Equations, Inequalities, Quadratics, and Finance

9

Free and always will be!

Focus: A set of questions and solutions for Year 9 students on Linear and Quadratic Equations, tailored to the Australian Curriculum under the strand 'Number and Algebra':

1. Linear Equations:

a) Solve the equation $4x - 3 = 13$ **.**

b) Solve for y in $2(y + 3) = 8$ **.**

2. Solving Systems of Linear Equations:

a) Solve the system of equations using substitution: $\begin{cases} y = 4x + 2 \\ 3x - y = 4 \end{cases}$.

b) Solve the system of equations using substitution: $\begin{cases} y = 3x + 2 \\ x - 2y = 6 \end{cases}$.

c) Solve the system of equations using elimination: $\begin{cases} 2x + 3y = 8 \\ 2x - y = 2 \end{cases}$.

3. Understanding Quadratic Equations:

a) What is the general form of a quadratic equation?

b) Identify the coefficients $a, b,$ and $c,$ in the quadratic equation $2x^2 - 4x + 1 = 0$.

4. Quadratic Equations - Factorisation:

a) Solve $x^2 - 5x + 6 = 0$ by factorising.

b) Factorise and solve $x^2 + 2x - 15 = 0$ **.**

5. Quadratic Equations - Quadratic Formula:

a) Use the quadratic formula to solve $x^2 - 6x + 5 = 0$ **.**

b) Solve $2x^2 + 5x - 3 = 0$ using the quadratic formula.

6. Solving Linear and Quadratic Equations Simultaneously:

Solve the system of equations: $\begin{cases} y = 6 \\ y = x^2 - 1 \end{cases}$

7. Real-World Application:

a) The perimeter of a rectangle is given by $2l + 2w = 40$. If the length is $10\,cm$ more than the width, find the dimensions of the rectangle.

b) A ball is thrown upward with an initial speed of $20\,m/s$. Its height after t seconds is given by $h = -5t^2 + 20t + 2$. When will the ball hit the ground? *(Tricky)*

8. Identifying Linear and Quadratic Equations

Graph a) $y = -x^2$ **and b)** $y = x + 1$ **:**

9. Sketching Graphs: Slope / Intercept form and Point / Intercept form.

a) Sketch the graph of the linear function $y = -2x + 3$ **. Indicate the** $slope$ **and** $y - intercept$ **.**

b) Given the quadratic function $y = x^2 + 3x - 1$ **: Determine the vertex of the parabola and the** $y - intercept$ **. Sketch the graph.**

10. Financial Applications (Simple Interest):

a) Calculate the simple interest earned on an investment of $\$25{,}000$ **at an interest rate of** 2% $per\ annum$ **for** $3\ years$**. Show your working using the formula** $I = PRT$**.**

b) Calculate the simple interest earned on an investment of $\$12{,}000$ **at an interest rate of** 4% $per\ annum$ **for** $9\ months$**. Show your working using the formula** $I = PRT$**.**

c) Saxon has $\$10{,}000$ to invest. He is considering two options:

Option A: A savings account offering $3.7\,\%$ simple interest per annum for $5\ years$.

Option B: A term deposit offering $4.2\,\%$ simple interest per annum for $3\ years$.

I) Write a linear equation for each option, where (A) is the total amount and (T) is time in years.
II) Calculate the total amount for each option.
III) Which option gives the highest return, and by how much more?

d) Sam borrows $\$8{,}000$ at $5\,\%$ simple interest $per\ \ month$. He plans to repay the loan after a year and a half.

I) Write a linear equation for the total amount (A) owed, where (T) is time in months.
II) Calculate the total amount owing after a year and a half.

Solutions

1a.

Add 3 to both sides :

$$4x - 3 = 13$$
$$4x - \cancel{3} + \cancel{3} = 13 + 3$$
$$4x = 16$$

Divide by 4 :

$$\frac{\cancel{4}x}{\cancel{4}} = \frac{16}{4}$$
$$x = 4\,.$$

b.

