Statistics, and Probability of Combined Events

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Focus: A set of questions and solutions for Year 9 students on Statistics, and Probability of Combined Events, tailored to the Australian Curriculum under the strand 'Statistics and Probability':

1. Independent Events:

a) If you roll a die and then flip a coin, what is the probability of rolling a 4 **and flipping heads?**

b) Two cards are drawn from a standard deck without replacement. What's the probability of drawing two aces?

2. Dependent Events:

a) A bag contains 5 red balls and 3 blue balls. One ball is drawn, not replaced, then another ball is drawn. What is the probability of drawing two red balls?

b) If you have a box with 4 black pens and 6 blue pens, what is the probability of picking a blue pen followed by a black pen without replacement?

3. Using Probability Trees:

a) Construct a probability tree for the scenario of flipping two coins and determine the probability of getting at least one head.

b) Use a probability tree to find the probability of getting two red balls when drawing two balls from a bag containing 2 red, 3 blue, and 1 green ball, with replacement.

4. Real-World Applications:

a) There's a $70\,\%$ chance of rain on Saturday and a $60\,\%$ chance on Sunday. What is the probability that it rains on both days?

b) A tennis player has a $60\,\%$ chance of winning a set on grass and an $80\,\%$ chance on clay. If he plays one set on each surface, what's the probability he wins both sets?

5. Conditional Probability:

In a class, 40% of students play football and 30% play both football and basketball. What is the probability that a student plays basketball given they play football?

6. Standard Deviation

a) What does a small standard deviation tell you about a dataset?

b) Given the dataset: $\{\ 50,\ 60,\ 108,\ 120,\ 240\ \}$ **, calculate the standard deviation.**

7. Two-Way Tables

A survey results in a two-way table showing that out of 75 people, 45 like tea (T), 40 like coffee (C), and 20 like both (T and C). How many like tea but not coffee? What is the probability that a randomly selected person likes only coffee? Use a two-way table to solve the problem.

Solutions

1a.

$$P(\text{Event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}} = P(A) = \frac{n(A)}{n(S)}$$

$$\text{Probability of rolling a } 4 \text{ on a die} = \frac{1}{6},$$

$$\text{Probability of flipping heads on a coin} = \frac{1}{2}.$$

Since these events are independent, the combined probability is :

$$\begin{aligned} P(A\ and\ B) &= P(A \cap B) \\ &= P(A) \times P(B). \end{aligned}$$

$$\begin{aligned} P(4 \text{ and Heads}) &= \frac{1}{6} \times \frac{1}{2} \\ &= \frac{1}{12} \approx 0.08\dot{3} \approx 8.\dot{3}. \end{aligned}$$

b.

$$P(\text{Event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}} = P(A) = \frac{n(A)}{n(S)}$$

$$\begin{aligned} \text{Probability of drawing an ace first} &= \frac{4}{52} \\ &= \frac{4 \div 4}{52 \div 4} \\ &= \frac{1}{13}. \end{aligned}$$

After drawing one ace, 3 aces remain out of 51 cards:

$$\text{Probability of drawing a second ace} = \frac{3}{51}.$$

Combined probability (since the events are dependent):

$$\text{Probability of } A \times \text{Probability of } B \text{ given } A = P(A) \times P(B|A).$$

$$\begin{aligned} P(\text{two aces}) &= \frac{1}{13} \times \frac{3}{51} \\ &= \frac{1 \times 3}{13 \times 51} \\ &= \frac{3 \div 3}{663 \div 3} \\ &= \frac{1}{221} \approx 0.0045 \approx 0.45\,\%. \end{aligned}$$

2a.

$$P(\text{Event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}} = P(A) = \frac{n(A)}{n(S)}$$

Combined probability (since the events are dependent):

$$\text{Probability of } A \times \text{Probability of } B \text{ given } A = P(A) \times P(B\,|\,A)\,.$$

$$\text{Probability of drawing a red ball first } = \frac{5}{8}\,.$$

With one red ball removed, there are now 4 red balls out of 7 :

$$\text{Probability of drawing a second red ball } = \frac{4}{7}\,.$$

Combined probability:

$$\begin{aligned} P(\text{two red}) &= \frac{5}{8} \times \frac{4}{7} \\ &= \frac{5 \times 4}{8 \times 7} \\ &= \frac{20}{56} \\ &= \frac{20 \div 4}{56 \div 4} \\ &= \frac{5}{14} \approx 0.357 \approx 35.7\,\%\,. \end{aligned}$$

b.

$$P(\text{Event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}} = P(A) = \frac{n(A)}{n(S)}$$

Combined probability (since the events are dependent):

Probability of A $\times$ Probability of B given A $= P(A) \times P(B\,|\,A)$.

$$\begin{aligned}\text{Probability of picking a blue pen first } &= \frac{6}{10}\\ &= \frac{6 \div 2}{10 \div 2}.\\ &= \frac{3}{5}.\end{aligned}$$

After picking one blue pen, there are 5 blue and 4 black pens left:

$$\text{Probability of picking a black pen next } = \frac{4}{9}.$$

Combined probability:

$$\begin{aligned}P(\text{blue then black}) &= \frac{3}{5} \times \frac{4}{9}\\ &= \frac{3 \times 4}{5 \times 9}\\ &= \frac{12}{45}\\ &= \frac{12 \div 3}{45 \div 3}\\ &= \frac{4}{15} = 0.26\dot{6} = 26.\dot{6}\,\%.\end{aligned}$$

3a.

$$P(A) = \frac{n(A)}{n(S)}$$

For each coin flip, there are two outcomes:
$Heads\ (H)$ or $Tails\ (T)$.

