Integers, Powers, and Prime Numbers 7

Free and always will be!

Instructions: Solutions are in low contrast light blue. Print one copy in full colour, then photocopy in black and white, solutions should disappear. If not, turn contrast down on the photocopier.

Focus: A set of questions and solutions for Year 7 students focused on the topic of Integers, tailored to the Australian Curriculum:

1. Basic Concepts of Integers

a) Define integers and list four examples.

Solution:
Integers are whole numbers that can be positive, negative, or zero.
Examples: -8, -1, 0, 10.

b) Sketch a simple number line and place the integers -3, 0, and 4 on it.

Solution:
Draw a horizontal line with zero at the centre. Place -3 to the left of zero, 0 at the centre, and 4 to the right of zero.

2. Comparison of Integers

a) Order these integers from highest to lowest: $-2, 5, -7, 0, 3$.

Solution:
From highest to lowest: 5, 3, 0, -2, -7.

b) True or False: -6 **is less than** -4.

Solution:
True, -6 is further to the left on the number line than -4.

3. Operations with Integers

a) Perform the addition: $-4 + 6$.

Solution:
-4 + 6 = 2 (Moving 6 steps right from -4 on the number line).

b) Subtract: $-9 - 5$.

Solution:
-9 - 5 = -14 (Moving 5 steps left from -9).

c) Find the product of -3 **and** -7.

Solution:
-3 × -7 = 21 (The product of two negatives is positive).

d) Divide -15 **by** 3.

Solution:
-15 ÷ 3 = -5 (A negative divided by a positive gives a negative result).

4. Word Problems Involving Integers

a) The temperature increased by 3 **degrees from** $-5^{\circ}C$**. What is the new temperature?**

Solution:
New temperature: -5 + 3 = -2°C.

b) A diver starts at -10 **metres below sea level and then descends another** 7 **metres. How deep is the diver now?**

Solution:
New depth: -10 - 7 = -17 metres.

5. Integer Patterns

Identify the next two numbers in the pattern: $-1, 2, -3, 4, -5, \ldots$

Solution:
The pattern alternates between positive and negative numbers increasing by 1 each time: 6, -7.

6. Integer Properties

Why does the product of two negative numbers yield a positive number?

Solution:
When multiplying two negatives, you can consider it as reversing the direction twice on the number ,line, which brings you back to the positive side.

7. Practical Application

A football team's score decreases by 7 points in the first quarter and then increases by 3 points in the second. If they started at 0, what is their score now?

Solution:
Score after two quarters: 0 - 7 + 3 = -4.

8. Do you know?

These are known as Roman numerals: I, II, III, IV, V, VI, VII, VIII, IX, X **.**
What are these set of numerals known as? $1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10$.

Solution:
Arabic numerals

9. Rounding and Estimation

In a school canteen, a student estimates the total cost of their lunch to budget their pocket money. A sandwich costs $\$3.49$, a juice costs $\$1.50$, and a fruit cup costs $\$0.95$.

a) Round each item's cost to the nearest dollar and estimate the total cost.

Solution:

a) Rounding and Estimation

Sandwich :

$\$3.49 \approx \$3$ (since $0.49 < 0.5$, round down).

Juice :

$\$1.50 \approx \$2$ (since $0.50 \geq 0.5$, round up).

Fruit cup:

$\$0.95 \approx \$1$ (since $0.95 > 0.5$, round up).

Estimated total:

$\$3 + \$2 + \$1 = \$6$.

b) Calculate the actual total cost and compare it to your estimate. Was your estimate reasonable? Explain.

Solution:

b) Actual Total and Comparison

Actual total:

$\$3.49 + \$1.50 + \$0.95$

$\$3.49 + \$1.50 = \$4.99$

$\$4.99 + \$0.95 = \$5.94$

Difference:

$\$6 - \$5.94 = \$0.06$.

Reasonableness: The estimate of $\$6$ is very close to the actual cost $\$5.94$, differing by only $\$0.06$. This is reasonable because rounding to the nearest dollar simplifies calculations for quick budgeting, and the small difference shows the estimate is practical for planning purposes. Rounding up most items slightly overestimated the total, which is safer for budgeting to avoid overspending.

10. Integers as products of powers of prime numbers

A student is organising a coding club and needs to understand number factorisation for a project. Express the number 40 as a product of powers of prime numbers using index notation. Show your working using a factor tree or division method.

11. Square Numbers and Square Roots

A student is designing a square garden bed for a school project and needs to calculate its area and side length. A square garden bed has an area of $36\,m^2$.

a) What is the side length of the garden bed?

a) Side Length for $36\,m^2$

$$\begin{aligned} \text{Area} &= \text{side}^2 \\ &= 16\,m^2 \end{aligned}$$

$$\begin{aligned} \text{Side length} &= \sqrt{36} \\ &= 6\,m \text{ (since } 6 \times 6 = 36\text{).} \end{aligned}$$

b) If the area were increased to $49\,m^2$, what would the new side length be?

b) Side Length for $49\,m^2$

$$\text{Area} = \text{side}^2 = 49\,m^2$$

$$\text{Side length} = \sqrt{49} = 7\,m \text{ (since } 7 \times 7 = 49\text{).}$$

c) Explain why 36 and 49 are square numbers and how this relates to the side lengths.

c) Explanation of Square Numbers

Square numbers are numbers that are the product of an integer multiplied by itself (e.g., $1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, 5^2 = 25$).

