Algebraic Techniques, and Scientific Notation

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Free and always will be!

Focus: A set of questions and solutions for Year 9 students on Algebraic Techniques, and Scientific Notation tailored to the Australian Curriculum under the strand 'Number and Algebra':

1. Expanding and Simplifying Expressions:

a) Expand and simplify $(x+3)(x-5)$ **.**

b) Expand $(2a-1)^2$ **.**

c) Simplify $2x - 7 + 4x + 11$ **.**

d) Simplify $4x^2 + x - 7 - 2x^2 + 5x + 11$ **.**

2. Factorising:

a) Factorise $x^2 + 7x + 12$ **.**

b) Factorise $2y^2 - 18$**.**

3. Perfect Squares and Difference of Two Squares:

a) Expand $(x + 3)^2$ **.**

b) Factorise $x^2 - 25$.

4. Completing the Square:

Complete the square for $x^2 + 8x$.

5. Solving Linear Equations:

a) Solve $3x - 7 = 2$ **.**

b) Solve $\frac{x}{2} + 3 = 5$ **.**

6. Solving Quadratic Equations:

a) Solve $x^2 - 5x + 6 = 0$ by factorisation .

b) Use completing the square to solve $x^2 + 4x - 2 = 0$.

7. Simplifying Algebraic Fractions:

a) Simplify $\dfrac{3x+6}{x^2-4}$.

b) Add $\dfrac{2}{x}+\dfrac{3}{x}$.

8. Practical Application:

The area of a rectangle is given by $x^2 + 8x + 12\,square\ metres$. What are the possible dimensions of the rectangle in terms of x?

9. Manipulating Algebraic Fractions:

Simplify $\dfrac{x}{x^2 - 2^2} - \dfrac{1}{x+2}$.

10. Scientific Notation:

a) Convert the Avogadro constant $N_A = 602{,}214{,}076{,}000{,}000{,}000{,}000{,}000 \ /mol$ **to scientific notation.**

b) Convert the proton to electron mass ratio $\dfrac{m_p}{m_e} = 1836.15$ **to scientific notation.**

c) Convert 0.00108 **to scientific notation.**

d) Convert the mass of an electron $m_e = 0.00000000000000000000000000000091094\,kg$ **to scientific notation.**

e) Convert the speed of light $c = 299{,}792{,}500 \ m/s$ **to scientific notation with one significant figure.**

11. Operations with Scientific Notation:

a) Multiply $(3 \times 10^3) \times (36 \times 10^2)$ **.**

b) Divide $\dfrac{12 \times 10^9}{3 \times 10^6}$ **.**

c) Simplify $(1 \times 10^3)^{36}$ **.**

d) Evaluate $(0.000108 \times 10^3)^0$ **.**

12. Practical Application:

The distance from Earth to the Moon is approximately $384{,}400\, kilometres$ **when the moon is at its closest point to Earth. Express this distance in scientific notation with three significant figures.**

Solutions

1a.

Use the distributive property (FOIL) / Crab Claw :

($+ \times - = -$) If signs are:
opposite → change to $-$
same → change to $+$

$$(x+3)(x-5) = (1x+3)(1x-5)$$
$$= 1x \cdot 1x + 1x \cdot (-5) + 3 \cdot 1x + 3 \cdot (-5)$$
$$= x^2 \boxed{-5x + 3x} - 15$$
$$= x^2 - 2x - 15\,.$$

b.

Use the square of a binomial :

$$(2a-1)^2 = (2a-1)(2a-1)$$

If signs are: same → change to $+$

$$= 4a^2 - 4a + 1\,.$$

c.

Combine like terms:

$$\boxed{2x} - 7 \boxed{+4x} + 11 = 2x + 4x - 7 + 11$$
$$= 6x + 4\,.$$

d.

