Probability, Tree Diagrams, and Venn diagrams

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Focus: A set of questions and solutions for Year 8 students on the Probability, Tree Diagrams, and Venn Diagrams, tailored to the Australian Curriculum under the strand 'Statistics and Probability':

1. Basic Probability:

a) If a fair die is rolled, what is the probability of rolling a number less than 4 ?

b) A bag contains 5 blue balls, 3 red balls, and 2 green balls. What is the probability of picking a red ball?

2. Complementary Events:

a) If the probability of winning a game is $\frac{2}{5}$, what is the probability of not winning?

b) The probability that it will rain tomorrow is 0.6. What is the probability that it will not rain?

3. Probability of Multiple Events:

a) If you flip two coins, what is the probability of getting at least one head?

b) You roll two dice. What's the probability of getting a sum of 7 or 11 ?

4. Conditional Probability:

a) A class has 30 students, 20 of whom are girls. If 10 girls and 5 boys are in the science club, what is the probability that a randomly selected student from the science club is a girl?

b) Given that $40\,\%$ of students in a school play basketball and $30\,\%$ play both basketball and tennis, what is the probability that a student plays tennis given they play basketball?

5. Independent and Dependent Events:

a) What is the probability of drawing two aces in a row from a well-shuffled deck of cards if the first card is not replaced?

b) If the probability of winning a game is 0.3 and you play twice, what's the probability of winning both games if the games are independent?

6. Real-World Applications:

a) A weather forecast predicts a $20\,\%$ chance of rain over the weekend. How should you interpret this probability?

b) In a manufacturing process, the probability of producing a defective item is 0.02. If you produce 100 items, how many would you expect to be defective?

7. Two-Way Tables, Tree Diagrams and Venn Diagrams:

A student is choosing their afternoon snack and has the following options:
Drink: Milk (M) or Soda (S)
Food: Apple (A) or Biscuit (B)

a) Use a two-way table to list all possible combinations of a food and a drink.

b) Draw a tree diagram to represent all possible combinations. Check the combinations match those found in question 7a).

c) Use a Venn diagram to show the outcomes where Apple and Milk are considered "healthy" choices, and identify the overlapping event.

d) If the student randomly selects a food and a drink, calculate the probability of:

i) Choosing an Apple and Milk. P(A ∩ B).

ii) Choosing a Biscuit or Soda. P(B ∪ S).

e) In a Venn diagram where set V represents students who play volleyball, and set N represents students who play netball, what does the region inside both circles represent and what does the region outside both circles represent?

f) Draw a Venn diagram for two sets, A and B, where:

Set A has 20 elements

Set B has 15 elements

7 elements are in both A and B

Solutions

1a.

The numbers less than 4 on a die are 1, 2 and 3.

There are 6 possible outcomes in total:

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{3 \div 3}{6 \div 3}$$

$$= \frac{1}{2}.$$

b.

$$\text{Number of red balls} = 3$$

$$\text{Total number of balls} = 5 + 3 + 2$$

$$= 10 :$$

$$\text{Probability} = \frac{3}{10}.$$

2a.

The sum of probabilities for complementary events is 1 :

$$\text{Probability of not winning} = 1 - \frac{2}{5}$$

$$= \frac{3}{5}.$$

b.

$$\text{Probability of not raining} = 1 - 0.6$$

$$= 0.4.$$

3a.

Possible outcomes are HH, HT, TH, TT.

Only TT gives no heads:

Probability of no heads $= \frac{1}{4}$, so the probability of at least one head is:

$$\to 1 - \frac{1}{4}$$

$$= \frac{3}{4}.$$

b.

Sum of 7 can be $(1,\ 6), (2,\ 5), (3,\ 4), (4,\ 3), (5,\ 2), (6,\ 1) = 6$ ways.

$$\begin{aligned}\text{Sum of 11 can be } (5,\ 6), (6,\ 5) &= 2 \text{ ways.}\\ \text{Total outcomes for two dice } &= 6 \times 6\\ &= 36\ :\\ \text{Probability } &= \frac{6}{36} + \frac{2}{36}\\ &= \frac{8}{36}\\ &= \frac{2}{9}.\end{aligned}$$

4a.

$$\begin{aligned}\text{Total in science club } &= 10 \text{ girls } + 5 \text{ boys}\\ &= 15 \text{ students:}\\ \text{Probability } &= \frac{\text{Number of girls in club}}{\text{Total in club}}\\ &= \frac{10}{15}\\ &= \frac{2}{3}.\end{aligned}$$

b.

Let's use the given percentages:

Probability of playing tennis given playing basketball:

$$\begin{aligned}&= \frac{\text{Probability of both}}{\text{Probability of basketball}}\\ &= \frac{0.30}{0.40}\\ &= 0.75.\end{aligned}$$

5a.

$$\text{Probability of first ace} = \frac{4}{52},$$

$$\text{Then the probability for the second ace given the first was an ace} = \frac{3}{51}$$

$$\begin{aligned}\text{Probability} &= \frac{4}{52} \times \frac{3}{51} \\ &= \frac{12}{2652} \\ &= \frac{1}{221}.\end{aligned}$$

b.

Since the events are independent:

$$\begin{aligned}\text{Probability} &= 0.3 \times 0.3 \\ &= 0.09.\end{aligned}$$

6a.

There's a $20\,\%$ chance it will rain at some point over the weekend, meaning 20 out of 100 similar weekends in the past would have rain.

b.

$$\begin{aligned}\text{Expected number of defective items} &= \text{Total items} \times \text{Probability of defect} \\ &= 100 \times 0.02 \\ &= 2.\end{aligned}$$

7a.

