



# Statistics, and Probability of Combined Events

# 9

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**Focus:** A set of questions and solutions for Year 9 students on Statistics, and Probability of Combined Events, tailored to the Australian Curriculum under the strand 'Statistics and Probability':

## 1. Independent Events:

a) If you roll a die and then flip a coin, what is the probability of rolling a 4 and flipping heads?

b) Two cards are drawn from a standard deck without replacement. What's the probability of drawing two aces?



## 2. Dependent Events:

a) A bag contains 5 red balls and 3 blue balls. One ball is drawn, not replaced, then another ball is drawn. What is the probability of drawing two red balls?

b) If you have a box with 4 black pens and 6 blue pens, what is the probability of picking a blue pen followed by a black pen without replacement?



### 3. Using Probability Trees:

a) Construct a probability tree for the scenario of flipping two coins and determine the probability of getting at least one head.

A large, empty rectangular box with a thin black border, intended for the student to draw a probability tree for the scenario of flipping two coins.



**b) Use a probability tree to find the probability of getting two red balls when drawing two balls from a bag containing 2 red, 3 blue, and 1 green ball, with replacement.**

A large, empty rectangular box with a light gray border, intended for the student to draw a probability tree diagram.



#### 4. Real-World Applications:

a) There's a 70 % chance of rain on Saturday and a 60 % chance on Sunday. What is the probability that it rains on both days?

b) A tennis player has a 60 % chance of winning a set on grass and an 80 % chance on clay. If he plays one set on each surface, what's the probability he wins both sets?



## 5. Conditional Probability:

In a class, 40 % of students play football and 30 % play both football and basketball. What is the probability that a student plays basketball given they play football?

## 6. Standard Deviation

a) What does a small standard deviation tell you about a dataset?



**b) Given the dataset: { 50, 60, 108, 120, 240 } , calculate the standard deviation.**



## 7. Two-Way Tables

A survey results in a two-way table showing that out of 75 people, 45 like tea ( $T$ ), 40 like coffee ( $C$ ), and 20 like both ( $T$  and  $C$ ). How many like tea but not coffee? What is the probability that a randomly selected person likes only coffee? Use a two-way table to solve the problem.





## Solutions

1a.

$$P(\text{Event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}} = P(A) = \frac{n(A)}{n(S)}$$

$$\text{Probability of rolling a 4 on a die} = \frac{1}{6},$$

$$\text{Probability of flipping heads on a coin} = \frac{1}{2}.$$

Since these events are independent, the combined probability is :

$$\begin{aligned} P(A \text{ and } B) &= P(A \cap B) \\ &= P(A) \times P(B). \end{aligned}$$

$$\begin{aligned} P(4 \text{ and Heads}) &= \frac{1}{6} \times \frac{1}{2} \\ &= \frac{1}{12} \approx 0.08\dot{3} \approx 8.\dot{3}\%. \end{aligned}$$

b.

$$P(\text{Event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}} = P(A) = \frac{n(A)}{n(S)}$$

$$\begin{aligned} \text{Probability of drawing an ace first} &= \frac{4}{52} \\ &= \frac{4 \div 4}{52 \div 4} \\ &= \frac{1}{13}. \end{aligned}$$

After drawing one ace, 3 aces remain out of 51 cards:

$$\text{Probability of drawing a second ace} = \frac{3}{51}.$$

Combined probability (since the events are dependent):

$$\text{Probability of } A \times \text{Probability of } B \text{ given } A = P(A) \times P(B|A).$$

$$\begin{aligned} P(\text{two aces}) &= \frac{1}{13} \times \frac{3}{51} \\ &= \frac{1 \times 3}{13 \times 51} \\ &= \frac{3 \div 3}{663 \div 3} \\ &= \frac{1}{221} \approx 0.0045 \approx 0.45\%. \end{aligned}$$



2a.

$$P(\text{Event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}} = P(A) = \frac{n(A)}{n(S)}$$