36 is a square number because $36 = 6^2$, and
49 is a square number because $49 = 7^2$.

In the context of the garden bed, the area is a square number because the shape is a square, so the area is the side length squared.

The side length is the square root of the area, which is an integer for square numbers (e.g., $\sqrt{36} = 6$, $\sqrt{49} = 7$). This relationship ensures the side lengths are whole numbers, making calculations straightforward.

12. Combining Estimation and Square Numbers

A student is planning a school event and needs to estimate the number of tiles for a square floor. A square floor has an area of approximately $90\, m^2$.

a) Estimate the side length of the floor by finding the square root of the nearest perfect square.

Estimate Side Length
Find perfect squares near 90 :
$9^2 = 81$, $10^2 = 100$.
90 is closer to 81 than 100 , $(100 - 90 = 10,\ 90 - 81 = 9)$.

Estimated side length:
$\sqrt{81} = 9\, m$.

b) Calculate the actual side length to one decimal place and compare it to your estimate.

Actual Side Length
Actual side length $= \sqrt{90}$

Estimate $\sqrt{90}$:
Since $9^2 = 81$, try 9.4^2 :
$9.4 \times 9.4 = 88.36$ (slightly less than 90).

Try $9.5^2 = 90.25$ (very close)
$\sqrt{90} \approx 9.5$, so to one decimal place : $9.5\, m$.

Comparison :
The estimate $9\, m$ is close to $9.5\, m$, differing by $0.5\, m$, which is reasonable for quick planning.

c) If each tile is $1\, m^2$, how many tiles are needed, and how does rounding affect this decision?

Actual Side Length
Actual side length $= \sqrt{90}$

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Comparison :
The estimate $9\, m$ is close to $9.5\, m$, differing by $0.5\, m$, which is reasonable for quick planning.

13. Patterns and Sequences

a)

I. What is the next number in the pattern: $3, 6, 12, 24, \ldots$?

II. Identify the rule for this sequence: $7, 14, 21, 28, \ldots$.

13a.

I. Each number is multiplied by 2 to get the next number ($3 \times 2 = 6$, $6 \times 2 = 12$, etc.), so the next number in the sequence is 48 .

II. Each number increases by 7 .

b) Write down the next three numbers in the sequence: $0, 1, 3, 6, 10, \ldots$ **.**

b.

The pattern follows the sum of the previous two numbers (like Fibonacci, but with different starting numbers): $1 + 3 = 4,\ 3 + 4 = 7,\ 4 + 7 = 11$. Next three numbers: $15,\ 22,\ 33$.

c) Describe the *pattern* for the sequence $1, 3, 9, 27, \ldots$ **.**

c.

Each number is multiplied by 3 to get the next one. The pattern can be described as: Multiply by 3 .

Solutions

1a.

Integers are whole numbers that can be positive, negative, or zero.
Examples: -8, -1, 0, 10.

b.

Draw a horizontal line with zero at the centre. Place -3 to the left of zero, 0 at the centre, and 4 to the right of zero.

2a.

From highest to lowest: 5, 3, 0, -2, -7.

b.

True, -6 is further to the left on the number line than -4.

3a.

-4 + 6 = 2 (Moving 6 steps right from -4 on the number line).

b.

-9 - 5 = -14 (Moving 5 steps left from -9).

c.

-3 × -7 = 21 (The product of two negatives is positive).

d.

-15 ÷ 3 = -5 (A negative divided by a positive gives a negative result).

4a.

New temperature: -5 + 3 = -2°C.

b.

New depth: -10 - 7 = -17 metres.

5.

The pattern alternates between positive and negative numbers increasing by 1 each time: 6, -7.

6.

When multiplying two negatives, you can consider it as reversing the direction twice on the number line, which brings you back to the positive side.

7.

Score after two quarters: 0 - 7 + 3 = -4 .

8.

Arabic numerals.

9a.

Rounding and Estimation

Sandwich :

$\$3.49 \approx \$3$ (since $0.49 < 0.5$, round down).

Juice :

$\$1.50 \approx \$2$ (since $0.50 \geq 0.5$, round up).

Fruit cup:

$\$0.95 \approx \$1$ (since $0.95 > 0.5$, round up).

Estimated total:

$\$3 + \$2 + \$1 = \$6$.

b.

Actual Total and Comparison

Actual total:

$\$3.49 + \$1.50 + \$0.95$

$$\$3.49 + \$1.50 = \$4.99$$
$$\$4.99 + \$0.95 = \$5.94$$

Difference:

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Reasonableness: The estimate of $\$6$ is very close to the actual cost $\$5.94$, differing by only $\$0.06$. This is reasonable because rounding to the nearest dollar simplifies calculations for quick budgeting, and the small difference shows the estimate is practical for planning purposes. Rounding up most items slightly overestimated the total, which is safer for budgeting to avoid overspending.