Distribute the 2 :

$$2(y + 3) = 8$$
$$2y + 6 = 8$$

Subtract 6 from both sides:

$$2y + \cancel{6} - \cancel{6} = 8 - 6$$
$$2y = 2$$

Divide by 2 :

$$\frac{\cancel{2}y}{\cancel{2}} = \frac{2}{2}$$
$$y = 1\,.$$

2a.

$$\begin{cases} y = 4x + 2 \\ 3x - y = 4 \end{cases}$$

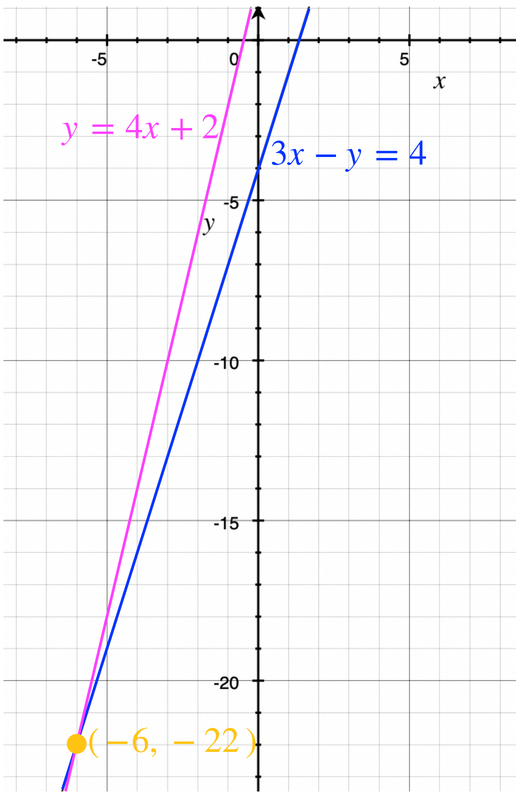
Substitute $y = 4x + 2$ into $3x - y = 4$:

$$3x - (4x + 2) = 4$$
$$3x - 4x - 2 = 4$$
$$-x - 2 = 4$$
$$-x \cancel{-2} \cancel{+2} = 4 + 2$$
$$-x = 6$$
$$-1x = 6$$
$$\frac{\cancel{-1}x}{\cancel{-1}} = \frac{6}{-1}$$
$$x = -6,$$

Substitute $x = -6$ into $y = 4x + 2$:

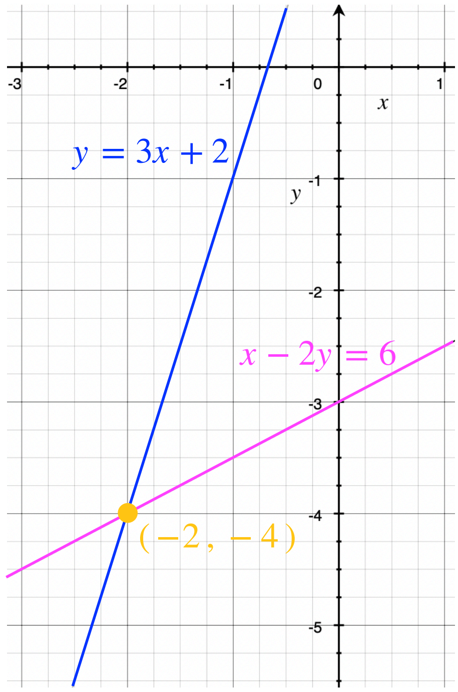
$$y = 4 \times (-6) + 2$$
$$= -24 + 2$$
$$y = -22,$$

Solution: $x = -6,\ y = -22.$



b.

$$\begin{cases} y = 3x + 2 \\ x - 2y = 6 \end{cases}$$



Substitute $y = 3x + 2$ into $x - 2y = 6$:

$$x - 2y = 6$$
$$x - 2(3x + 2) = 6$$
$$x - 6x - 4 = 6$$
$$1x - 6x - 4 = 6$$
$$-5x - 4 = 6$$
$$-5x \cancel{-4} \cancel{+4} = 6 + 4$$
$$-5x = 10$$
$$\frac{\cancel{-5}x}{\cancel{-5}} = \frac{10}{-5}$$
$$x = \frac{10}{-5},$$
$$x = -2,$$

Substitute $x = -2$ into $y = 3x+2$:

$$y = 3 \times (-2) + 2$$
$$y = -6 + 2$$
$$y = -4$$
$$y = -4,$$

Solution: $x = -2,\ y = -4.$

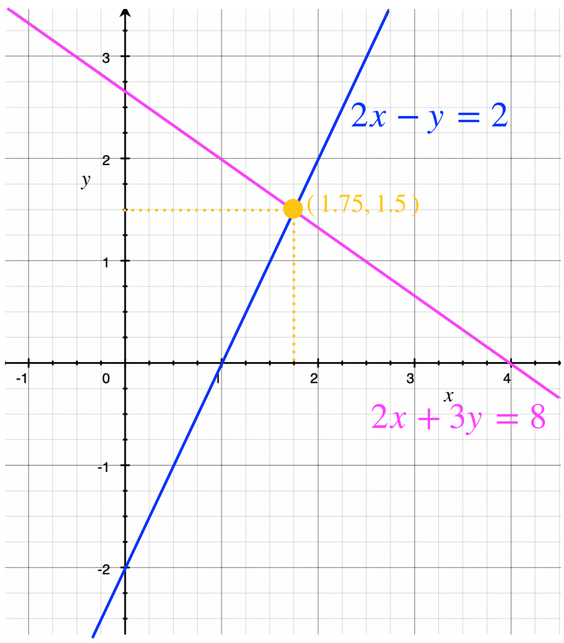
c.

$$\begin{cases} 2x + 3y = 8 \\ 2x - y = 2 \end{cases}$$

If signs are:
opposite → change to −
same → change to +

Subtract the second equation from the first:

$$(2x + 3y) - (2x - y) = 8 - 2$$
$$2x + 3y - 2x + y = 6$$
$$\cancel{2x} + 3y \cancel{-2x} + y = 6$$
$$4y = 6$$
$$\frac{\cancel{4}y}{\cancel{4}} = \frac{6}{4}$$
$$y = \frac{6}{4}$$
$$y = 1.5\,,$$



Substitute $y = 1.5$ into $2x - y = 2$:

$$2x - 1.5 = 2$$
$$2x = 3.5$$
$$\frac{\cancel{2}x}{\cancel{2}} = \frac{3.5}{2}$$
$$x = \frac{3.5}{2}$$
$$x = 1.75\,,$$

Solution:

$$x = 1.75,\ y = 1.5\,.$$

3a.

Factorise:

$$x^2 - 5x + 6 = 0$$

$$\_ \times \_ = 6 \ and\ \_ + \_ = -5$$
$$\rightarrow -2 \times -3 = 6 \ and\ -2 + -3 = -5$$

$$(x-2)(x-3) = 0$$

Set each factor equal to zero:

$x - 2 = 0$ or $x - 3 = 0$

So, the solutions are:

$x = 2$ or $x = 3$.

b.

Find two numbers that multiply to -15 and add to 2 :

$$\_ \times \_ = -15 \ and \ \_ + \_ = 2$$
$$\rightarrow 5 \times -3 = -15 \ and \ 5 + -3 = 2$$

These numbers are 5 and -3 :

$$x^2 + 2x - 15 = 0$$
$$(x+5)(x-3) = 0$$

Solve:

$$x + 5 = 0 \ and \ x - 3 = 0$$
$$x = -5 \ and \ x = 3 .$$

4a.

The general form of a quadratic equation is:

$ax^2 + bx + c = 0$, where a $\neq 0$.

b.

$$2x^2 - 4x + 1 = 0$$
$$a = +2, b = -4, c = +1 .$$

5a.

$$x^2 - 6x + 5 = 0$$

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ where $a = 1, b = -6, c = 5$:

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1}$$
$$x = \frac{6 \pm \sqrt{36 - 20}}{2}$$
$$x = \frac{6 \pm \sqrt{16}}{2}$$
$$x = \frac{6 \pm 4}{2}$$

Solutions: $x = \frac{10}{2} = 5 \ and \ x = \frac{2}{2} = 1 .$

b.

$$2x^2 + 5x - 3 = 0$$

Here, $a = 2, b = 5, c = -3$:

$$x = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 2 \cdot (-3)}}{2 \cdot 2}$$

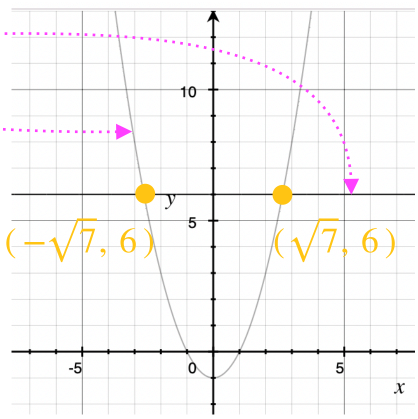
$$x = \frac{-5 \pm \sqrt{25 + 24}}{4}$$

$$x = \frac{-5 \pm \sqrt{49}}{4}$$

$$x = \frac{-5 \pm 7}{4}$$

Solutions: $x = \frac{2}{4} = 0.5 \ and \ x = \frac{-12}{4} = -3$.