A two-way table lists all possible combinations of the two events: Food (Apple or Biscuit) and Drink (Milk or Soda).
Two-way table:

		Drink	
		Milk (M)	**Soda (S)**
Food	**Apple (A)**	A, M	A, S
	Biscuit (B)	B, M	B, S

The possible combinations are:
Apple and Milk (A, M)
Apple and Soda (A, S)
Biscuit and Milk (B, M)
Biscuit and Soda (B, S)
Total combinations = 4.

b.

Drink: Milk (M) or Soda (S) **Food: Apple (A) or Biscuit (B)**

A tree diagram branches from the first event (Food) to the second event (Drink).

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        Start
        /   \
       A     B          Food
      / \   / \
     M   S M   S        Drink
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(A,M) (A,S) (B,M) (B,S)

From A: Branch to M (A, M) or S (A, S)
From B: Branch to M (B, M) or S (B, S)

The tree diagram matches the 4 combinations: (A, M), (A, S), (B, M), (B, S).

c.

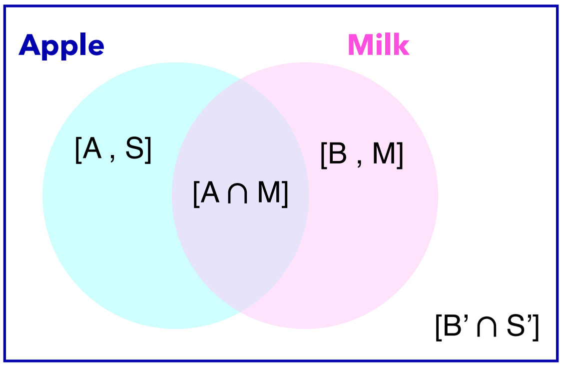
For the Venn diagram, "healthy" choices are Apple (A) for food and Milk (M) for drink. Two circles represent Apple and Milk, with overlap for both.

Left circle (Apple): Includes (A, M) and (A, S)
Right circle (Milk): Includes (A, M) and (B, M)

Overlap: Intersection of (A ∩ M) (both Apple and Milk)
Outside both circles: Intersection of (B' ∩ S') (neither Apple nor Milk)

Drink: Milk (M) or Soda (S) **Food: Apple (A) or Biscuit (B)**

Venn diagram:



The overlapping event is (A ∩ M), where both healthy choices occur.

d.

i)

$$P(Apple\ and\ Milk) = \frac{Favourable\ Outcome}{Total\ Number\ of\ Outcomes}$$

$$P(A \cap M) = \frac{1}{4} = 0.25 = 25\,\%\,.$$

ii)

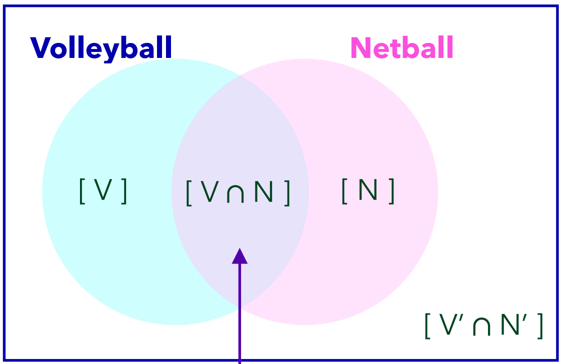
$$P(Biscuit\ or\ Soda) = \frac{Favourable\ Outcome}{Total\ Number\ of\ Outcomes}$$

$$P(B \cup S) = \frac{2}{4} = 0.50 = 50\,\%\,.$$

e.

N - Netball N' - Not Netball V - Volleyball V' - Not Volleyball

Inside: Intersection of students who play both volleyball and netball. [V ∩ N]
Outside: Intersection of students who do not volleyball nor netball. [V' ∩ N']



(∩) = Intersection

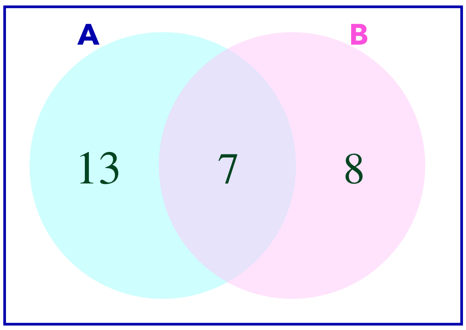
f.

Description for drawing: Two overlapping circles, labelled A and B,

Circle A contains 20 elements, Circle B contains 15 elements, and there is an overlap of 7 elements,

Therefore, 7 elements are in A ∩ B, 13 elements are only in A ($20 - 7$), and 8 elements are only in B ($15 - 7$).

Two-way table:



Additional Notes for Teachers:

Learning Outcomes:
Students should grasp the concepts of probability, understand how to calculate probabilities for simple and compound events, and appreciate conditional and independent probabilities.

Teaching Strategies:
Use games, dice, coins, or cards for practical demonstrations of probability. Discuss real-life scenarios like weather predictions or quality control in manufacturing.

Assessment:
Assess through problems where students must calculate probabilities, interpret results, and solve for conditional or independent events.

Resources:
Probability games or simulators can engage students. Use physical objects for hands-on probability experiments or software for more complex scenarios.

This set of questions aligns with the Australian Curriculum for Year 8, aiming to develop students' understanding of probability in both theoretical and practical contexts.