Combined probability (since the events are dependent):

$$\text{Probability of } A \times \text{Probability of } B \text{ given } A = P(A) \times P(B|A).$$

$$\text{Probability of drawing a red ball first} = \frac{5}{8}.$$

With one red ball removed, there are now 4 red balls out of 7 :

$$\text{Probability of drawing a second red ball} = \frac{4}{7}.$$

Combined probability:

$$\begin{aligned} P(\text{two red}) &= \frac{5}{8} \times \frac{4}{7} \\ &= \frac{5 \times 4}{8 \times 7} \\ &= \frac{20}{56} \\ &= \frac{20 \div 4}{56 \div 4} \\ &= \frac{5}{14} \approx 0.357 \approx 35.7\%. \end{aligned}$$



b.

$$P(\text{Event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}} = P(A) = \frac{n(A)}{n(S)}$$

Combined probability (since the events are dependent):

$$\text{Probability of } A \times \text{Probability of } B \text{ given } A = P(A) \times P(B|A).$$

$$\begin{aligned} \text{Probability of picking a blue pen first} &= \frac{6}{10} \\ &= \frac{6 \div 2}{10 \div 2} \\ &= \frac{3}{5}. \end{aligned}$$

After picking one blue pen, there are 5 blue and 4 black pens left:

$$\text{Probability of picking a black pen next} = \frac{4}{9}.$$

Combined probability:

$$\begin{aligned} P(\text{blue then black}) &= \frac{3}{5} \times \frac{4}{9} \\ &= \frac{3 \times 4}{5 \times 9} \\ &= \frac{12}{45} \\ &= \frac{12 \div 3}{45 \div 3} \\ &= \frac{4}{15} = 0.26\dot{6} = 26.\dot{6} \% . \end{aligned}$$



3a.

$$P(A) = \frac{n(A)}{n(S)}$$

For each coin flip, there are two outcomes:

*Heads (H) or Tails (T).*

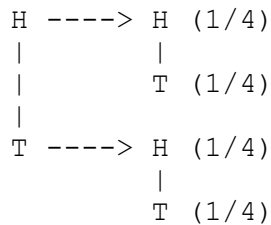
First flip:

$$P(H) = \frac{1}{2} \text{ or } P(T) = \frac{1}{2},$$

Second flip:

$$P(H) = \frac{1}{2} \text{ or } P(T) = \frac{1}{2} \text{ for each outcome of the first flip.}$$

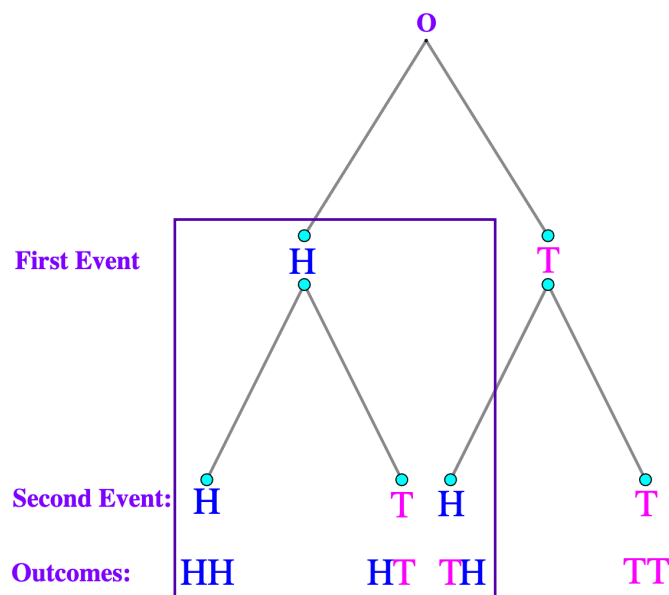
Here's the tree:



The scenarios for at least one head are:

*HH, HT, TH, ~~TT~~*

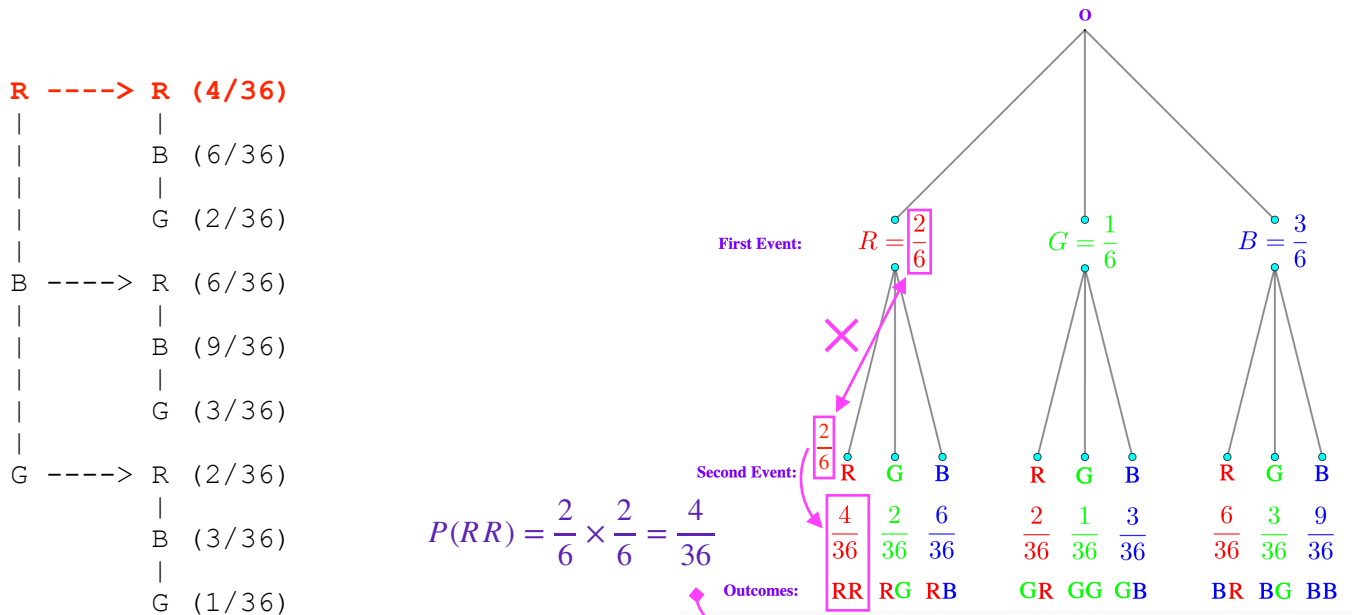
$$\begin{aligned}
 \text{Total probability} &= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \\
 &= \frac{1 + 1 + 1}{4} \\
 &= \frac{3}{4} = 0.75 = 75\%.
 \end{aligned}$$





b.

For each draw, there are 6 possible outcomes (since we replace the ball after each draw):



$$P(\text{Event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}} = P(A) = \frac{n(A)}{n(S)}$$

Combined probability (since the events are independent):

$$\text{Probability of } A \times \text{Probability of } B = P(A) \times P(B).$$

Probability of drawing two red balls with replacement:

$$\begin{aligned} P(RR) &= P(R) \times P(R) = \frac{2}{6} \times \frac{2}{6} \\ &= \frac{2 \times 2}{6 \times 6} \\ &= \frac{4}{36} \\ &= \frac{4 \div 4}{36 \div 4} \\ &= \frac{1}{9} \approx 0.11\bar{1} \approx 11.\bar{1} \% \end{aligned}$$



4a.

Combined probability (assuming the events are independent):

$$\text{Probability of } A \times \text{Probability of } B = P(A) \times P(B).$$

$$70\% = \frac{70}{100} = 0.7,$$

$$60\% = \frac{60}{100} = \frac{3}{5} = 0.6.$$

Assuming these probabilities are independent:

$$\begin{aligned} P(\text{Rain on both}) &= 0.70 \times 0.60 \\ &= 0.42 = 42\%. \end{aligned}$$

b.