6. Solve the system of equations: $\begin{cases} y = 6 \\ y = x^2 - 1 \end{cases}$



Since $y = y$, i.e. both equations are of the form $y = \ldots$,

Set the equations equal to each other :

$$6 = x^2 - 1$$

$$6 + 1 = x^2 - \cancel{1} + \cancel{1}$$

$$7 = x^2$$

$$\sqrt{7} = \cancel{\sqrt{x^2}}$$

$$\pm\sqrt{7} = x$$

$$x = \pm\sqrt{7},$$

We already know the value of $y = 6$, from the original equation, check by substituting $x = \pm\sqrt{7}$ into $y = x^2 - 1$

For y, substitute $x = \pm\sqrt{7}$ into $y = x^2 - 1$:

If $x = +\sqrt{7}$, $y = (\sqrt{7})^2 - 1 = 7 - 1 = 6$.

If $x = -\sqrt{7}$, $y = (-\sqrt{7})^2 - 1 = 7 - 1 = 6$.

As $y = 6$ in both cases, this confirms we have the correct solutions.

Final solutions :

$\rightarrow (\sqrt{7}, 6) \ and \ (-\sqrt{7}, 6)$.

$\approx (2.646, 6) \ and \ (-2.646, 6)$.

7a.

Let w be the width. Then $l = w + 10$

Substituting into the perimeter equation:

$$2l + 2w = 40$$
$$2(w + 10) + 2w = 40$$

Simplify:

$$2w + 20 + 2w = 40$$
$$4w + \cancel{20} - \cancel{20} = 40 - 20$$
$$4w = 20$$
$$\frac{\cancel{4}w}{\cancel{4}} = \frac{20}{4}$$
$$w = 5\,.$$

Therefore,

$$l = w + 10$$
$$= 5 + 10$$
$$l = 15\,.$$

The dimensions are $5\,cm$ (width) and $15\,cm$ (length).

b.

The ball hits the ground when $h = 0$:

$$h = -5t^2 + 20t + 2$$
$$0 = -5t^2 + 20t + 2$$

Multiply through by -1 to simplify:

$$0 \times -1 = \left[-5t^2 + 20t + 2\right] \times -1$$
$$\frac{0}{5} = \frac{5t^2 - 20t - 2}{5}$$
$$0 = \frac{5t^2}{5} - \frac{20t}{5} - \frac{2}{5}$$
$$0 = 1t^2 - 4t - \frac{2}{5}$$
$$0 = t^2 - 4t - \frac{2}{5}$$

Use the quadratic formula with $a = 1, b = -4, c = -\frac{2}{5}$:

$$t = \frac{4 \pm \sqrt{16 + \frac{8}{5}}}{2}$$
$$t = \frac{4 \pm \sqrt{\frac{80}{5} + \frac{8}{5}}}{2}$$
$$t = \frac{4 \pm \sqrt{\frac{88}{5}}}{2}$$
$$t = \frac{4 \pm \frac{2\sqrt{22}}{\sqrt{5}}}{2}$$
$$t = 2 \pm \frac{\sqrt{22}}{\sqrt{5}}$$

Since time can't be negative, we discard the negative solution:

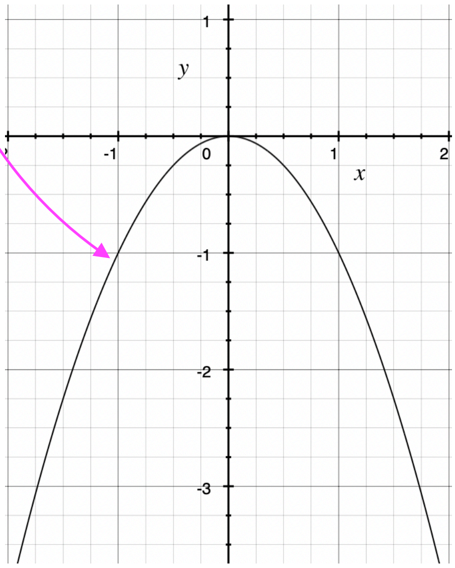
$$t \approx 2 + \frac{\sqrt{22}}{\sqrt{5}}$$
$$\approx 4.24\, seconds.$$

