UNIT TEST #022 | Australian Curriculum Mathematics | UTM9T3U6

Trigonometry, and Transformations

9 Unit Test

ACACIA TUTORING

Free and always will be!

Instructions: Read all questions carefully to ensure you understand what is being asked. When completing your official tests / exams, your grade will be based upon your: **understanding, fluency, reasoning, and problem solving,** so ensure you show all lines of working and draw accurate, labelled diagrams where necessary. (ACiQ|9.0 Mathematics standard elaborations found on final page (general assessment marking standards)). [Practise tests are marked out of a score of 10]. For multiple choice questions, tick or fill in the circle next to the corresponding letter under the question.

Check your work if you have time. *Remember:* you don't have to start at question one, it's always best to firstly look through the test, highlight the easy looking questions and complete them first, then secondly, go back through and work on the harder questions. Good luck! And remember to breathe!

$$\sum = \frac{\quad}{10} = \quad \%$$

Part 1: Multiple Choice (2 marks)

Question 1:

a) In a right-angled triangle, the side opposite the right angle is called:

A. The hypotenuse **B.** The adjacent **C.** The opposite **D.** The base

○ A ○ B ○ C ○ D

Space for question 1a…

b) Which equation below represents the sine rule?

A. $\frac{a}{\sin A} = \frac{b}{\sin B}$ **B.** $\frac{a}{\sin A} = \frac{b}{\sin C}$ **C.** $\frac{c}{\sin A} = \frac{c}{\sin B}$ **D.** $\frac{c}{\sin C} = \frac{b}{\sin C}$

○ **A** ○ **B** ○ **C** ○ **D**

Space for question 1b...

Question 2:

a) If $\sin(\theta) = \frac{3}{5}$ **, what is** $\cos(\theta)$ **in a right-angled triangle where the hypotenuse is** 5 **?**

A. $\frac{3}{5}$ **B.** $\frac{4}{5}$ **C.** $\frac{5}{3}$ **D.** $\frac{5}{4}$

○ **A** ○ **B** ○ **C** ○ **D**

Space for question 2a...

b) What happens to the coordinates of point (x, y) **when it is rotated** 90° **about the origin?**

A. $(x, y) \to (x, -y)$ **B.** $(x, y) \to (y, -x)$ **C.** $(x, y) \to (y, x)$ **D.** $(x, y) \to (-y, x)$

○ **A** ○ **B** ○ **C** ○ **D**

Space for question 2b...

Part 2: Short Answer (4 marks)

Question 3:

a) Define the $sine$, $cosine$, and $tangent$ of an angle in a right-angled triangle.

b) In triangle ABC, $AC = 10\,cm$, $BC = 8\,cm$, and $\angle BCA = 25^\circ$. Find the length of AB using the cosine rule $c^2 = a^2 + b^2 - 2ab\cos(C)$.

Question 4:

a) Calculate $\tan(\theta)$ **for a right-angled triangle where the opposite side is** $4\,cm$ **and the adjacent side is** $3\,cm$**.**

b) Area Rule: $Area = \frac{1}{2}ab\sin(C)$

Find the area of a triangle with side lengths $a = 5$**,** $b = 5$**, and the angle between** a **and** b **is** 20°

Part 3: Problem Solving (4 marks)

Question 5:

a) In a right-angled triangle, one angle is $30°$, and the hypotenuse is $10\,cm$. Find the lengths of the other two sides.

b) A landscape architect in Cairns is designing a pavilion for a community park. The pavilion's floor plan includes a rectangular signboard represented on a Cartesian plane, with vertices at $A(1, 1)$, $B(5, 1)$, $C(5, 3)$, and $D(1, 3)$. To fit the pavilion's orientation with the park's layout, the architect plans to apply a rotation of 90° anticlockwise about the origin.

Determine the coordinates of the vertices of the signboard after a 90° anticlockwise rotation about the origin. Show your working and verify that the shape remains congruent.

Question 6:

a) A ladder is leaning against a wall at an angle of 60° with the ground. If the base of the ladder is $4\,metres$ from the wall, how long is the ladder?

b) A renewable energy company in Brisbane is planning a solar panel installation on a rectangular rooftop. The panel's position is represented on a Cartesian plane with vertices at $E(2, 2)$, $F(6, 2)$, $G(6, 4)$, and $H(2, 4)$. To optimise sunlight exposure, the company considers reflecting the panel's position over the line $y = 1$.

