Angles, Shapes, and Transformations 7

Free and always will be!

Focus: A set of questions and solutions for Year 7 students on Angles, Shapes, and Transformations, tailored to the Australian Curriculum under the strand 'Measurement and Space':

1. Types of Angles

a) Identify the types of angles in the following set of angles: { $30^\circ, 90^\circ, 150^\circ, 180^\circ$ **}.**

b) What type of angle is formed by two lines that intersect to form a 175° **angle?**

2. Angle Relationships

a) If two angles are supplementary and one angle is 120°, what is the measure of the other angle?

b) Two angles are complementary. If one angle measures 35°, what is the measure of the other?

3. Angles in Triangles

a) Find the missing angle in a triangle where the known angles are 50° and 60° .

b) If one angle in an isosceles triangle is 40°, what are the other two angles if the triangle has two equal sides?

4. Properties of Shapes

a) How many sides does a pentagon have? What's the sum of its interior angles?

b) Identify the properties of a rectangle that distinguish it from a general parallelogram.

5. Practical Applications

a) A road sign is shaped like an equilateral triangle. If one angle is 60°, what are the other angles?

b) You're designing a kite. If two angles are right angles, what could be the sum of the other two angles?

6. Symmetry

a) How many lines of symmetry does a regular hexagon have?

b) Does a scalene triangle have any lines of symmetry?

7. Translation

a) Describe what happens when point $A(3,2)$ is translated 4 units right and 1 unit down.

b) Translate the triangle with vertices at $(1,1)$, $(1,4)$, and $(4,1)$, 2 units left and 3 units up. What are the new coordinates?

8. Reflection

a) Reflect the point $(2 , 3)$ over the x-axis.

b) Reflect the line segment from $(1, 2)$ to $(3, 2)$ over the y-axis.

9. Rotation

a) Rotate point $B(4, 0)$, 90 degrees counterclockwise around the origin.

b) Describe the image of the point $(2, -1)$ after a 180-degree rotation about the origin.

10. Enlargement (Dilation)

a) Enlarge the point $(3, 3)$ by a scale factor of 2 centred at the origin.

b) If a rectangle with vertices at $(0, 0), (2, 0), (2, 1), (0, 1)$ is dilated by a factor of 3 with the centre of dilation at $(0, 0)$, what are the new vertices?

11. Combined Transformations

a) What are the coordinates of point $P(1, 1)$ after reflecting it across the x-axis and then translating it 2 units to the right?

b) Describe the transformation that takes point $Q(-2, 3)$ to $Q'(2, -3)$.

Solutions

1a.

30° - Acute angle (less than 90°)
90° - Right angle (exactly 90°)
150° - Obtuse angle (greater than 90° but less than 180°)
180° - Straight angle (exactly 180°)

b.

This is an obtuse angle since it's greater than 90° but less than 180° .

2a.

Supplementary angles add up to 180°.
Therefore, the other angle is:

$$180^\circ - 120^\circ = 60^\circ.$$

b.

Complementary angles sum to 90°.
Thus, the other angle is:

$$90^\circ - 35^\circ = 55^\circ.$$

3a.

The sum of angles in a triangle is 180° .
Therefore, the missing angle is:

$$180^\circ - 50^\circ - 60^\circ = 70^\circ$$

b.

Since it's isosceles, the two equal angles must be the same. If one angle is 40°, and the sum of angles is 180°:
Let the two equal angles be x . Then:

$$40^\circ + x + x = 180^\circ$$
$$\cancel{40^\circ} \cancel{-40^\circ} + 2x = 180^\circ - 40^\circ$$
$$\frac{\cancel{2}x}{\cancel{2}} = \frac{140^\circ}{2}$$
$$x = 70^\circ$$

So, the angles are:
40°, 70°, and 70° .

4a.

A pentagon has 5 sides.
The sum of the interior angles of a polygon with n sides is $(n-2) \times 180^\circ$:

Sum of angles in a pentagon:

$$= (n-2) \times 180^\circ$$
$$= (5-2) \times 180^\circ$$
$$= 3 \times 180^\circ$$
$$= 540^\circ.$$

b.

A rectangle has:

All angles are right angles (90°).
Opposite sides are equal in length and parallel.
Diagonals are equal in length and bisect each other.

5a.

Since it's equilateral, all angles are equal, so the other angles are also 60°.

b.

Kites have two pairs of adjacent sides equal. With two right angles (90° each), the sum of the other two angles must be:

$$360^\circ - 90^\circ - 90^\circ$$
$$= 180^\circ.$$

6a.

A regular hexagon has 6 lines of symmetry.

b.

No, a scalene triangle has no lines of symmetry because all sides and angles are different.

7a.

New coordinates:

x starts at 3 and increases by 4:
$3 + 4 = 7$

y starts at 2 and decreases by 1:
$2 - 1 = 1$

Therefore, point A moves to $A'(7,1)$.

b.