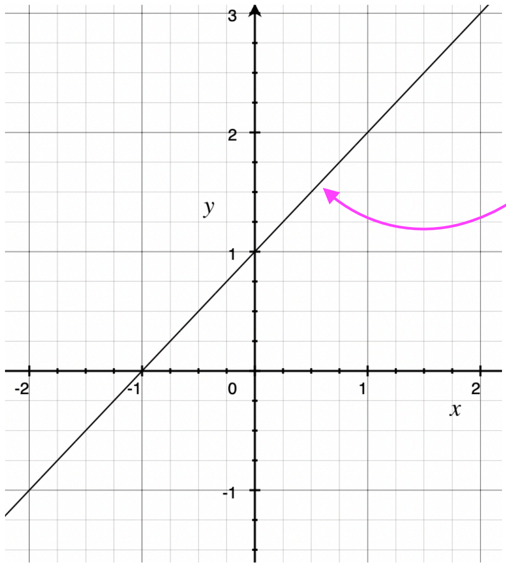
8.

a) A parabola opening upwards.
$y = -x^2 = -1x^2$ represents a quadratic function with a negative coefficient (-1) for x^2, hence it opens downwards.

$y = 1x + 1 = \frac{1}{1}x + 1$

b) A straight line passing through the $y - axis$ at $y = +1$, with a slope of $\frac{1}{1} = \frac{Rise}{Run}$.





9a.

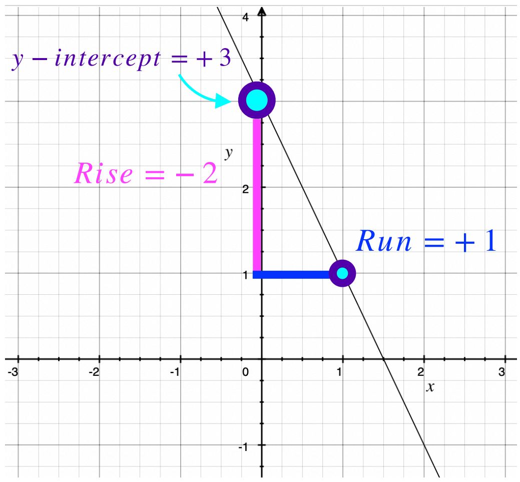
$$y = -2x + 3$$
$$y = mx + c$$

$Slope\ (m):$

$$m = -2 = \frac{-2}{+1} = \frac{rise}{run}.$$

$y - intercept\ (c):$

$$c = +3.$$



[Description for drawing on graph paper with labelled axes :
Draw a point at $x = 3$, then go down 2 and across 1, draw a point here, then draw a straight line between the two points.]

b.

Vertex, use the formula :

$$x = -\frac{b}{2a} \text{ where } a = 1 \text{ and } b = 3:$$

$$x = -\frac{3}{2 \times (1)}$$

$$= \frac{-3}{2} = -1.5 \,.$$

Substituting $x = -1.5$ into the equation for y :

$$y = (-1.5)^2 + 3(-1.5) - 1$$

$$= 2.25 - 4.5 - 1$$

$$= -3.25 \,.$$

Substituting $x = 0$ into the equation for y :

$$y = (0)^2 + 3(0) - 1$$

$$= 0 + 0 - 1$$

$$= -1 \,.$$

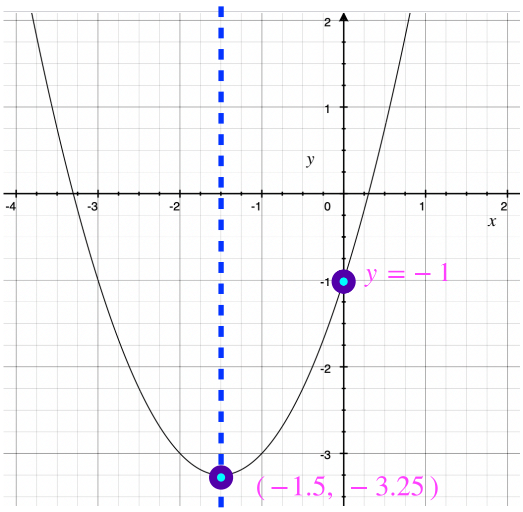
So, the vertex is at $(-1.5, \ -3.25)$, and the $y-intercept$ is at $y = -1$.

[Description for sketching:

Draw a parabola opening upwards with the vertex at $(-1.5, \ -3.25)$.

The parabola crosses the $y-axis$ at $y = -1$ (when $x = 0$)

and has symmetry about $x = -1.5$.]



10a.

