Simultaneous Equations, and Circles

9

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Focus: A set of questions and solutions for Year 9 students on Simultaneous Equations, tailored to the Australian Curriculum under the strand 'Number and Algebra':

1. Solving by Substitution:

a) Solve the system: $x + y = 5$ **and** $2x - y = 1$ **.**

b) Solve: $3a + b = 10$ **and** $a - b = 4$ **.**

2. Solving by Elimination:

a) Solve the system: $2x + y = 9$ **and** $x - y = 3$ **.**

b) Solve: $5m - 2n = 14$ **and** $3m + 2n = 6$ **.**

3. Real-World Application:

a) The sum of two numbers is 17 **, and their difference is** 7 **. Find the numbers.**

b) One cinema ticket costs $\$12$ **, and one popcorn bag costs** $\$5$ **. If a group bought** 10 **items in total for** $\$90$ **, how many tickets and popcorn bags did they buy? List assumptions and reasons for possible errors.**

4. Basic Properties of Circles:

a) Define the terms radius, diameter, and circumference of a circle.

b) If the radius of a circle is $5\ cm$, what is its diameter and circumference?

5. Area of a Circle:

a) Calculate the area of a circle with a radius of $4\ cm$.

b) If the area of a circle is $154\,cm^2$, what is its radius? (Use $\pi \approx 3.14$)

6. Arc Length and Sector Area:

a) Find the length of an arc that subtends an angle of 60° in a circle with a radius of $10\,cm$.

b) Calculate the area of a sector with an angle of 45° in a circle where the radius is $8\,cm$.

7. Circle Theorems:

a) In a circle, if an angle subtended by an arc at the centre is 100° , what is the measure of the angle subtended by the same arc at the circumference?

b) Two tangents from the same point outside a circle are equal in length. If the tangents from point P to circle C are $6\,cm$ each, and the distance from P to the centre of the circle is $10\,cm$, what is the radius of the circle?

8. Cyclic Quadrilaterals:

a) If three angles of a cyclic quadrilateral are 80°, 100°, and 120°, what is the measure of the fourth angle?

b) Prove that the opposite angles of a cyclic quadrilateral sum to 180° .

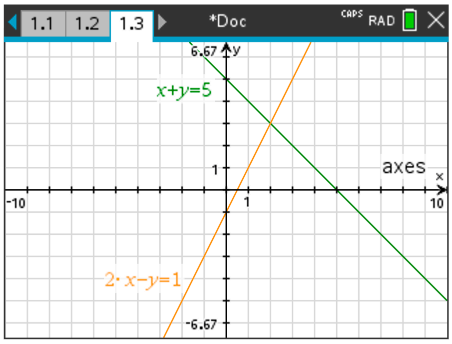
9. Real-World Applications:

a) A circular garden has a radius of $7\,metres$. How much fencing is needed to go around it?

b) A pizza with a diameter of $30\,cm$ is cut into 6 equal slices. What is the area of one slice?

Solutions

1a.



$$\begin{cases} x+y &= 5 \\ 2x-y &= 1 \end{cases}$$

From the first equation, solve for y :

$$y = 5 - x$$

Substitute $y = 5 - x$ into the second equation:

$$2x - (5 - x) = 1$$
$$2x - 5 + x = 1$$
$$3x = 6$$
$$x = 2\,.$$

Substitute $x = 2$ back into $y = 5 - x$:

$$y = 5 - 2$$
$$y = 3\,.$$

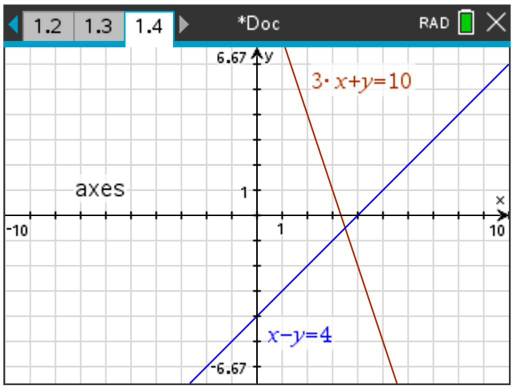
Solution: $x = 2\,,\ y = 3\,.$

b.

