Integers, Powers, and Prime Numbers 7

Free and always will be!

Focus: A set of questions and solutions for Year 7 students focused on the topic of Integers, tailored to the Australian Curriculum:

1. Basic Concepts of Integers:

a) What are integers? Give three examples.

Solution:
Integers are whole numbers, including positive numbers, negative numbers, and zero. Examples: $-5, 0, 7$.

b) How are integers represented on a number line?

Solution:
Integers are represented on a number line with zero in the middle, negative numbers to the left (decreasing towards -∞), and positive numbers to the right (increasing towards +∞).

2. Comparison of Integers:

a) Arrange the following integers in ascending order: $-4,\ 3, -1,\ 0,\ 6$.

Solution:
Ascending order: $-4, -1, 0, 3, 6$.

b) Which integer is greater, -5 or -3?

Solution:
-3 is greater than -5 because -3 is closer to zero on the number line.

3. Operations with Integers:

a) Add: $-3+5$.

Solution:
$-3+5=2$ (Move 5 steps to the right from -3).

b) Subtract: $8-12$.

Solution:
$8-12=-4$ (Move 12 steps to the left from 8).

c) Multiply: -2×4.

Rules for $\times$ and $\div$, If signs are: $\begin{cases}\text{opposite} \rightarrow \text{change to } - \\ \text{same} \rightarrow \text{change to } + \end{cases}$

Solution:
$-2\times 4=-8$ (A negative times a positive is negative).

d) Divide: $-10\div 2$. **Rules for multiplying (and dividing) positive and negative numbers =**

If signs are:
Opposite
$(+\times - = -)$
$(-\times + = -)$
Same
$(+\times + = +)$
$(-\times - = +)$

Solution:
$-10\div 2=-5$ (A negative divided by a positive is negative).

4. Word Problems Involving Integers:

a) The temperature was $2^{\circ}C$ at midday but dropped by 7 degrees by midnight. What was the temperature at midnight?

Solution:
Temperature at midnight: $2-7=-5^{\circ}C$.

b) A submarine is at $-50\,metres$ below sea level. It ascends $20\ metres$. What is its new depth?

Solution:
New depth of the submarine:
$-50+20=-30\,metres$.

5. Integer Patterns:

Generate the next three numbers in the sequence: $-3,-1,1,3,\ \ldots$

Solution:
The pattern increases by 2 each time: $5,7,9$.

6. Integer Properties:

Explain why the sum of two negative integers results in a negative integer.

Solution:
When you add two negative numbers, you are essentially moving left on the number line (in the negative direction) twice, which always results in a position further to the left (or more negative) than either starting point.

7. Practical Application:

A golf score is calculated where going below par (negative score) is good. If a player starts at 0 and shoots -2 on the first hole and -1 on the second, what is their score after two holes?

Solution:
Golf score after two holes: $0 + (-2) + (-1) = -3$.

8. Rounding and Estimation

In a school canteen, a student estimates the total cost of their lunch to budget their pocket money. A sandwich costs $\$4.85$, a juice costs $\$2.95$, and a fruit cup costs $\$1.75$.

a) Round each item's cost to the nearest dollar and estimate the total cost.
b) Calculate the actual total cost and compare it to your estimate. Was your estimate reasonable? Explain.

Solution:
a) Rounding and Estimation
Sandwich :
$\$4.85 \approx \$5$ (since $0.85 > 0.5$, round up).
Juice :
$\$2.95 \approx \$3$ (since $0.95 > 0.5$, round up).
Fruit cup:
$\$1.75 \approx \$2$ (since $0.75 > 0.5$, round up).
Estimated total:
$\$5 + \$3 + \$2 = \$10$.

b) Actual Total and Comparison
Actual total:
$\$4.85 + \$2.95 + \$1.75$

$$\$4.85 + \$2.95 = \$7.80$$
$$\$7.80 + \$1.75 = \$9.55$$

Difference:
$\$10 - \$9.55 = \$0.45$.

Reasonableness: The estimate of $\$10$ is close to the actual cost $\$9.55$, differing by only $\$0.45$. This is reasonable because rounding to the nearest dollar simplifies calculations for quick budgeting, and the small difference shows the estimate is practical for planning purposes. Rounding up each item slightly overestimated the total, which is safer for budgeting to avoid overspending.

9. Integers as products of powers of prime numbers

A student is organising a coding club and needs to understand number factorisation for a project. Express the number 72 as a product of powers of prime numbers using index notation. Show your working using a factor tree or division method.

Solution:
Method: Use a factor tree to find prime factors.

Step 1, Break down 72 :

$$72 = 2 \times 36$$
$$36 = 2 \times 18$$
$$18 = 2 \times 9$$
$$9 = 3 \times 3$$

Step 2, Collect prime factors :

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

Step 3, Write in index notation :

2 appears 3 times : 2^3

3 appears 2 times : 3^2

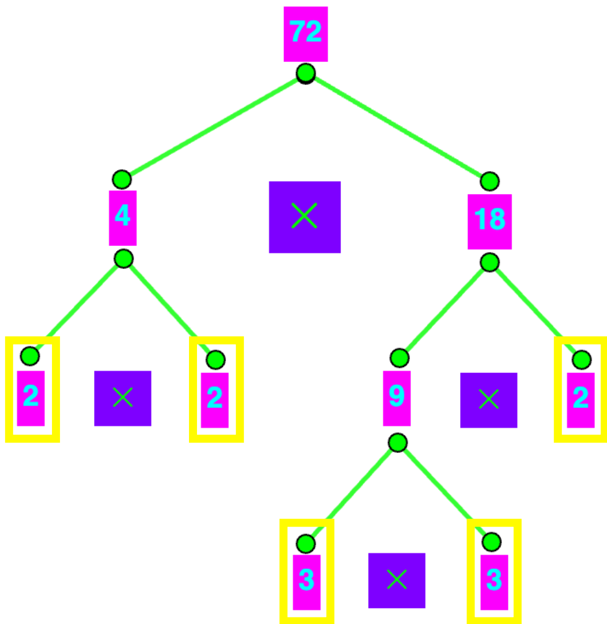
Final answer :

$$72 = 2^3 \times 3^2$$

Verification :

$$2^3 = 8,\ 3^2 = 9$$

$8 \times 9 = 72$, confirming correctness.