$P = 25{,}000$ (principal)
$R = 2\,\%$
$= \dfrac{2}{100}$
$= 0.02$ (rate as a decimal in years)
$T = 3$ (time in years)

$I = PRT$

$$I = 25{,}000 \times 0.02 \times 3$$
$$= 500 \times 3$$
$$= 1{,}500\,.$$

The simple interest earned is $\$1{,}500$.

b.

$P = 12{,}000$ (principal)
$R = 4\,\%$
$= \dfrac{4}{100}$
$= 0.04$ (rate as a decimal per year)

Divide by 12 to convert to per month:
$\rightarrow 0.04 \div 12$
$= \dfrac{0.04}{12} \; \left(= \dfrac{1}{300} \right)$ (rate in months)
≈ 0.00333

$T = 9$ (time in months)

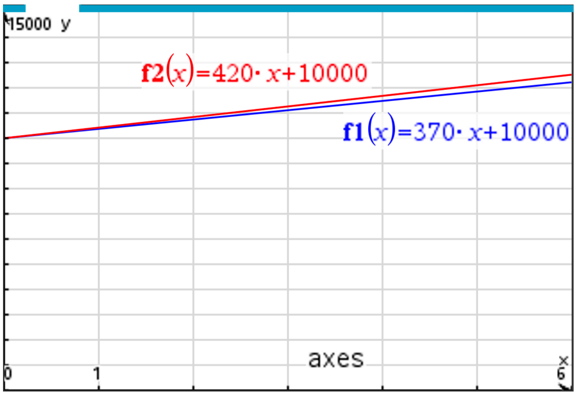
$I = PRT$

$$I = 12{,}000 \times \left(\frac{0.04}{12} \right) \times 9$$
$$= 40 \times 9$$
$$= 360\,.$$

The simple interest earned is $\$360$.

c.

I) Write the linear equations: The total amount $A = P + I$, where $I = PRT$.



Option A:

$P = 10{,}000,\ R = 3.7\,\% = 0.037,\ (T)$ is variable.

$I = 10{,}000 \times 0.037 \times T$

$I = 370T$

$A = 10{,}000 + 370T.$ *blue line*

Option B:

$P = 10{,}000,\ R = 4.2\,\% = 0.042,\ (T)$ is variable.

$I = 10{,}000 \times 0.042 \times T$

$I = 420T,$

$A = 10{,}000 + 420T.$ *red line*

II) Calculate the total amount:

Option A:

$T = 5\ years$

$A = 10{,}000 + 370 \times 5$

$= 10{,}000 + 1{,}850$

$A = 11{,}850$

Total amount $= \$11{,}850.$

Option B:

$T = 3\ years$

$A = 10{,}000 + 420 \times 3$

$= 10{,}000 + 1{,}260$

$A = 11{,}260$

Total amount $= \$11{,}260.$

III) Compare and find the difference:

Option A: $11,850

Option B: $11,260

Difference:

$\rightarrow 11{,}850 - 11{,}260 = 590.$

Option A gives the highest return by $590 more than Option B.
Answer: Sarah should choose Option A, earning $220 more.

d.

I) Write the linear equations:

$A = P + I$, where $I = PRT$.
$P = 8{,}000$, $R = 5\,\% = 0.05\ per\ month$, (T) is variable.

$I = 8{,}000 \times 0.05 \times T$
$I = 400T$, (Time in months)

$A = 8{,}000 + 400T$. (Time in months)

II) Calculate the total amount after a year and a half:

$$T = 1.5\ years$$
$$= 1.5\ \cancel{years} \times 12\ \frac{months}{\cancel{year}}$$
$$= 18\ months.$$

Remember, time and rate MUST be in the same units!

$$A = 8{,}000 + 400T$$

$$A = 8{,}000 + 400 \times 18$$
$$= 8{,}000 + 7{,}200$$
$$A = 15{,}200.$$

Total amount $= \$15{,}200$.

Additional Notes for Teachers:

Learning Outcomes:
Students should solve linear and quadratic equations using various methods, understand how to apply these equations in real-world contexts, and recognise when each method is most appropriate.

Teaching Strategies:
Use interactive software or real-life scenarios to link algebra with physical phenomena. Encourage students to check solutions by back-substitution or graphing.

Assessment:
Assess through problems that require students to choose between methods (factorisation, quadratic formula) based on the equation's form or context.

Resources:
Graphing calculators or software for visual learning, algebra tiles for physical manipulation of equations, and real-world problems for application.

This set of questions aligns with the Australian Curriculum for Year 9, focusing on deepening students' skills in solving linear and quadratic equations.