$$\begin{cases} 3a+b = 10 \\ a-b \ \ = 4 \end{cases}$$

Express b in terms of a from the second equation:

$$b = a - 4$$



Substitute $b = a - 4$ into the first equation

$$3a + (a - 4) = 10$$
$$4a - 4 = 10$$
$$4a = 14$$
$$a = 3.5\,.$$

Substitute $a = 3.5$ back into $b = a - 4$:

$$b = 3.5 - 4$$
$$b = -0.5\,.$$

Solution: $a = 3.5\,,\ b = -0.5\,.$

2a.

$$\begin{cases} 2x + y = 9 \\ x - y \quad = 3 \end{cases}$$

Add both equations to eliminate y :

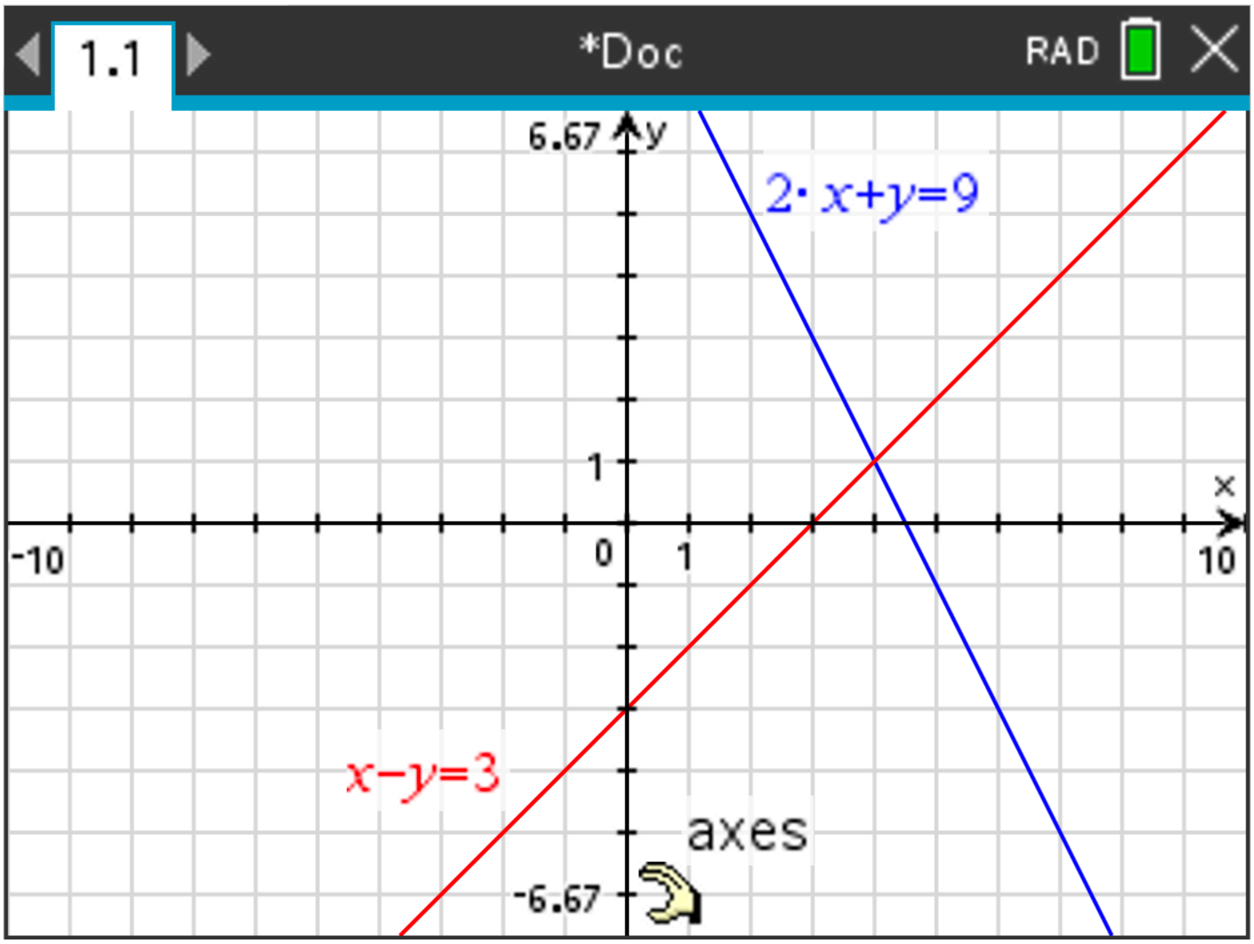
$$(2x + y) + (x - y) = 9 + 3$$
$$2x + y + x - y = 9 + 3$$
$$2x + x \cancel{+y} \cancel{-y} = 9 + 3$$
$$3x = 12$$
$$x = 4$$