I) Find the coordinates of the vertices of the panel after reflecting over the line $y = 1$.
II) If the reflected panel is then dilated by a scale factor of 1.5 with the origin as the centre, calculate the coordinates of the new vertices. Explain how these transformations help optimise the solar panel's placement and efficiency.

Solutions

1a. (0.5 marks)

A. The hypotenuse.

- This is the longest side in a right-angled triangle, opposite the $90 - degree$ angle.

b. (0.5 marks)

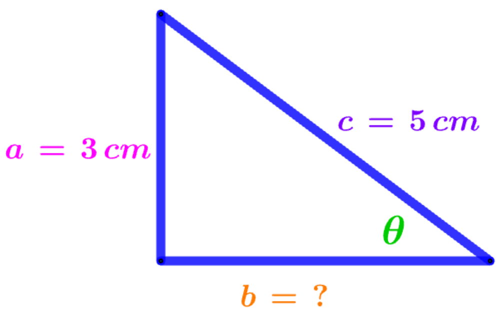
A. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$.

2a. (0.5 marks)

B. $\dfrac{4}{5}$.

Using the Pythagorean identity, $a^2 + b^2 = c^2$, with $\sin(\theta) = \dfrac{3}{5} = \dfrac{opposite}{hypotenuse}$

$\rightarrow a = 3$ and $c = 5$, we need to find $b\ (adjacent)$:



$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
\rightarrow 3^2 + b^2 &= 5^2 \\
9 + b^2 &= 25 \\
\cancel{9} \cancel{-9} + b^2 &= 25 - 9 \\
b^2 &= 25 - 9 \\
b^2 &= 16 \\
\cancel{\sqrt{b^2}} &= \sqrt{16} \\
b &= \sqrt{16} \\
b &= 4 .
\end{aligned}
$$

Thus,

$$\cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{4}{5} .$$

b. (0.5 marks)

D. $(x,\ y) \rightarrow (-y,\ x)$

3a. (1 mark)

Sine (sin) of an angle θ is the ratio of the length of the opposite side to the hypotenuse :

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

Cosine (cos) of an angle θ is the ratio of the length of the adjacent side to the hypotenuse :

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Tangent (tan) of an angle θ is the ratio of the length of the opposite side to the adjacent side:

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}.$$

b. (1 mark)

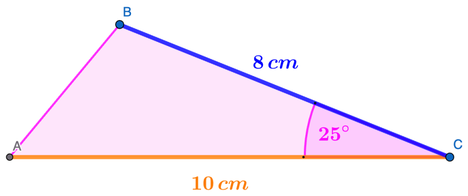
Here, $AB = c,$

$b = 10,$

$a = 8,$

and $\cos(25^\circ) \approx 0.9063$

Cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$



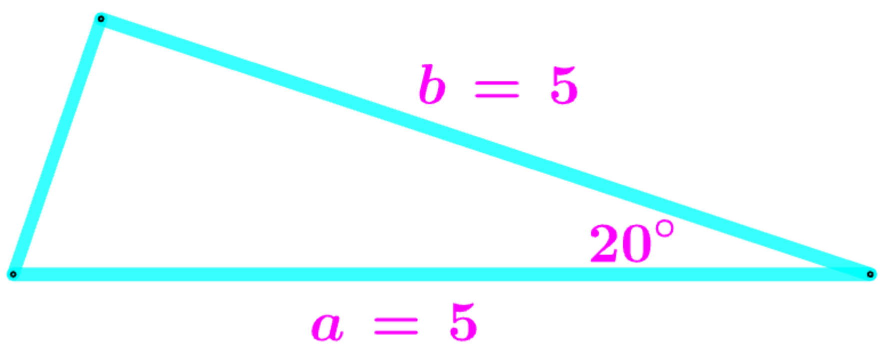
$$\begin{aligned} c^2 &\approx 8^2 + 10^2 - 2 \times 8 \times 10 \times 0.9063 \\ &\approx 64 + 100 - 16 \times 9.063 \\ &\approx 164 - 145 \\ c^2 &\approx 19 \\ \cancel{\sqrt{c^2}} &\approx \sqrt{19} \\ c = AB &\approx 4.36\,cm\,. \end{aligned}$$

