



Rates, Ratios, and Algebraic Expressions

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Focus: A set of questions and solutions for Year 7 students on Ratios and Rates.

1. Understanding Ratios

a) Write the ratio of 5 boys to 8 girls in its simplest form.

b) A recipe calls for 2 cups of flour for every 3 cups of sugar. What is the ratio of flour to sugar?

2. Simplifying Ratios

a) Simplify the ratio 15 : 25 .



b) If there are 6 red balls for every 10 blue balls, simplify this ratio.

3. Ratios in Different Contexts

a) If the ratio of cats to dogs in a shelter is 4 : 7 , and there are 28 dogs, how many cats are there?

b) A map scale is given as $1\text{ cm} : 5\text{ km}$. If a distance on the map measures 3 cm , what is the actual distance?



4. Rates

a) If a car travels 150 km in 3 hours , what is its average speed in km/h ?

b) A factory produces 1200 widgets in 5 days . What is the rate of production in widgets per day?

5. Comparing Rates

a) Which is the better deal: 5 apples for $\$2$ or 8 apples for $\$3$?



b) If one machine can print 100 pages in 20 minutes and another can print 150 pages in 30 minutes, which machine is faster?

6. Word Problems Involving Rates

a) If a train travels at a constant speed of 60 km/h , how far will it travel in 2.5 hours ?

b) A tap drips at a rate of 1 drop every 3 seconds . How many drops would there be in 15 minutes ?



7. Basic Concepts of Algebraic Expressions

a) Define an algebraic expression and provide two examples.

b) Write an expression for "a number multiplied by 6 and then added to 8".

8. Simplifying Expressions

a) Simplify the expression $3x + 7x - 2x$.



b) Simplify $5y - 2z + 3y + z$.

9. Evaluating Expressions

a) Evaluate $2a + 3$ **when** $a = -4$.

b) Find the value of $4p - 5$ **when** $p = 2$.

10. Expanding Expressions

a) Expand $4(2x - 3)$.



b) Expand and simplify $3(2a + b) - a$.

11. Writing Expressions from Word Problems

a) Write an expression for "the sum of a number and its square".

b) A ticket to an event costs t dollars. If there is a discount of \$5, write an expression for the cost after the discount.



12. Collecting Like Terms

a) Collect like terms in the expression $3m + 4n - 2m + 5n$.

b) Simplify $6 - 2x + 3x + 4$.

13. Algebraic Expressions in Context

a) The perimeter of a square is given by $4s$ where s is the side length. If the side length increases by 2 cm , write the new expression for the perimeter.



b) A car's fuel consumption is described by $12 + 2d$ where d is the distance in kilometres. Write an expression for the fuel consumption after traveling an additional 5 kilometres.

**Solutions****1a.**

The ratio 5 : 8 is **already in its simplest form** since 5 and 8 have no common factors other than 1 .

b.

The ratio is **2 : 3**.

2a.

Divide both parts by the greatest common divisor (GCD), which is 5 :

$$\frac{15}{5} : \frac{25}{5}$$

$$= 3 : 5.$$

b.

Simplify 6 : 10 by dividing both by their GCD, which is 2 :

$$\frac{6}{2} : \frac{10}{2}$$

$$= 3 : 5.$$

3a.

4 *cats* to 7 *dogs*, so 28 *dogs* gives:

For every 4 cats, there are 7 dogs. So, if there are 28 dogs:

$$= \frac{4 \text{ cats}}{7 \text{ dogs}} \times 28 \text{ dogs}$$

$$= \frac{4 \times 28}{7}$$

$$\text{or} = \frac{4}{7} \times 28$$

$$= 16 \text{ cats.}$$

$$\begin{array}{ccc} 4 \text{ cats} & : & 7 \text{ dogs} \\ \times ? \downarrow & & \downarrow \times ? \\ \# \text{ cats} & : & 28 \text{ dogs} \end{array}$$

OR

$$\begin{array}{ccc} 4 \text{ cats} & : & 7 \text{ dogs} \\ \times 4 \downarrow & & \downarrow \times 4 \\ 16 \text{ cats} & : & 28 \text{ dogs} \end{array}$$

$$= 4 \times 4$$

$$= 16 \text{ cats.}$$

b.