Substitute $x = 4$ into one of the original equations, say

$$x - y = 3$$

$$4 - y = 3$$
$$4 \cancel{-y} \cancel{+y} = 3 \ +y$$
$$4 = 3 + y$$
$$4 \ -3 = \cancel{3} \cancel{-3} \ +y$$
$$1 = y$$
$$y = 1$$

Solution: $x = 4$, $y = 1$.



b.

$$\begin{cases} 5m - 2n = 14 \\ 3m + 2n = 6 \end{cases}$$

Add both equations to eliminate n :

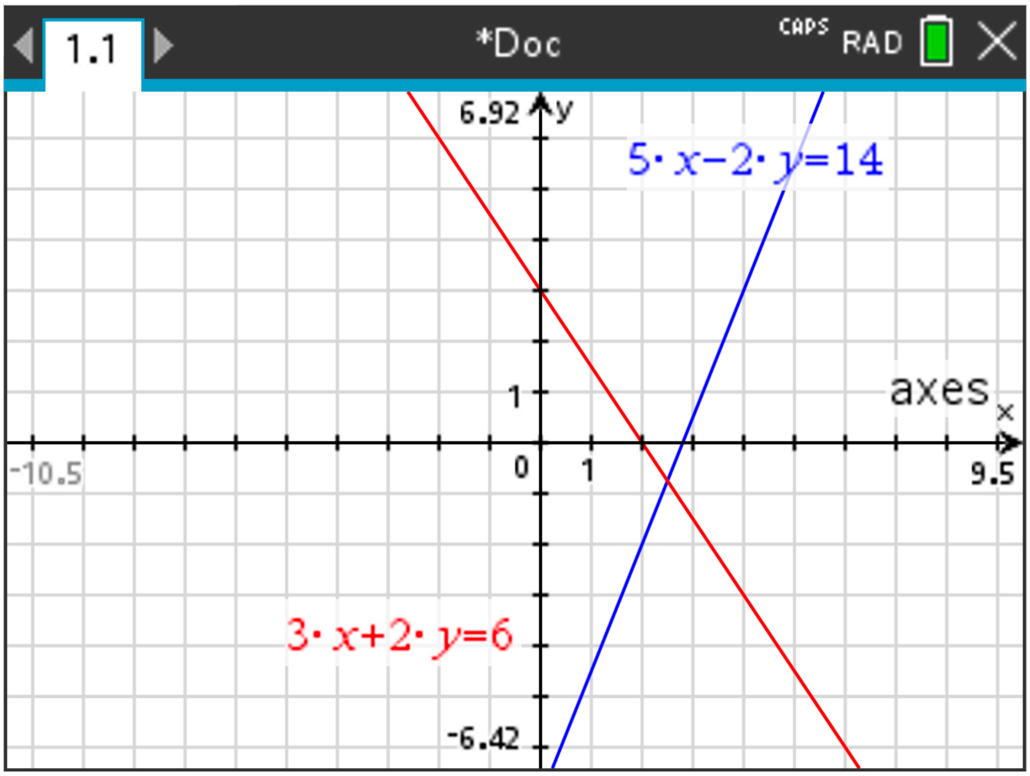
$$(5m - 2n) + (3m + 2n) = 14 + 6$$
$$5m - 2n + 3m + 2n = 14 + 6$$
$$5m + 3m \cancel{-2n + 2n} = 20$$
$$8m = 20$$
$$\frac{\cancel{8}m}{\cancel{8}} = \frac{20}{8}$$
$$m = 2.5\,.$$