4a. (1 mark)

$$\begin{aligned} \tan(\theta) &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{4}{3}. \end{aligned}$$

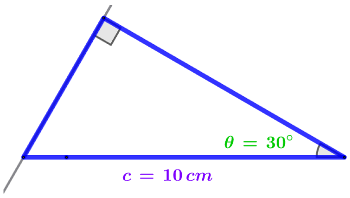
b. (1 mark)

$$\begin{aligned} Area &= \frac{1}{2}ab\sin(C) \\ &= \frac{1}{2} \cdot 5 \cdot 5 \cdot \sin(20) \\ &= 12.5\sin(20) \\ &\approx 4.28\ units\,. \end{aligned}$$



5a. (0.5 marks)

For a 30° angle in a right-angled triangle:



$$\sin(30^{\circ}) = \frac{opposite}{hypotenuse} = \frac{1}{2},$$

$$\frac{opposite}{10} = \frac{1}{2},$$

$$\frac{opposite}{\cancel{10}} \times \cancel{10} = \frac{1}{2} \times 10,$$

$$opposite = \frac{1}{2} \times 10$$

$$= 5\,cm\,.$$

$$\cos(30^{\circ}) = \frac{adjacent}{hypotenuse} = \frac{\sqrt{3}}{2},$$

$$\frac{adjacent}{10} = \frac{\sqrt{3}}{2},$$

$$\frac{adjacent}{\cancel{10}} \times \cancel{10} = \frac{\sqrt{3}}{2} \times 10,$$

$$adjacent = \frac{\sqrt{3}}{2} \times 10$$

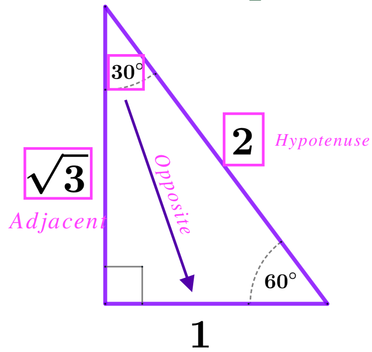
$$= \frac{\sqrt{3} \times 10}{2}$$

$$adjacent = \sqrt{3} \times 5$$

$$= 5\sqrt{3}\,cm\,.$$

$$\cos(\theta) = \frac{Adj.}{Hyp.}$$

$$\cos(30^{\circ}) = \frac{\sqrt{3}}{2}$$



b. (1.5 marks)

A 90° anticlockwise rotation about the origin transforms a point (x, y) to $(-y, x)$.
Calculate the new coordinates for each vertex:

$A(1, 1) \rightarrow (-1, 1) = A'$.
$B(5, 1) \rightarrow (-1, 5) = B'$.
$C(5, 3) \rightarrow (-3, 5) = C'$.
$D(1, 3) \rightarrow (-3, 1) = D'$.

To confirm the shape remains congruent, check that the side lengths (and angles) are preserved using the formula for distance between two points $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$:

Original side AB :

$$\text{Distance from } A(1, 1) \text{ to } B(5, 1) = \sqrt{(5-1)^2 + (1-1)^2} = \sqrt{4^2} = 4.$$

Rotated side $A'B'$:

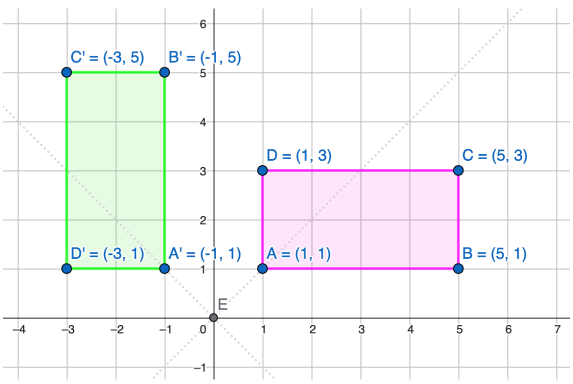
$$\text{Distance from } A'(-1, 1) \text{ to } B'(-1, 5) = \sqrt{(-1-(-1))^2 + (5-1)^2} = \sqrt{0 + 4^2} = 4.$$

