F_WORKSHEET # 014 | Australian Curriculum Year 8 Mathematics | FWM8T3U6

ACACIA TUTORING

Angle Relationships, Congruence, and Similarity 8

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Focus: A set of questions and solutions for Year 8 students on Angle Relationships, Congruence, and Similarity tailored to the Australian Curriculum under the strand 'Measurement and Geometry':

1. Complementary and Supplementary Angles:

a) If one angle is $45°$, what is its complement?

b) Find the supplement of an angle measuring $130°$.

2. Angles Formed by Intersecting Lines:

a) Two lines intersect, forming four angles. If one angle is 60°, what are the measures of the other three angles?

b) If two angles are adjacent and form a straight line, one being 105°, what is the measure of the other angle?

3. Angles in Triangles:

a) In a triangle, one angle is 50° and another is 60°. What is the measure of the third angle?

b) The base angles of an isosceles triangle are 40° each. What is the measure of the vertex angle?

4. Angles in Parallel Lines and Transversals:

a) If a transversal intersects two parallel lines, and one angle formed is $70°$, what are the measures of the other interior angles?

b) Given one angle is $150°$ in a setup where a transversal cuts two parallel lines, find all other interior angles.

5. Practical Application:

a) A street sign is falling over and makes a 45° angle the ground. What is the measure of the supplementary angle (between the sign and the ground)?

b) A bridge has a support beam that makes a 30° angle with the vertical post. What angle does the beam make with the horizontal?

6. Congruence:

a) Two triangles have side lengths of $3\,cm$, $4\,cm$, and $5\,cm$ each. Are these triangles congruent?

b) If two triangles have one angle of $90°$, one side adjacent to the right angle of $6\,cm$, and the hypotenuse of $10\,cm$, are they congruent?

7. Similarity:

a) Two triangles have angles measuring $30°$, $60°$, and $90°$. Are these triangles similar?

b) If one triangle has sides in the ratio $1:2:3$ and another triangle has sides in the ratio $2:4:6$, are these triangles similar?

8. Scale Factor and Similarity:

a) If two similar triangles have corresponding sides in the ratio $1:3$, what is the scale factor of enlargement from the smaller to the larger triangle?

b) Triangle A has an area of $16\,cm^2$ and is similar to Triangle B with a scale factor of 2. What is the area of Triangle B?

9. Congruence in Quadrilaterals:

a) Are two squares with side lengths of $5\,cm$ each congruent?

b) If two rectangles have one side of $4\,cm$ and the other side of $8\,cm$, are they congruent?

10. Real-World Applications:

a) A map has a scale where $1\,cm$ represents $10\,km$. If the distance between two cities on the map is $3\,cm$, how far apart are they in reality?

b) A poster is a scaled-up version of a photo where each dimension is tripled. If the photo is $10\,cm$ by $15cm$, what are the dimensions of the poster?

11. Problem Solving with Similarity:

If a tree casts a shadow of $12\,metres$ when a $2-metre$ stick casts a shadow of $3\,metres$, how tall is the tree?

Solutions

1a.

Complementary angles add up to 90° :

$$\text{Complement} = 90^\circ - 45^\circ$$
$$= 45^\circ .$$

b.

Supplementary angles sum to 180° :

$$\text{Supplement} = 180^\circ - 130^\circ$$
$$= 50^\circ .$$

2a.

Opposite angles (or vertical angles) are equal,
and adjacent angles are supplementary:
The opposite angle to 60° is also 60° .
The adjacent angles to the 60° angle would be

$$\rightarrow 180^\circ - 60^\circ$$
$$= 120^\circ \text{ each.}$$

So, the angles are $60^\circ, 60^\circ, 120^\circ,$ and 120° .

b.

Adjacent angles on a straight line sum to 180° :

$$\text{Other angle} = 180^\circ - 105^\circ$$
$$= 75^\circ .$$

3a.

The sum of angles in a triangle is 180° :

$$\text{Third angle} = 180^\circ - 50^\circ - 60^\circ$$
$$= 70^\circ .$$

b.

In an isosceles triangle, the base angles are equal,
and the sum of the angles is 180° :

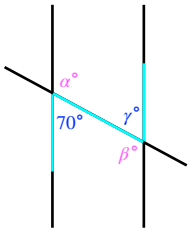
$$\text{Vertex angle} = 180^\circ - 40^\circ - 40^\circ$$
$$= 100^\circ .$$

4a.

For parallel lines cut by a transversal:
Consecutive interior angles would be supplementary to: 70° ,

$$\text{so } \alpha = 180^\circ - 70^\circ$$
$$\alpha = 110^\circ .$$

Vertical angles are the same as the given angle, so other angles are $\gamma = 70^\circ,$
and $\beta = 110^\circ$



b.