First flip:

$P(H) = \frac{1}{2}$ or $P(T) = \frac{1}{2}$,

Second flip:

$P(H) = \frac{1}{2}$ or $P(T) = \frac{1}{2}$ for each outcome of the first flip.

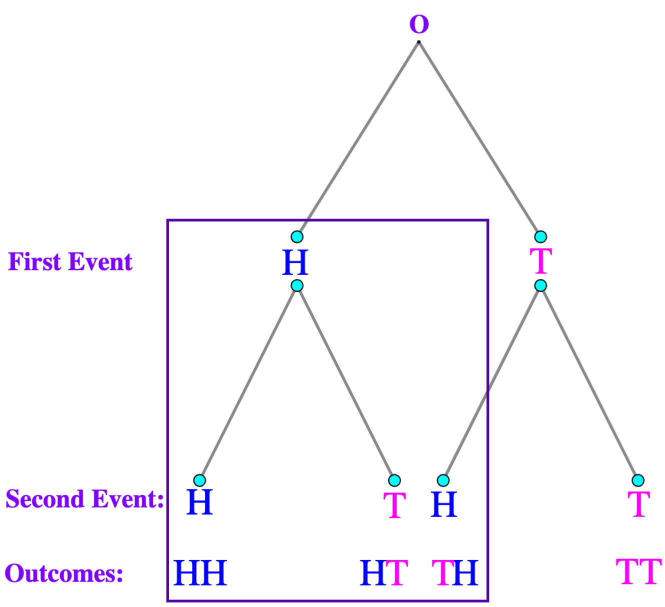
Here's the tree:

```
H ----> H (1/4)
|       |
|       T (1/4)
|
T ----> H (1/4)
        |
        T (1/4)
```

The scenarios for at least one head are:

$$HH,\ HT,\ TH,\ \cancel{TT}$$

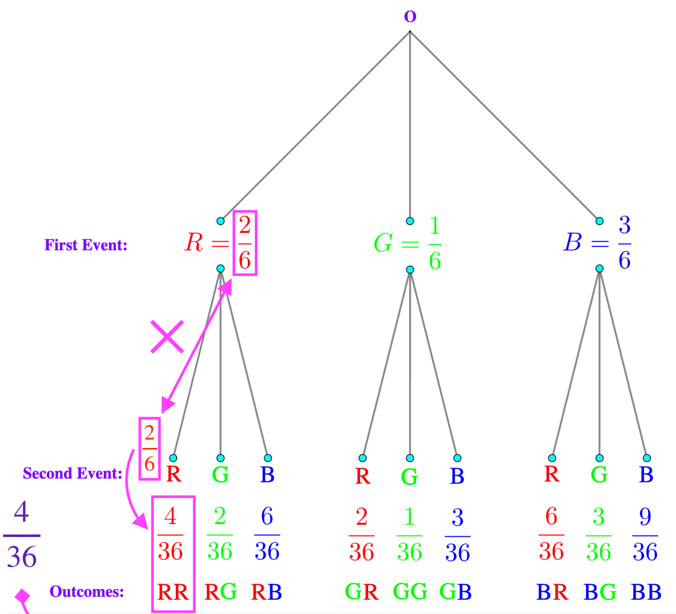
$$\begin{aligned}\text{Total probability} &= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \\ &= \frac{1+1+1}{4} \\ &= \frac{3}{4} = 0.75 = 75\,\%.\end{aligned}$$



b.

For each draw, there are 6 possible outcomes (since we replace the ball after each draw):

```
R ----> R (4/36)
|       |
|       B (6/36)
|       |
|       G (2/36)
|
B ----> R (6/36)
|       |
|       B (9/36)
|       |
|       G (3/36)
|
G ----> R (2/36)
        |
        B (3/36)
        |
        G (1/36)
```



$$P(RR) = \frac{2}{6} \times \frac{2}{6} = \frac{4}{36}$$

$$P(\text{Event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}} = P(A) = \frac{n(A)}{n(S)}$$

Combined probability (since the events are independent):

$$\text{Probability of } A \times \text{Probability of } B = P(A) \times P(B)\,.$$

Probability of drawing two red balls with replacement:

$$\begin{aligned} P(RR) = P(R) \times P(R) &= \frac{2}{6} \times \frac{2}{6} \\ &= \frac{2 \times 2}{6 \times 6} \\ &= \frac{4}{36} \\ &= \frac{4 \div 4}{36 \div 4} \\ &= \frac{1}{9} \approx 0.11\dot{1} \approx 11.\dot{1}\,\%\,. \end{aligned}$$

4a.