1 *cm* represents 5 *km*, so 3 *cm* represents:

$$\begin{array}{ccc} 1 \text{ cm} & : & 5 \text{ km} \\ \times 3 \downarrow & & \downarrow \times 3 \\ 3 \text{ cm} & : & 15 \text{ km} \end{array}$$

$$= 5 \times 3$$

$$= 15 \text{ km.}$$



4a.

$$\begin{aligned}\text{Average speed} &= \frac{\text{distance}}{\text{time}} \\ &= \frac{150 \text{ km}}{3 \text{ hours}} \\ &= 50 \text{ km/h}.\end{aligned}$$

b.

$$\begin{aligned}\text{Rate of production} &= \frac{1200 \text{ widgets}}{5 \text{ days}} \\ &= 240 \text{ widgets / day}.\end{aligned}$$

5a.

$$\begin{aligned}\text{Rate 1: } \frac{5 \text{ apples}}{2 \text{ dollars}} &= 2.5 \text{ apples/dollar} \\ \text{Rate 2: } \frac{8 \text{ apples}}{3 \text{ dollars}} &\approx 2.67 \text{ apples/dollar}\end{aligned}$$

The second deal (8 apples for \$3) gives more apples per dollar, so it's a better deal.

b.

$$\begin{aligned}\text{Machine 1 rate: } \frac{100 \text{ pages}}{20 \text{ minutes}} &= 5 \text{ pages / minute} \\ \text{Machine 2 rate: } \frac{150 \text{ pages}}{30 \text{ minutes}} &= 5 \text{ pages / minute}\end{aligned}$$

Both machines have the same speed.

6a.

$$\begin{aligned}\text{Distance} &= \text{Speed} \times \text{Time} \\ &= 60 \text{ km/h} \times 2.5 \text{ h} \\ &= 150 \text{ km}.\end{aligned}$$

b.

First convert minutes to seconds:

$$15 \text{ minutes} \times 60 \text{ seconds per minute} = 900 \text{ seconds}.$$

$$\text{Number of drops} \times 3 \text{ seconds / drop} = 900 \text{ seconds}$$

$$\begin{aligned}\text{Number of drops} &= \frac{900 \text{ seconds}}{3 \text{ seconds / drop}} \\ &= 300 \text{ drops}.\end{aligned}$$



7a.

An algebraic expression is a combination of numbers, variables, and operations which does not include an equality or inequality sign.

Examples:

$$4x - 3$$

$$y^2 + 5y$$

b.

If the number is n , the expression is $6n + 8$.

8a.

Combine like terms:

$$3x + 7x - 2x$$

$$= (3 + 7 - 2)x$$

$$= 8x.$$

b.

Group like terms:

$$5y + 3y - 2z + z$$

$$= 8y - z.$$

9a.

$$\text{Substitute } a = -4 : 2(-4) + 3$$

$$= -8 + 3$$

$$= -5.$$

b.

$$\text{Substitute } p = 2 : 4(2) - 5$$

$$= 8 - 5$$

$$= 3.$$

10a.

Use the distributive property: $4(2x - 3)$
 $= 8x - 12.$

$$a(b + c) = ab + ac$$

b.

First expand: $3(2a + b)$
 $= 6a + 3b.$

$$a(b + c) = ab + ac$$

Then combine with the remaining term:

$$\begin{aligned} & (6a) + 3b - a \\ & = 5a + 3b. \end{aligned}$$

11a.

If the number is x , the expression is $x + x^2$.

b.

The cost after the discount would be $t - 5$.



12a.

Group and combine: $(3m - 2m) + (4n + 5n)$
 $= m + 9n .$

b.

Combine constants and like terms: $(6 + 4) - 2x + 3x$
 $= 10 + x .$

13a.

If the new side length is $s + 2$,
 the new perimeter is $4(s + 2)$
 $= 4s + 8 .$

$$a(b + c) = ab + ac$$

b)

If the car travels an additional 5 km,
 the new distance is $d + 5$,

so the new fuel consumption is $12 + 2(d + 5)$
 $= (12 + 2d + 10)$
 $= 22 + 2d .$

$$a(b + c) = ab + ac$$



Additional Notes for Teachers

Learning Outcomes:

Students should understand how to work with ratios and rates, convert between them, and apply them in real-life scenarios. Students should master the manipulation of algebraic expressions, including writing, simplifying, and evaluating them.

Teaching Strategies:

Use visual aids like ratio tables or rate diagrams. Engage students with practical problems involving cooking recipes, travel distances, or production rates. Use practical examples like costs, dimensions, or scientific formulas to contextualise algebra. Encourage students to model real-life scenarios with expressions.

Assessment:

Evaluate through problem sets where students must calculate, compare, or apply ratios and rates. Observation of students' ability to interpret and use rates in different contexts. Test students' abilities through varied problems involving both abstract algebra and applied contexts. Focus on process as well as accuracy.

Resources:

Encourage the use of calculators for complex calculations. Use online or classroom simulations for rate problems, like speed or production scenarios. Algebra tiles or software for visualising the manipulation of expressions. Encourage group activities where students solve problems collaboratively.

This set of questions aligns with the Australian Curriculum for Year 7, enhancing students' abilities to work with and understand ratios, rates and algebraic expressions through practical application, problem-solving, and reasoning.

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