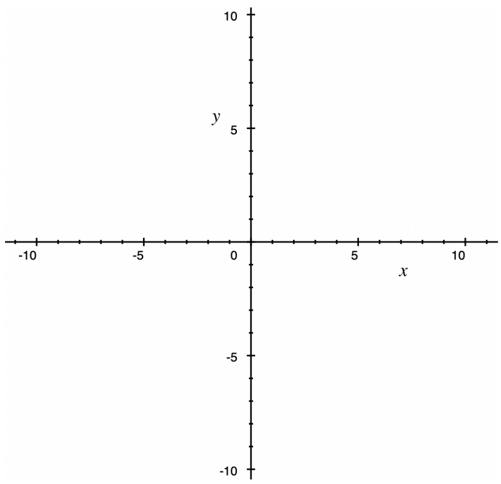
For each vertex:

Remember:

(**x** , **y**)
(left (–) $\leftrightarrow$ right (+) **,** up (+) $\leftrightarrow$ down (–)**)**

$$(1,1) \rightarrow (1-2,\ 1+3) = (-1,4)$$
$$(1,4) \rightarrow (1-2,\ 4+3) = (-1,7)$$
$$(4,1) \rightarrow (4-2,\ 1+3) = (2,4)$$

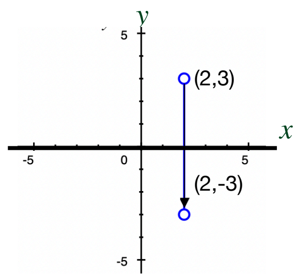
New vertices: $(-1,4)$, $(-1,7)$, $(2,4)$.



8a.

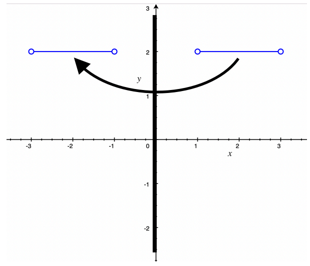
Remember for a reflection in the x-axis, the x-coordinate stays the same, but the y-coordinate changes sign: $(x, y) \rightarrow (x,\ -y)$.

$(2, 3) \rightarrow (2, -3)$.



b.

Remember for a reflection in the y-axis, the x-coordinates change sign, but the y-coordinates remain the same: $(x, y) \to (-x, y)$.



$$(1, 2) \to (-1, 2)$$
$$(3, 2) \to (-3, 2)$$

New line segment: from $(-1, 2)$ to $(-3, 2)$.

9a.

Rotaion Coordinate Rules (around the origin)

90° counterclockwise or 270° clockwise
$(x, y) \to (-y, x)$
180° clockwise or 180° counterclockwise
$(x, y) \to (-x, -y)$
90° clockwise or 270° counterclockwise
$(x, y) \to (y, -x)$

A 90-degree counterclockwise rotation around the origin swaps the coordinates with the x-coordinate becoming the y-coordinate with a negative sign:

$$B(4, 0) \to B'(-0, 4)$$
$$= B(4, 0) \to B'(0, 4)$$

b.

A 180-degree rotation changes both coordinates to their negatives:

$$(2, -1) \to (-2, 1).$$

10a.

Multiply both coordinates by the scale factor:

$$(3, 3) \to (3 \times 2, 3 \times 2)$$
$$= (6, 6).$$

b.

Multiply each coordinate by 3:

$$(0, 0) \to (0, 0)$$
$$(2, 0) \to (6, 0)$$
$$(2, 1) \to (6, 3)$$
$$(0, 1) \to (0, 3)$$

11a.

Reflect across the x-axis: $(1, 1) \to (1, -1)$
Translate 2 units right: $(1, -1) \to (3, -1)$

Final coordinates: $(3, -1)$.

b.

This involves a reflection over the y-axis (which changes the x-coordinate to its opposite) followed by a reflection over the x-axis (which changes the y-coordinate to its opposite) **OR** a 180° clockwise rotation around the origin:

Reflection over y-axis: $(-2, 3) \to (2, 3)$,
Reflection over x-axis: $(2, 3) \to (2, -3)$.

Additional Notes for Teachers

Learning Outcomes:
Students should identify and understand different types of angles, their relationships, and the properties of basic geometric shapes. Students should understand how to perform basic transformations like translation, reflection, rotation, and dilation on points and shapes.

Teaching Strategies:
Use physical models or digital tools for interactive exploration of angles and shapes. Encourage students to draw and measure angles to see relationships firsthand. Use physical manipulatives or digital software for students to see transformations in action. Encourage drawing transformations on graph paper to visualise changes.

Assessment:
Evaluate through tasks that involve recognising angles, calculating unknown angles, and identifying properties of shapes in varied contexts. Assess through exercises where students describe or perform transformations, identify types of transformations based on results, and solve problems involving combined transformations.

Resources:
Use protractors for angle measurement, geometric software for creating and exploring shapes, or art projects where students create symmetrical patterns. Graph paper for manual plotting, transformation software or apps for interactive learning, and physical objects like mirrors for demonstrating reflection.

This set of questions aligns with the Australian Curriculum for Year 7, enhancing students' understanding of angles, geometric shapes, and transformations through both theoretical and applied learning.