10.

Method: Use a factor tree to find prime factors.

Step 1, Break down 40 :

$$40 = 4 \times 10$$
$$10 = 2 \times 5$$

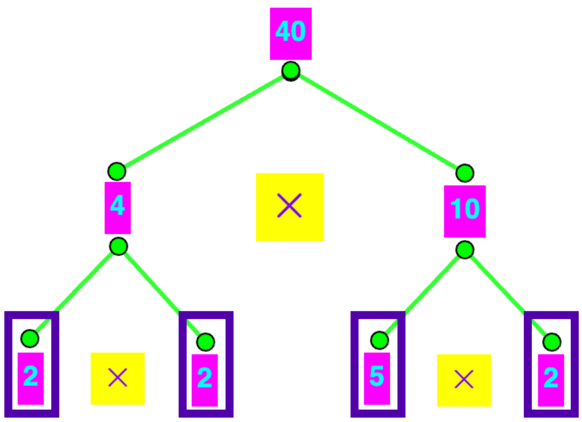
Step 2, Collect prime factors :

$$40 = 5 \times 2 \times 2 \times 2$$

Step 3, Write in index notation :

2 appears 3 times : 2^3

5 appears once : 5^1



Final answer :

$$40 = 2^3 \times 5^1$$

Verification :

$$2^3 = 8,\ 5^1 = 5$$

$8 \times 5 = 40$, confirming correctness.

11a.

Side Length for $36\,m^2$

Area $=$ side^2
$= 16\,m^2$

Side length $= \sqrt{36}$
$= 6\,m$ (since $6 \times 6 = 36$).

b.

Side Length for $49\,m^2$

Area $=$ side^2
$= 49\,m^2$

Side length $= \sqrt{49}$
$= 7\,m$ (since $7 \times 7 = 49$).

c.

Explanation of Square Numbers

Square numbers are numbers that are the product of an integer multiplied by itself (e.g., $1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, 5^2 = 25$).

36 is a square number because $36 = 6^2$, and
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In the context of the garden bed, the area is a square number because the shape is a square, so the area is the side length squared.

The side length is the square root of the area, which is an integer for square numbers (e.g.,$\sqrt{36} = 6,\ \sqrt{49} = 7$). This relationship ensures the side lengths are whole numbers, making calculations straightforward.

12a.

Estimate Side Length
Find perfect squares near 90 :
$9^2 = 81$, $10^2 = 100$.
90 is closer to 81 than 100 , $(100 - 90 = 10,\ 90 - 81 = 9)$.

Estimated side length:
$\sqrt{81} = 9\,m$.

b.

Actual Side Length

Actual side length $= \sqrt{90}$

Estimate $\sqrt{90}$:
Since $9^2 = 81$, try 9.4^2 :
$9.4 \times 9.4 = 88.36$ (slightly less than 90).

Try $9.5^2 = 90.25$ (very close)
$\sqrt{90} \approx 9.5$, so to one decimal place : $9.5\, m$.

Comparison :
The estimate $9\, m$ is close to $9.5\, m$, differing by $0.5\, m$, which is reasonable for quick planning.

c.

Number of Tiles and Rounding

Area $= 90\, m^2$, each tile $= 1\, m^2$, so 90 tiles are needed exactly.

If using the estimated side length $9\, m$, the estimated area is $9 \times 9 = 81\, m^2$, suggesting 81 tiles.

Rounding effect:
The estimate underestimates by 9 tiles. In practice, is it better to always round up and order 100 tiles, to ensures enough coverage, as it's better to have a slight surplus than a shortage when tiling a floor because tiles are inevitably broken in the building process.

13a.

I. Each number is multiplied by 2 to get the next number ($3 \times 2 = 6$, $6 \times 2 = 12$, etc.), so the next number in the sequence is 48 .
II. Each number increases by 7 .

b.

The pattern follows the sum of the previous two numbers (like Fibonacci, but with different starting numbers): $1 + 3 = 4,\ 3 + 4 = 7,\ 4 + 7 = 11$. Next three numbers: $15,\ 22,\ 33$.

c.

Each number is multiplied by 3 to get the next one. The pattern can be described as: Multiply by 3 .

Additional Notes for Teachers

Learning Outcomes:
Students should show proficiency in understanding, comparing, and operating with integers, as well as applying these concepts in real-world contexts.

Teaching Strategies:
Use visual aids like number lines for operations. Incorporate real-life contexts like weather changes or financial balances for engagement.

Assessment:
Monitor students' ability to correctly order integers, perform operations, and solve problems involving integers. Look for understanding through discussion of patterns and properties.

Resources:
Leverage interactive online tools for number line activities, or create classroom games involving integer operations.

This set of questions supports the Australian Curriculum for Year 7 Mathematics, enhancing students' skills in understanding, fluency, problem-solving, and reasoning with integers.