Combined probability (assuming the events are independent):

$$\text{Probability of } A \times \text{Probability of } B = P(A) \times P(B)\,.$$

$$70\,\% = \frac{7\not{0}}{10\not{0}} = 0.7\,,$$
$$60\,\% = \frac{6\not{0}}{10\not{0}} = \frac{3}{5} = 0.6\,.$$

Assuming these probabilities are independent:

$$P(\text{Rain on both}) = 0.70 \times 0.60$$
$$= 0.42 = 42\,\%\,.$$

b.

Combined probability (assuming the events are independent):

$$\text{Probability of } A \times \text{Probability of } B = P(A) \times P(B)\,.$$

$$60\,\% = \frac{6\not{0}}{10\not{0}} = \frac{3}{5} = 0.6\,,$$
$$80\,\% = \frac{8\not{0}}{10\not{0}} = \frac{4}{5} = 0.8\,.$$

Since the outcomes of the matches on different surfaces are independent:

$$P(\text{Winning both}) = 0.60 \times 0.80$$
$$= 0.48 = 48\,\%\,.$$

5.

$$40\,\% = \frac{4\not{0}}{10\not{0}} = \frac{2}{5} = 0.4\,,$$
$$30\,\% = \frac{3\not{0}}{10\not{0}} = 0.3\,.$$

Conditional probability formula:

$$P(B\,|\,A) = \frac{P(A \cap B)}{P(A)}$$

Probability of playing basketball given playing football, is conditional:

$$P(\text{Basketball}\,|\,\text{Football}) = \frac{P(\text{Both Football and Basketball})}{P(\text{Football})}$$
$$= \frac{0.3\not{0}}{0.4\not{0}}$$
$$= \frac{3}{4} = 0.75 = 75\,\%\,.$$

6a.

A small standard deviation indicates that the data points are condensed around a small range of values, suggesting low variability or dispersion from the mean. This means the data is highly consistent or predictable.

b.

$$\{\,50,\ 60,\ 108,\ 120,\ 240\,\}$$

$$\text{Mean} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\text{Sum of scores}}{\text{Number of Scores}}$$

$$\text{Mean} = \mu = \frac{50 + 60 + 108 + 120 + 240}{5} = \frac{578}{5}$$

$$\mu = 115.6\,.$$

$$\text{Variance (for population)} = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{n} = \frac{(50 - 115.6)^2 + (60 - 115.6)^2 + (108 - 115.6)^2 + (120 - 115.6)^2 + (240 - 115.6)^2}{5}$$

Deviations from the mean:

$$50 - 115.6 = -65.6,\ 60 - 115.6 = -55.6,\ 108 - 115.6 = -7.6,\ 120 - 115.6 = 4.4,\ 220 - 115.6 = 104.4\,.$$

Squared deviations:

$$(-65.6)^2 = 4{,}303.36,\ (-55.6)^2 = 3{,}091.36,\ (-7.6)^2 = 57.76,\ 4.4^2 = 19.36,\ 104.4^2 = 10{,}899.36\,.$$

$$\text{Sum of squared deviations} = 4{,}303.36 + 3{,}091.36 + 57.76 + 19.36 + 10{,}899.36 = 18{,}371.2\,.$$

$$\text{Variance (for population)} = \frac{18{,}371.2}{5} = 3{,}674.24\,.$$

$$\text{Standard Deviation} = \sigma = \sqrt{\text{Variance}} = \sqrt{3{,}674.24} \approx 60.62\,.$$

7.

Two-way Table:

	Like Tea	Dislike Tea	**Total**
Like Coffee	20	20	40
Dislike Coffee	25	10	35
Total	45	30	75

$$\text{Like tea but not coffee} = 45 - 20 = 25.$$

$$\text{Like only coffee} = 40 - 20 = 20.$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$\text{Probability of liking only coffee} = \frac{20}{75} = \frac{20 \div 5}{75 \div 5} = \frac{4}{15} = 0.26\dot{6} = 26.\dot{6}\,\%.$$

Additional Notes for Teachers:

Learning Outcomes:
Students should be able to calculate probabilities for combined events, understanding the difference between independent and dependent events, and how to use probability trees and conditional probability.

Teaching Strategies:
Use games or simulations to demonstrate probability concepts. Discuss how conditional probability relates to everyday scenarios like sports or weather forecasting.

Assessment:
Evaluate through problems involving multiple events, where students must decide whether events are independent or dependent and apply the appropriate probability rules.

Resources:
Physical or digital tools for probability simulations, probability tree diagram generators, or real-world data for more engaging scenarios.

This set of questions aligns with the Australian Curriculum for Year 9, focusing on developing a detailed understanding of combined event probabilities in statistics and probability.