Substitute $m = 2.5$ into one of the equations, say :

$$3m + 2n = 6$$

$$3(2.5) + 2n = 6$$
$$3 \times (2.5) + 2n = 6$$
$$7.5 + 2n = 6$$
$$\cancel{+7.5} \cancel{-7.5} + 2n = 6 - 7.5$$
$$2n = 6 - 7.5$$
$$2n = -1.5$$
$$\frac{\cancel{2}n}{\cancel{2}} = \frac{-1.5}{2}$$
$$n = -0.75\,.$$

Solution: $m = 2.5$, $n = -0.75$.



3a.

Let the numbers be x and y :

$$x + y = 17$$
$$x - y = 7$$

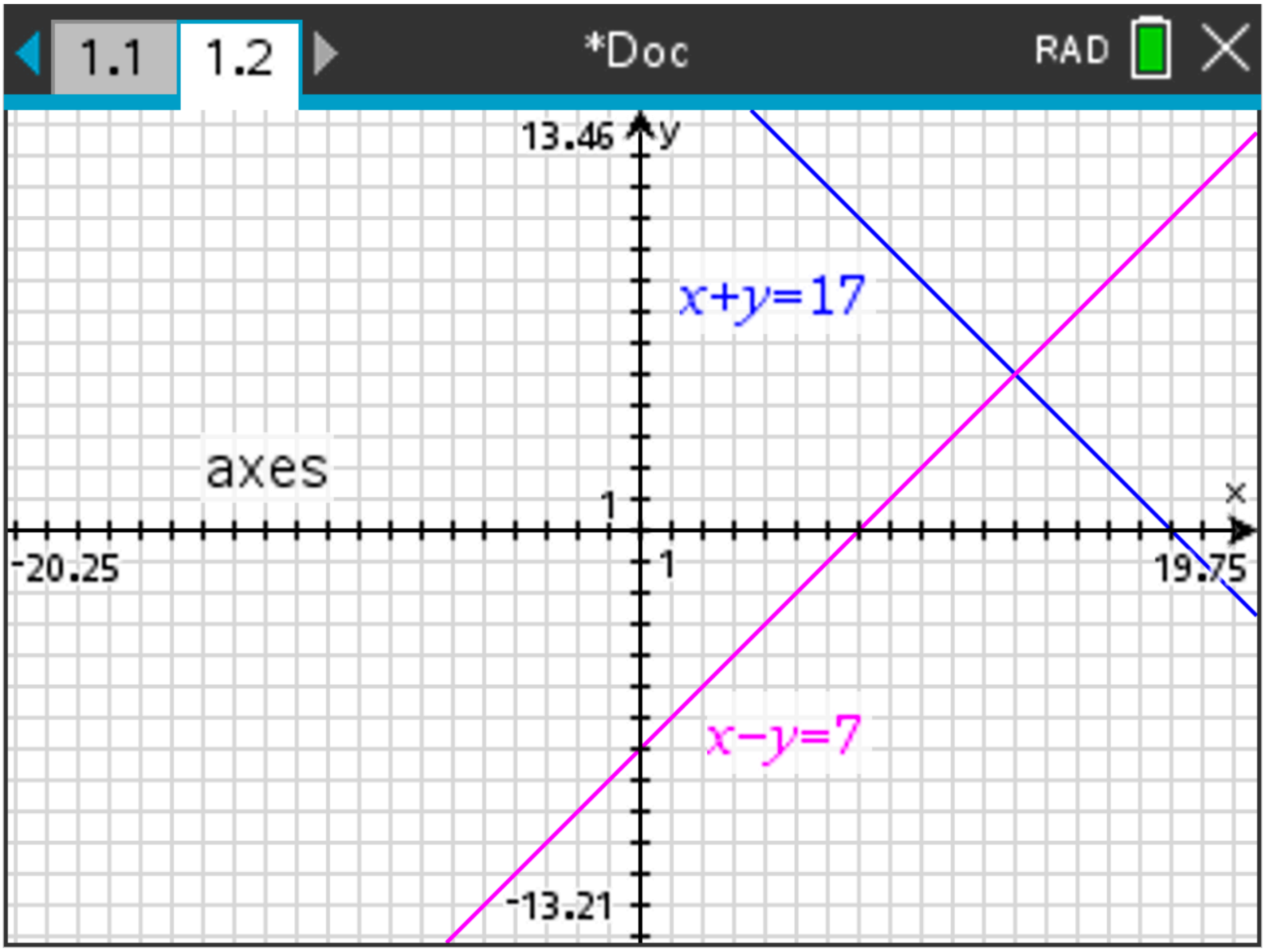
Add these equations:

$$x + x + y + -y = 17 + 7$$
$$x + x \cancel{+ y - y} = 17 + 7$$
$$2x = 24$$
$$\frac{\cancel{2}x}{\cancel{2}} = \frac{24}{2}$$
$$x = 12\,.$$

Substitute $x = 12$ into the first equation:

$$x + y = 17$$
$$12 + y = 17$$
$$\cancel{+12}\ \cancel{-12} + y = 17 - 12$$
$$y = 5\,.$$

The numbers are 12 and 5 .



b.

Let t be the number of tickets and p be the number of popcorn bags:

$$12t + 5p = 90$$
$$t + p = 10$$

Solve by substitution:

From $t + p = 10$,

$$p = 10 - t$$

Substitute into the first equation:

$$12t + 5(10 - t) = 90$$
$$12t + 50 - 5t = 90$$
$$7t = 40$$
$$t = \frac{40}{7} \approx 5.71\,.$$

Since t must be a whole number, we test $t = 5$ and $t = 6$:

For $t = 5$, $p = 10 - 5 = 5$:

Check: $12(5) + 5(5) = 60 + 25 = 85$ (not 90) .

For $t = 6$, $p = 10 - 6 = 4$:

Check: $12(6) + 5(4) = 72 + 20 = 92$ (not 90) .

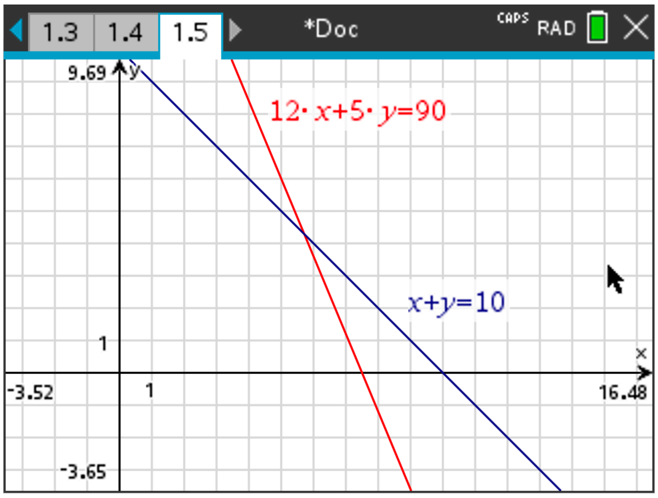
There's no whole number solution, indicating a potential error in the assumptions made when setting up the system of equations. Or a possible error in an assumption about whole numbers.