Combine like terms:

$$\boxed{4x^2} + x - 7 \boxed{-2x^2} + 5x + 11 = 4x^2 - 2x^2 + 1x + 5x - 7 + 11$$
$$= 2x^2 + 6x + 4\,.$$

2a.

$$x^2 + 7x + 12$$

$$\_\times\_ = 12 \; and \; \_+\_ = 7$$
$$\rightarrow 3 \times 4 = 12 \; and \; 3 + 4 = 7$$

$$= (x+3)(x+4)\,.$$

OR

$$x^2 \;+7x \;+\; 12$$

$$\_\times\_ = +12 \; and \; \_+\_ = +7$$
$$\rightarrow 3 \times 4 = +12 \; and \; 3 + 4 = +7$$

$$= (x \;+\; 3)(x \;+\; 4)$$

$$= (x+3)(x+4)\,.$$

b.

Factor out the greatest common factor (2) :

$$\begin{aligned} 2y^2 - 18 &= 2 \cdot y^2 + 2 \cdot (-9) \\ &= 2 \cdot (y^2 + -9) \\ &= 2 \cdot (y^2 - 9) \\ &= 2(y^2 - 9)\,, \\ &= 2(y^2 - 3^2) \quad \text{(Difference between two squares)} \\ &= 2(y+3)(y-3) \\ &= 2(y+3)(y-3)\,. \end{aligned}$$

3a.

Re-write, expand, then collect like terms:

$$(x+3)^2 = (x+3)(x+3) \quad \textit{Use Crab Claw}$$

$$\begin{aligned} &= x \cdot x + x \cdot 3 + 3 \cdot x + 3 \cdot 3 \\ &= x^2 + 3 \cdot x + 3 \cdot x + 9 \\ &= x^2 + 6 \cdot x + 9 \\ &= x^2 + 6x + 9\,. \end{aligned}$$

OR

Using rule for perfect squares:

$$\begin{aligned} (a+b)^2 &= a^2 + 2ab + b^2 \\ (x+3)^2 &= x^2 + 2 \cdot x \cdot 3 + 3^2 \\ &= x^2 + 6x + 9\,. \end{aligned}$$

Rules for perfect squares:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$

b.

This is a difference of two squares:

$$\begin{aligned} x^2 - 25 &= x^2 - 5^2 \\ &= (x+5)(x-5)\,. \end{aligned}$$

Rule for difference of two squares:

$$a^2 - b^2 = (a+b)(a-b)$$

Remember, $(+b - b = 0)$, so by adding and subtracting $b\ (=\frac{a}{2})$,

4. we aren't changing the equation, just making it look different.

Take half of the coefficient of $x = (8/2) = 4$, square it $4^2 = (16)$,

Then, add and subtract it:

$$\rightarrow x^2 + 8x = x^2 + 8x + (\frac{8}{2})^2 - (\frac{8}{2})^2$$
$$= x^2 + 8x + 4^2 - 4^2$$
$$= [x^2 + 8x + 4^2] - 16,$$

Using rule for perfect squares:

Rule for perfect squares:

$(a+b)^2 = a^2 + 2ab + b^2$

$a^2 + 2ab + b^2 = (a+b)^2$

$$[x^2 + 8x + 4^2] - 16 = [x^2 + 2 \cdot x \cdot 4 + 4^2] - 16$$
$$= [(x+4)^2] - 16.$$
$$= (x+4)^2 - 16.$$

OR

Complete the square for $x^2 + 8x$.

Using the rule for completing the square:

$$x^2 + ax = \left[\left(x + \left(\frac{a}{2}\right)\right)^2\right] - \left(\frac{a}{2}\right)^2$$

$$x^2 + ax = [x + (\frac{8}{2})]^2 - (\frac{8}{2})^2$$
$$= [x+4]^2 - 4^2.$$
$$= (x+4)^2 - 16.$$

Rule for completing the square:

$$x^2 + ax = x^2 + ax + (\frac{a}{2})^2 - (\frac{a}{2})^2 \quad \left[= x^2 + ax + b^2 - b^2, \rightarrow b = \frac{a}{2}.\right]$$
$$= [x^2 + 2 \cdot x \cdot (\frac{a}{2}) + (\frac{a}{2})^2] - (\frac{a}{2})^2$$
$$= [x^2 + 2ab + (\frac{a}{2})^2] - (\frac{a}{2})^2$$
$$= [\text{Perfect Square}] - \left(\frac{a}{2}\right)^2$$
$$x^2 + ax = \left[\left(x + \left(\frac{a}{2}\right)\right)^2\right] - \left(\frac{a}{2}\right)^2$$

Rule for perfect squares:

$(a+b)^2 = a^2 + 2ab + b^2$

$a^2 + 2ab + b^2 = (a+b)^2$

5a.