Original side BC :

$$\text{Distance from } B(5, 1) \text{ to } C(5, 3) = \sqrt{(5-5)^2 + (3-1)^2} = \sqrt{2^2} = 2.$$

Rotated side $B'C'$:

$$\text{Distance from } B'(-1, 5) \text{ to } C'(-3, 5) = \sqrt{(-3-(-1))^2 + (5-5)^2} = \sqrt{(-2)^2} = 2.$$



Similarly, sides CD and DA match $C'D'$ and $D'A'$.
Angles remain 90° since the rectangle's structure is preserved (rotations preserve angles).
Thus, the rotated shape $A'B'C'D'$ is congruent to $ABCD$.

6a. (0.5 marks)

Using the cosine ratio for the 60° angle (adjacent/hypotenuse):

$$\cos(60^\circ) = \frac{adjacent}{hypotenuse} = \frac{4}{\text{length of ladder}}$$

$$\text{And, } \cos(60^\circ) = \frac{1}{2}:$$

$$\frac{1}{2} = \frac{4}{\text{length of ladder}}$$

$$\frac{1}{2} \times (\text{ Length of Ladder }) = \frac{4}{\cancel{\text{length of ladder}}} \times \cancel{(\text{ Length of Ladder })}$$

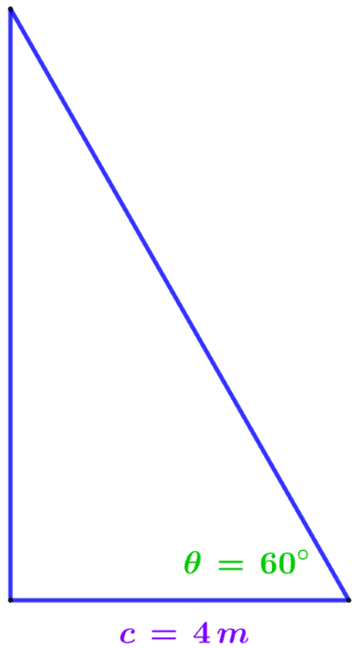
$$\frac{1}{2} \times \text{Length of Ladder} = 4$$

$$\frac{1 \times \text{Length of Ladder}}{2} = 4$$

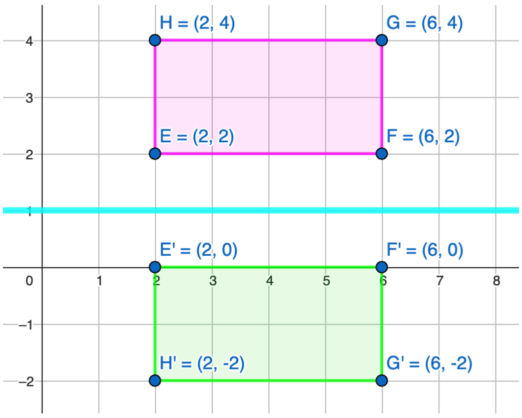
$$\frac{\text{Length of Ladder}}{\cancel{2}} \times \cancel{2} = 4 \times 2$$

$$\text{Length of ladder} = 4 \times 2$$

$$= 8\,metres.$$



b. (1.5 marks)



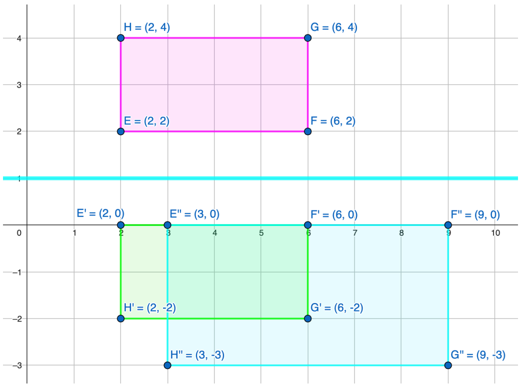
Part I: Reflection

Reflection over the line $y = 1$ transforms a point (x, y) to $(x, 2 - y)$
the x-position remains the same, and the y-position reflects over the line ,
as the line $y = 1$ is horizontal, and the $y - coordinate$ changes based on its distance from $y = 1$.