Assumptions:

–Only tickets and popcorn have been purchased.
–Everyone bought a ticket and one item each.
–1 ticket = 1, and 1 item = 1 (other items may have been chosen which may have cost more, and / or other items may include more than one piece etc.).

Possible Errors:

–May not have just bought popcorn and tickets, could have been other items.
–Someone may have just bought other sweets and left.
–Someone may have bought a ticket but no sweets etc.



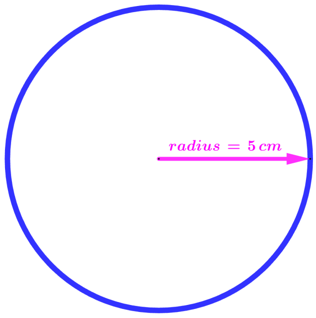
4a.

Radius: The distance from the centre of the circle to any point on the circle's edge.
Diameter: The distance across the circle through its centre, equal to twice the radius.
Circumference: The perimeter or the distance around the circle.

b.

$$r = 5\,cm\,.$$
$$D = 2r\,.$$
$$C = 2\pi r\,.$$

$$D = ?\,.$$
$$C = ?\,.$$



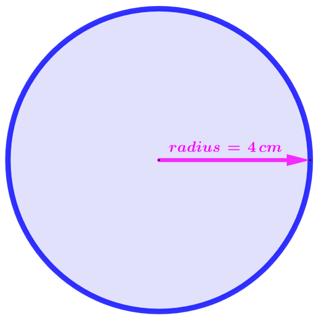
$$\text{Diameter } = 2 \times \text{ radius} = 2 \times 5\,cm = 10\,cm\,.$$

$$\text{Circumference } = 2\pi r = 2\pi \times 5\,cm = 10\pi\,cm \approx 31.42\,cm\,.$$

5a.

$$r = 4\,cm\,.$$
$$A = \pi r^2$$

$$A = ?$$



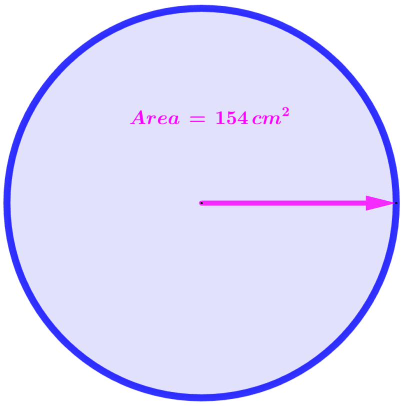
Area :

$$A = \pi r^2 = \pi \times (4\,cm)^2 = \pi \times 4^2\,cm^2 = 16\pi\,cm^2 \approx 50.27\,cm^2\,.$$

b.

$$A = 154\,cm^2.$$
$$A = \pi r^2$$
$$r = ?.$$

$$154 = 3.14 \times r^2$$
$$\frac{154}{3.14} = \frac{\cancel{3.14} \times r^2}{\cancel{3.14}}$$
$$r^2 = \frac{154}{3.14}$$
$$r^2 \approx 49$$
$$\cancel{\sqrt{r^2}} \approx \sqrt{49}$$
$$r \approx \sqrt{49}$$
$$\approx 7\,cm.$$

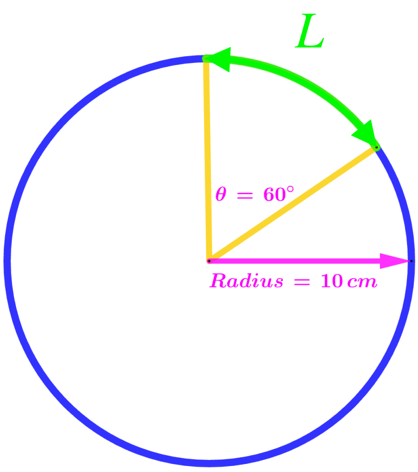


6a.