Add 7 to both sides :

$$3x\cancel{-7}\cancel{+7}=2+7$$
$$3x=9,$$

Divide by 3 :

$$\frac{\cancel{3}x}{\cancel{3}}=\frac{9}{3}$$
$$x=3.$$

b.

Subtract 3 from both sides :

$$\frac{x}{2}+\cancel{3}\cancel{-3}=5-3$$
$$\frac{x}{2}=2,$$

Multiply by 2 :

$$\frac{x}{\cancel{2}}\times\cancel{2}=2\times 2$$
$$x=4.$$

6a.

Factorise:

$$x^2-5x+6=0$$
$$\rightarrow (x-2)(x-3)=0$$

$$x^2+ax=\left[\left(x+\left(\frac{a}{2}\right)\right)^2\right]-\left(\frac{a}{2}\right)^2$$

$$(x-2)(x-3)=0$$
$$\frac{(x-2)\cancel{(x-3)}}{\cancel{(x-3)}}=\frac{0}{(x-3)},\ \frac{0}{(x-3)}=0$$
$$(x-2)=0.$$

$$(x-2)(x-3)=0$$
$$\frac{\cancel{(x-2)}(x-3)}{\cancel{(x-2)}}=\frac{0}{(x-2)},\ \frac{0}{(x-2)}=0$$
$$(x-3)=0.$$

$$\rightarrow x-2=0\ or\ x-3=0$$

$$x-2=0$$
$$x\cancel{-2}\cancel{+2}=0+2$$
$$x=2.$$

$$x-3=0$$
$$x\cancel{-3}\cancel{+3}=0+3$$
$$x=3.$$

This gives:

$$x=2\ or\ x=3.$$

b.

Complete the square:

Take half of the coefficient of $x = (4/2)$, square it $= (4)$,

Then, add and subtract it:

$$x^2 + 4x + 4 - 4 - 2 = 0 + \cancel{4} - \cancel{4}$$

$$[x^2 + 4x + 4] - 4 - 2 = 0 \quad [x^2 + (2+2)\cdot x + (2 \times 2)] - 4 - 2 = 0$$

$$(x+2)^2 - 6 = 0 \quad [(x+2)(x+2)] - 6 = 0$$

$$(x+2)^2 - \cancel{6} + \cancel{6} = 0 + 6$$

$$(x+2)^2 = 6$$

$$\cancel{\sqrt{(x+2)^{\cancel{2}}}} = \sqrt{6}$$

$$x + 2 = \pm\sqrt{6}$$

Solutions:

$$x = +\sqrt{6} - 2,$$

$$AND$$

$$x = -\sqrt{6} - 2.$$

OR

Continued over page…

$$x^2 + 4x - 2 = 0$$

Complete the square:

$$x^2 + 4x + 4 - 4 - 2 = 0$$
$$[x^2 + 4x + 4] - 4 - 2 = 0$$
$$[x^2 + 4x + 2^2] - 4 - 2 = 0$$
$$[x^2 + 2 \cdot x \cdot 2 + 2^2] - 4 - 2 = 0$$
$$(x+2)^2 - 6 = 0$$
$$(x+2)^2 = 6$$
$$\cancel{\sqrt{(x+2)^2}} = \sqrt{6}$$
$$x + 2 = \pm\sqrt{6}$$

Rule for perfect squares:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$a^2 + 2ab + b^2 = (a+b)^2$$

Solutions:

$$x + 2 = \sqrt{6}$$
$$x \cancel{+2} \cancel{-2} = +\sqrt{6} \; -2$$
$$x = +\sqrt{6} - 2,$$

$$AND$$

$$x + 3 = -4$$
$$x \cancel{+2} \cancel{-2} = -\sqrt{6} \; -2$$
$$x = -\sqrt{6} - 2.$$

7a.