Calculate the new coordinates :

$E(2, 2) : y = 2$, so new $y - coordinate = 2 - 2 = 0$, giving $E'(2, 0)$.
$F(6, 2) : y = 2$, so new $y - coordinate = 2 - 2 = 0$, giving $F'(6, 0)$.
$G(6, 4) : y = 4$, so new $y - coordinate = 2 - 4 = -2$, giving $G'(6, -2)$.
$H(2, 4) : y = 4$, so new $y - coordinate = 2 - 4 = -2$, giving $H'(2, -2)$.

Part II: Dilation



Dilation with a scale factor of 1.5 from the origin multiplies each coordinate by 1.5, transforming (x, y) to $(1.5x, 1.5y)$.

Calculate the new coordinates :

$$E'(2, 0) : (1.5 \times 2, 1.5 \times 0) = (3, 0) = E''.$$
$$F'(6, 0) : (1.5 \times 6, 1.5 \times 0) = (9, 0) = F''.$$
$$G'(6, -2) : (1.5 \times 6, 1.5 \times -2) = (9, -3) = G''.$$
$$H'(2, -2) : (1.5 \times 2, 1.5 \times -2) = (3, -3) = H''.$$

The reflection over $y = 1$ repositions the panel to a part of the roof with better sunlight exposure, perhaps aligning it to face the sun's path in Brisbane's northern orientation. The dilation increases the panel's size by a factor of 1.5, covering a larger area to capture more solar energy, thus improving efficiency. The reflection ensures the panel remains a rectangle, preserving its structural integrity, while the dilation scales it proportionally, maintaining compatibility with the rooftop's dimensions and maximising energy output without altering the panel's shape.

$$\sum = \frac{\quad}{10} = \quad \%$$

General Assessment Marking Standards

Remember: When your official tests are marked, they won't be a score out of 10, they will be a grade (A,B,C,D,E) based on the following standards:

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Year 9 Mathematics standard elaborations

		A	B	C	D	E
		The folio of student work contains evidence of the following:				
Mathematical proficiencies	Understanding	accurate and consistent identification, representation, description and connection of mathematical concepts and relationships in complex unfamiliar, complex familiar, and simple familiar situations	accurate identification, representation, description and connection of mathematical concepts and relationships in complex familiar and simple familiar situations	identification, representation, description and connection of mathematical concepts and relationships in simple familiar situations	partial identification, representation and description of mathematical concepts and relationships in some simple familiar situations	fragmented identification, representation and description of mathematical concepts and relationships in isolated and obvious situations
	Fluency	choice, use and application of comprehensive facts, definitions, and procedures to find solutions in complex unfamiliar, complex familiar, and simple familiar situations	choice, use and application of effective facts, definitions, and procedures to find solutions in complex familiar and simple familiar situations	choice, use and application of facts, definitions, and procedures to find solutions in simple familiar situations	choice and use of partial facts, definitions, and procedures to find solutions in some simple familiar situations	choice and use of fragmented facts, definitions and procedures to find solutions in isolated and obvious situations
	Reasoning	comprehensive explanation of mathematical thinking, strategies used, and conclusions reached in complex unfamiliar, complex familiar, and simple familiar situations	detailed explanation of mathematical thinking, strategies used, and conclusions reached in complex familiar and simple familiar situations	explanation of mathematical thinking, strategies used, and conclusions reached in simple familiar situations	partial explanation of mathematical thinking, strategies used, and conclusions reached in some simple familiar situations	fragmented explanation of mathematical thinking, strategies used, and conclusions reached in isolated and obvious situations
	Problem-solving	purposeful use of problem-solving approaches to find solutions to problems.	effective use of problem-solving approaches to find solutions to problems.	use of problem-solving approaches to find solutions to problems.	partial use of problem-solving approaches to make progress towards finding solutions to problems.	fragmented use of problem-solving approaches to make progress towards finding solutions to problems.

Key shading emphasises the qualities that discriminate between the A–E descriptors