For parallel lines cut by a transversal:

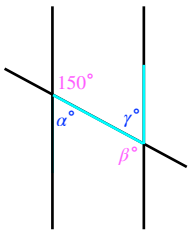
Consecutive interior angles would be supplementary to: 150°,

so $\alpha = 180^\circ - 150^\circ$

$\alpha = 30^\circ$,

Vertical angles are the same as the given angle, so other angles are $\gamma = 30^\circ$,

and $\beta = 150^\circ$.



5a.

Since the ground is a straight line (180°),

the angle on the other side of the sign will be: $180^\circ - 45^\circ$

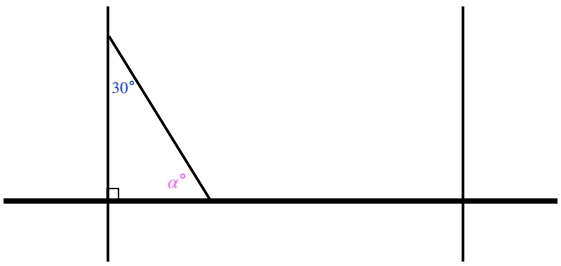
$= 135^\circ$.

b.

Since the beam makes a 30° angle with the vertical:

All angles in a triangle add to 180°, so the angle with the horizontal $= 180^\circ - 90^\circ - 30^\circ$

$= 60^\circ$.



6a.

By the Side-Side-Side (SSS) criterion, if all three corresponding sides of two triangles are equal, the triangles are congruent.

Since both triangles have the same side lengths, they are congruent.

b.

These triangles match the Right Angle-Hypotenuse-Side (RHS) criterion for congruence, where if two right triangles have a hypotenuse and one leg of equal length, they are congruent.

Thus, they are congruent.

7a.

Since the angles of both triangles are the same, by the Angle-Angle (AA) similarity criterion, where two corresponding angles of two triangles are equal,

Therefore, the triangles are similar.

b.

Both sets of ratios simplify to $1:2:3$,
indicating that the triangles have corresponding sides proportional.

Therefore, they are similar.

8a.

The scale factor is the ratio of corresponding sides, which is 3.

Thus, the enlargement from the smaller to the larger triangle has a scale factor of 3.

b.

When the scale factor is 2, the linear dimensions double,
but the area scales by the square of the scale factor:

$$\begin{aligned}\text{Area of Triangle } B &= 16\,cm^2 \times 2^2 \\ &= 16\,cm^2 \times 4 \\ &= 64\,cm^2.\end{aligned}$$

9a.

All sides of a square are equal, and if two squares have the same side length,
they are congruent by the Side-Side-Side-Side $(SSSS)$ criterion for quadrilaterals.

Thus, they are congruent.

b.

Since all corresponding sides of the rectangles are equal,
they are congruent by the Side-Side-Side-Side $(SSSS)$ criterion for rectangles.

Therefore, they are congruent.

10a.

$$\begin{aligned}\text{Using the scale, the real distance} &= 3\,\cancel{cm} \times 10\,km/\cancel{cm} \\ &= 30\,km.\end{aligned}$$

b.

$$\begin{aligned}\text{Width of the poster} &= 10\,cm \times 3 \\ &= 30\,cm.\end{aligned}$$

$$\begin{aligned}\text{Height of the poster} &= 15\,cm \times 3 \\ &= 45\,cm.\end{aligned}$$

11.

The tree and the stick form similar triangles because the sun's rays are parallel, creating corresponding angles.

$$\frac{\text{tree height}}{\text{stick height}} = \frac{\text{tree shadow}}{\text{stick shadow}}$$

$$\frac{x}{2} = \frac{12}{3}$$

$$\frac{x}{\cancel{2}} \times \cancel{2} = 4 \times 2$$

$$x = 4 \times 2$$

$$= 8\,metres.$$

Additional Notes for Teachers:

Learning Outcomes:
Students should understand the concepts of congruence and similarity, recognise the criteria for these relationships, and apply them to solve problems. Students should recognise and calculate angles based on their relationships, including complementary, supplementary, angles in triangles, and angles formed by parallel lines with transversals.

Teaching Strategies:
Use visual aids, physical models, or digital tools to illustrate congruent and similar shapes. Real-life scenarios like architecture or photography can be engaging contexts. Use visual aids like diagrams or physical manipulatives to illustrate these relationships. Real-world contexts like architecture or navigation can make these concepts more tangible.

Assessment:
Evaluate through exercises that require students to identify, compare, and solve problems involving congruent and similar figures. Assess through problems that require students to identify, calculate, or describe angle relationships in various geometric configurations.

Resources:
Geometry software for exploring shape transformations, scale drawings, or physical kits for constructing congruent and similar shapes. Use protractors for physical measurement, geometry software for interactive learning, or drawing exercises to practice visualising and solving angle problems.

This set of questions aligns with the Australian Curriculum for Year 8, focusing on developing a deep understanding of angle relationships, congruence, and similarity in geometric shapes.