Factorise numerator and denominator :

$$\frac{3\cancel{(x+2)}}{\cancel{(x+2)}(x-2)},$$

Cancel the common factor $x + 2$ (assuming $x \neq -2$)

$$= \frac{3}{x-2}.$$

b.

Since the denominators are the same, add the numerators :

$$\frac{2}{x} + \frac{3}{x} = \frac{2+3}{x}$$
$$= \frac{5}{x}.$$

8.

Factorise the area to find dimensions:

$$x^2 + 8x + 12$$
$$= (x + 2)(x + 6)\,,$$
$$= (x + 2) \times (x + 6)\,.$$
$$\text{Area} = \text{Length} \times \text{Width}$$

Possible dimensions:

$$\text{Length} = (x + 2)\,m\,,$$
$$\text{Width} = (x + 6)\,m\,.$$

9.

$$\frac{x}{x^2 - 2^2} - \frac{1}{x+2} = \frac{x}{x^2 - 2^2} - \frac{1}{x+2} \times \frac{(x-2)}{(x-2)}$$
$$= \frac{x}{(x+2)(x-2)} - \frac{1(x-2)}{(x+2)(x-2)}$$
$$= \frac{x \times 1}{(x+2)(x-2)} - \frac{(x-2) \times 1}{(x+2)(x-2)}$$
$$= \left[x - (x-2)\right] \times \left[\frac{1}{(x+2)(x-2)}\right]$$
$$= \frac{[x - (x-2)] \times 1}{(x+2)(x-2)}$$
$$\frac{x - x + 2}{(x+2)(x-2)}$$
$$= \frac{2}{(x+2)(x-2)}\,,$$

Simplify further:

$$\rightarrow \frac{2}{(x+2)(x-2)}$$
$$= \frac{2}{x^2 - 2^2}$$
$$= \frac{2}{x^2 - 4}\,.$$

10a.

$602{,}214{,}076{,}000{,}000{,}000{,}000{,}000$ in scientific notation is:

$$N_A = 6.02214076 \times 10^{23}\ /mol\,.$$

b.

$1{,}836.15$ in scientific notation is:

$$1{,}836.15$$

$$\frac{m_p}{m_e} = 1.83615 \times 10^3.$$

c.

0.00108 in scientific notation is:

$$0.00108$$

$$1.08 \times 10^{-3}.$$

d.

$0.00000000000000000000000000000091094$ in scientific notation is:

$$m_e = 9.91094 \times 10^{-31}\ kg.$$

e.

First, round $299{,}792{,}500$ to one significant figure:

$= 300{,}000{,}000$

$$= 300{,}000{,}000$$

$$c = 3 \times 10^8\ m/s.$$

11a.

Multiply the numbers and add the exponents:

$$3 \times 36 = 108$$

$$10^3 \times 10^2 = 10^{3+2}$$

$$= 10^5$$

Result: 108×10^5.

b.

Divide the numbers and subtract the exponents:

$$\frac{12}{3} = 4$$

$$10^9 \div 10^6 = 10^{9-6}$$

$$= 10^3$$

Result: 4×10^3.

c.

Apply the power to each term:

$$(1 \times 10^3)^{36} = 1^{36} \times 10^{3 \times 36}$$
$$= 1 \times 10^{108}.$$

d.

Anything to the power of zero equals one. $(0.000108 \times 10^3)^0 = 1$.

12.

Firstly round $384{,}400\,km$ to three significant figures:
$384{,}000\,km$, then convert to scientific notation:
$3.84 \times 10^5\,km$.

Additional Notes for Teachers:

Learning Outcomes:
Students should be proficient in various algebraic techniques including expanding, factorising, solving equations, and manipulating algebraic fractions.

Teaching Strategies:
Use visual aids like algebra tiles for expanding and factorising. Encourage students to check their solutions by substituting back into the original equations.

Assessment:
Test through a mix of straightforward algebra problems and more complex scenarios that require combining several techniques.

Resources:
Algebra software for interactive problem-solving, graph paper for manual plotting, or algebra games to make learning engaging.

This set of questions aligns with the Australian Curriculum for Year 9, focusing on enhancing students' algebraic manipulation skills.