$$L = \frac{\theta}{360} \times 2\pi r.$$
$$\theta = 60^\circ.$$
$$r = 10\,cm.$$
$$L = ?.$$

Arc length :

$$L = \frac{\theta}{360} \times 2\pi r$$
$$L = \frac{60}{360} \times 2\pi \times 10$$
$$L = \frac{60 \div 60}{360 \div 60} \times 20\pi$$
$$= \frac{1}{6} \times 20\pi$$
$$\approx 10.47\,cm.$$

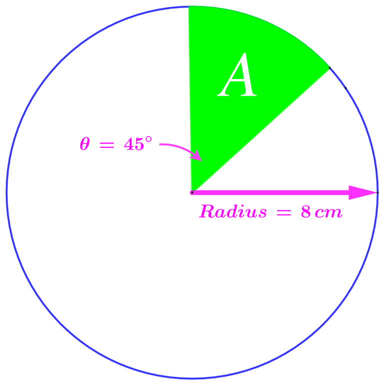


b.

$$\theta = 45^\circ .$$
$$r = 8\,cm .$$
$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

$$A = ? .$$

Sector Area :

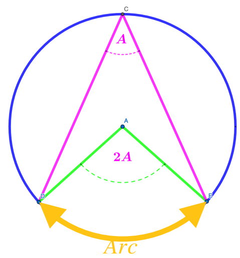


$$\begin{aligned}
A &= \frac{\theta}{360} \times \pi r^2 \\
A &= \frac{45}{360} \times \pi \times 8^2 \\
A &= \frac{45 \div 45}{360 \div 45} \times \pi \times 8^2 \\
&= \frac{1}{8} \times 64\pi \\
&= \frac{64}{8}\pi \\
&= 8\pi \\
&\approx 25.13\,cm^2 .
\end{aligned}$$

7a.

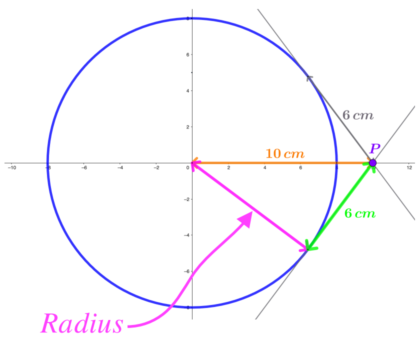
The angle at the circumference is half the angle at the centre:

$$\begin{aligned}
\text{Angle at circumference} &= \frac{\text{Angle at Centre}}{2} \\
&= \frac{100^\circ}{2} \\
&= 50^\circ .
\end{aligned}$$



b.

Let r be the radius.
Using the right triangle formed by the radius, tangent,
and the line from the external point to the centre:



$$\begin{aligned}
a^2 + b^2 &= c^2 \\
r^2 + 6^2 &= 10^2 \\
r^2 + 36 &= 100 \\
r^2 \cancel{+36} \cancel{-36} &= 100 - 36 \\
r^2 &= 64 \\
\cancel{\sqrt{r^2}} &= \sqrt{64} \\
r &= 8\,cm .
\end{aligned}$$

8a.

Opposite angles in a cyclic quadrilateral sum to 180° :

The sum of all angles in a quadrilateral is 360° :

$$\begin{aligned} 360^\circ - (80^\circ + 100^\circ + 120^\circ) &= 360^\circ - 300^\circ \\ &= 60^\circ . \end{aligned}$$

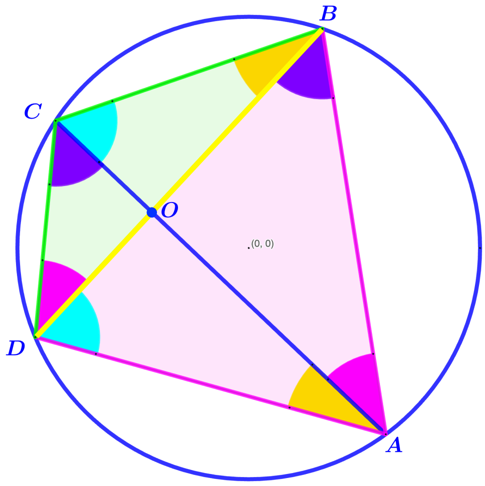
b.