Combined probability (assuming the events are independent):

$$\text{Probability of } A \times \text{Probability of } B = P(A) \times P(B).$$

$$60\% = \frac{60}{100} = \frac{3}{5} = 0.6,$$

$$80\% = \frac{80}{100} = \frac{4}{5} = 0.8.$$

Since the outcomes of the matches on different surfaces are independent:

$$\begin{aligned} P(\text{Winning both}) &= 0.60 \times 0.80 \\ &= 0.48 = 48\%. \end{aligned}$$

5.

$$40\% = \frac{40}{100} = \frac{2}{5} = 0.4,$$

$$30\% = \frac{30}{100} = 0.3.$$

Conditional probability formula:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Probability of playing basketball given playing football, is conditional:

$$\begin{aligned} P(\text{Basketball} | \text{Football}) &= \frac{P(\text{Both Football and Basketball})}{P(\text{Football})} \\ &= \frac{0.30}{0.40} \\ &= \frac{3}{4} = 0.75 = 75\%. \end{aligned}$$

**6a.**

A small standard deviation indicates that the data points are condensed around a small range of values, suggesting low variability or dispersion from the mean. This means the data is highly consistent or predictable.

**b.**

$$\{ 50, 60, 108, 120, 240 \}$$

$$\begin{aligned} \text{Mean} &= \frac{\sum_{i=1}^n x_i}{n} \\ &= \frac{\text{Sum of scores}}{\text{Number of Scores}} \end{aligned}$$

$$\begin{aligned} \text{Mean} = \mu &= \frac{50 + 60 + 108 + 120 + 240}{5} \\ &= \frac{578}{5} \\ \mu &= 115.6. \end{aligned}$$

$$\begin{aligned} \text{Variance (for population)} &= \sum_{i=1}^n \frac{(x_i - \mu)^2}{n} \\ &= \frac{(50 - 115.6)^2 + (60 - 115.6)^2 + (108 - 115.6)^2 + (120 - 115.6)^2 + (240 - 115.6)^2}{5} \end{aligned}$$

Deviations from the mean:

$$50 - 115.6 = -65.6, 60 - 115.6 = -55.6, 108 - 115.6 = -7.6, 120 - 115.6 = 4.4, 240 - 115.6 = 104.4.$$

Squared deviations:

$$(-65.6)^2 = 4,303.36, (-55.6)^2 = 3,091.36, (-7.6)^2 = 57.76, 4.4^2 = 19.36, 104.4^2 = 10,899.36.$$

$$\begin{aligned} \text{Sum of squared deviations} &= 4,303.36 + 3,091.36 + 57.76 + 19.36 + 10,899.36 \\ &= 18,371.2. \end{aligned}$$

$$\begin{aligned} \text{Variance (for population)} &= \frac{18,371.2}{5} \\ &= 3,674.24. \end{aligned}$$

$$\begin{aligned} \text{Standard Deviation} = \sigma &= \sqrt{\text{Variance}} = \sqrt{3,674.24} \\ &\approx 60.62. \end{aligned}$$



7.

**Two-way Table:**

	Like Tea	Dislike Tea	Total
Like Coffee	20	20	40
Dislike Coffee	25	10	35
Total	45	30	75

$$\begin{aligned}\text{Like tea but not coffee} &= 45 - 20 \\ &= 25.\end{aligned}$$

$$\begin{aligned}\text{Like only coffee} &= 40 - 20 \\ &= 20.\end{aligned}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$\begin{aligned}\text{Probability of liking only coffee} &= \frac{20}{75} \\ &= \frac{20 \div 5}{75 \div 5} \\ &= \frac{4}{15} = 0.26\dot{6} = 26.\dot{6} \%. \end{aligned}$$





## Additional Notes for Teachers:

### Learning Outcomes:

Students should be able to calculate probabilities for combined events, understanding the difference between independent and dependent events, and how to use probability trees and conditional probability.

### Teaching Strategies:

Use games or simulations to demonstrate probability concepts. Discuss how conditional probability relates to everyday scenarios like sports or weather forecasting.

### Assessment:

Evaluate through problems involving multiple events, where students must decide whether events are independent or dependent and apply the appropriate probability rules.

### Resources:

Physical or digital tools for probability simulations, probability tree diagram generators, or real-world data for more engaging scenarios.

This set of questions aligns with the Australian Curriculum for Year 9, focusing on developing a detailed understanding of combined event probabilities in statistics and probability.

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