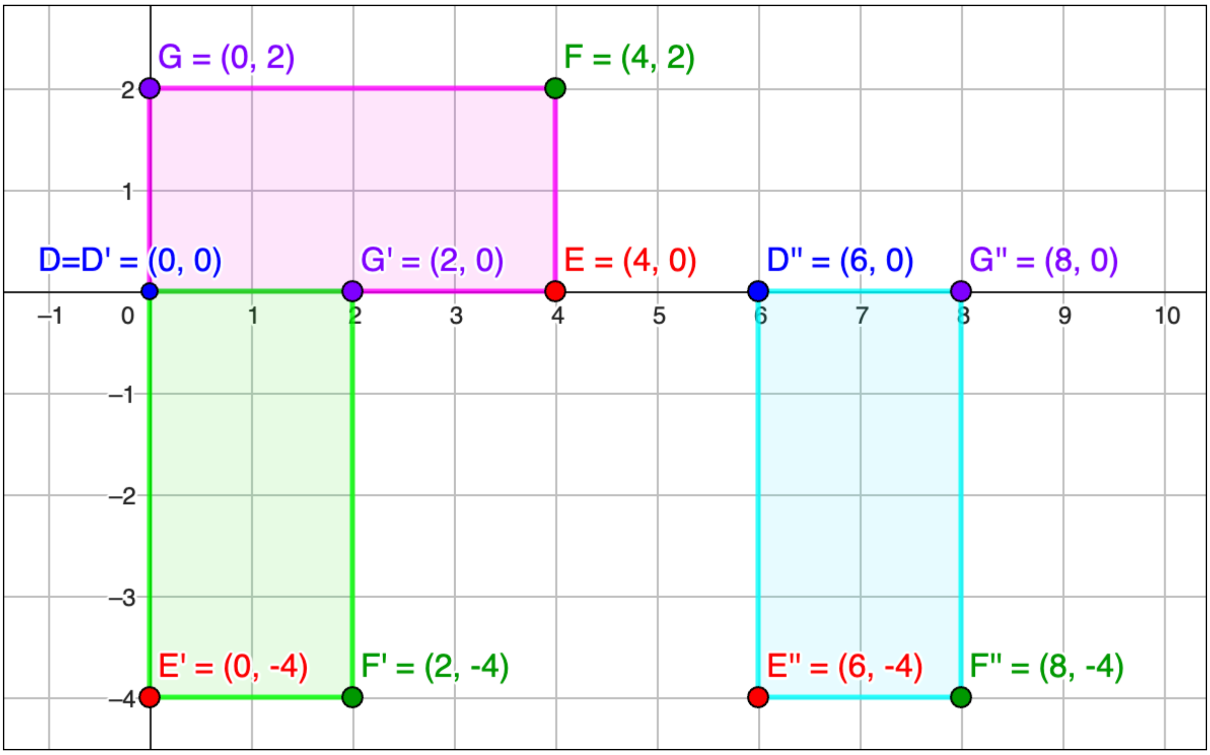
Original Image



90° Clockwise Rotation



Translation 6 units to the right



Step 1: Rotate 90° clockwise about the origin.

A 90° clockwise rotation about the origin is represented by $(x, y) \rightarrow (y, -x)$.

For $D(0, 0)$: $(0, 0)$, so $D' = (0, 0)$.

For $E(4, 0)$: $(0, -4)$, so $E' = (0, -4)$.

For $F(4, 2)$: $(2, -4)$, so $F' = (2, -4)$.

For $G(0, 2)$: $(2, 0)$, so $G' = (2, 0)$.

The intermediate rectangle has vertices :

$D'(0, 0)$, $E'(0, -4)$, $F'(2, -4)$, $G'(2, 0)$.

Step 2: Translate 6 units to the right.

A translation 6 units to the right is represented by $(x, y) \rightarrow (x + 6, y)$.

For $D'(0, 0)$: $(0 + 6, 0) = (6, 0)$, so $D'' = (6, 0)$.

For $E'(0, -4)$: $(0 + 6, -4) = (6, -4)$, so $E'' = (6, -4)$.

For $F'(2, -4)$: $(2 + 6, -4) = (8, -4)$, so $F'' = (8, -4)$.

For $G'(2, 0)$: $(2 + 6, 0) = (8, 0)$, so $G'' = (8, 0)$.

Step 3: Real-world application.

This transformation could be used to create a repeating pattern on product packaging, such as a cereal box, where the logo is rotated and shifted to form a visually appealing, tessellating design. This enhances brand recognition by maintaining the logo's shape while adding dynamic visual interest.

Answer: The coordinates of the final rectangle are

$D''(6, 0)$, $E''(6, -4)$, $F''(8, -4)$, $G''(8, 0)$.

One application is creating a repeating logo pattern on product packaging to enhance visual appeal and brand recognition.

Additional Notes for Teachers:

Learning Outcomes:

Students should be able to calculate volumes and surface areas of basic prisms and cylinders, understand how these concepts apply to composite shapes, and solve problems involving real-life applications.

Teaching Strategies:

Use physical models or 3D visualisation software to help students understand these concepts spatially. Engage them with practical problems like designing containers or calculating material needs.

Assessment:

Assess through exercises where students must calculate volume or surface area, interpret results, and apply these calculations to scenarios involving different geometric shapes.

Resources:

3D models, virtual manipulatives, or apps for interactive learning. Encourage students to calculate volumes and areas of everyday objects to connect theory with practice.

This set of questions aligns with the Australian Curriculum for Year 8, focusing on deepening understanding of volume and surface area in geometry.