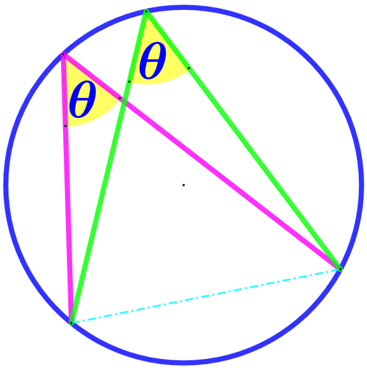
Consider a cyclic quadrilateral $ABCD$ with opposite angles $\angle A$ and $\angle C$, and $\angle B$ and $\angle D$.

The angles subtended by the same arc at the circumference are equal:

$\angle A + \angle C$ and $\angle B + \angle D$ are formed by the arcs BC and AD respectively.

Since each pair shares the same arc, they sum to 180° as per the property of angles at the circumference and centre.

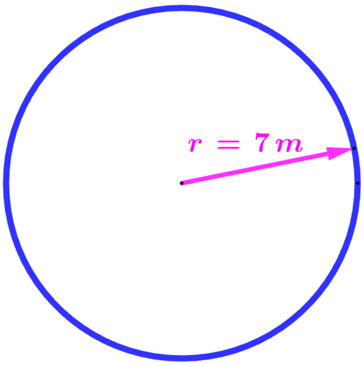




9a.

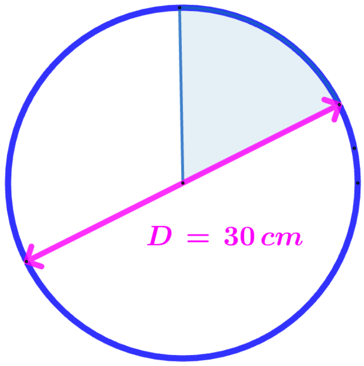
Fencing length $=$ Circumference

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi \times 7\,m \\ &= 14\pi\ m \\ &\approx 43.98\,metres . \end{aligned}$$



b.

$$
\begin{aligned}
\text{Radius} &= 15\,cm\,. \\
A &= \pi r^2 \\
\text{Area of one Slice} &= \frac{\text{Area of Pizza}}{6}
\end{aligned}
$$



$$
\begin{aligned}
\text{Area of the whole pizza} &= \pi \times 15^2 \\
&= 225\pi \\
&\approx 706.86cm^2\,.
\end{aligned}
$$

$$
\begin{aligned}
\text{Area of one slice} &= \frac{225\pi}{6} \\
&= 37.5\pi \\
&\approx 117.81\,cm^2\,.
\end{aligned}
$$

Additional Notes for Teachers:

Learning Outcomes:

Students should be able to solve systems of linear equations using substitution and elimination methods, understanding when each method might be more efficient. Students should understand the properties of circles, calculate areas, circumferences, and use circle theorems in problem-solving.

Teaching Strategies:

Use visual aids or software to illustrate the concept of intersecting lines representing solutions. Engage students with practical problems where systems of equations appear naturally. Use physical models, drawing circles with compasses, or digital tools for interactive learning. Discuss real-life applications like architecture or design.

Assessment:

Test through problems that vary in complexity, from simple systems to those requiring strategic choice between methods, and real-life applications. Assess through problems that require students to apply formulas, understand theorems, and interpret results in context.

Resources:

Use graphing calculators or software to verify solutions graphically. Encourage peer teaching where students explain their methods to each other. Use apps or software for dynamic geometry to explore circle properties. Provide circle templates or use pizza cutting as a visual aid for sectors.

This set of questions aligns with the Australian Curriculum for Year 9, focusing on developing proficiency in solving simultaneous equations, and circles.