10. Square Numbers and Square Roots

A student is designing a square garden bed for a school project and needs to calculate its area and side length. A square garden bed has an area of $16\,m^2$.

a) What is the side length of the garden bed?
b) If the area were increased to $25\,m^2$, what would the new side length be?
c) Explain why 16 and 25 are square numbers and how this relates to the side lengths.

Solution:

a) Side Length for $16\,m^2$

$$\begin{aligned}\text{Area} &= \text{side}^2\\ &= 16\,m^2\end{aligned}$$

$$\begin{aligned}\text{Side length} &= \sqrt{16}\\ &= 4\,m \text{ (since } 4 \times 4 = 16).\end{aligned}$$

b) Side Length for $25\,m^2$

$$\begin{aligned}\text{Area} &= \text{side}^2\\ &= 25\,m^2\end{aligned}$$

$$\begin{aligned}\text{Side length} &= \sqrt{25}\\ &= 5\,m \text{ (since } 5 \times 5 = 25).\end{aligned}$$

c) Explanation of Square Numbers

Square numbers are numbers that are the product of an integer multiplied by itself (e.g., $1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, 5^2 = 25$).

16 is a square number because $16 = 4^2$, and
25 is a square number because $25 = 5^2$.

In the context of the garden bed, the area is a square number because the shape is a square, so the area is the side length squared.

The side length is the square root of the area, which is an integer for square numbers (e.g., $\sqrt{16} = 4$, $\sqrt{25} = 5$). This relationship ensures the side lengths are whole numbers, making calculations straightforward.

11. Combining Estimation and Square Numbers

A student is planning a school event and needs to estimate the number of tiles for a square floor. A square floor has an area of approximately $99\,m^2$.

a) Estimate the side length of the floor by finding the square root of the nearest perfect square.
b) Calculate the actual side length to one decimal place and compare it to your estimate.
c) If each tile is $1\,m^2$, how many tiles are needed, and how does rounding affect this decision?

Solution:
a) Estimate Side Length
Find perfect squares near 99 :
$9^2 = 81$, $10^2 = 100$.
100 is closer to 99 than 81 , $(100 - 99 = 1,\ 99 - 81 = 18)$.

Estimated side length:
$\sqrt{100} = 10\,m$.

b) Actual Side Length
Actual side length $= \sqrt{99}$

Estimate $\sqrt{99}$:
Since $10^2 = 100$, try 9.9^2 :
$9.9 \times 9.9 = 98.01$ (slightly less than 99).

Try $9.95^2 \approx 99.0025$ (very close)
$\sqrt{99} \approx 9.95$, so to one decimal place : $9.9\,m$.

Comparison :
The estimate $10\,m$ is close to $9.9\,m$, differing by $0.1\,m$, which is reasonable for quick planning.

c) Number of Tiles and Rounding
Area $= 99\,m^2$, each tile $= 1\,m^2$, so 99 tiles are needed exactly.

If using the estimated side length $10\,m$, the estimated area is $10 \times 10 = 100\,m^2$, suggesting 100 tiles.

Rounding effect:
The estimate overestimates by 1 tile. In practice, ordering 100 tiles ensures enough coverage, as it's better to have a slight surplus than a shortage when tiling a floor.

12. Patterns and Sequences

a)

I. What is the next number in the pattern: $2, 4, 8, 16, \dots$?

II. Identify the rule for this sequence: $5, 10, 15, 20, \dots$.

Solution:

I. Each number is multiplied by 2 to get the next number ($2 \times 2 = 4$, $4 \times 2 = 8$, etc.), so the next number in the sequence is 32 .

II. Each number increases by 5 .

b) Write down the next three numbers in the sequence: $1, 3, 6, 10,$ **… .**

Solution:

The pattern follows the sum of the previous two numbers (like Fibonacci, but with different starting numbers): $1 + 3 = 4,\ 3 + 4 = 7,\ 4 + 7 = 11$. Next three numbers: $15, 22, 33$.

c) Describe the *pattern* for the sequence $3, 6, 12, 24,$ **… .**

Solution:

Each number is multiplied by 2 to get the next one. The pattern can be described as: Multiply by 2 .

d) Consider the sequence $10, 7, 4, 1,$ **… :**

I. Write the next three terms.

II. Describe the *rule (equation)* for this sequence in words.

Solution:

I. Next three terms:

The pattern decreases by 3 each time:

$$1 - 3 = -2$$
$$-2 - 3 = -5$$
$$-5 - 3 = -8$$

Hence, the next three terms are: $-2, -5, -8$.

II. Rule: Subtract 3 from the previous number.

e) A pattern starts with 2 **and each subsequent term is obtained by adding** 3 **to the previous term.**

I. Write the first 5 terms of this sequence and then,

II. Write an algebraic *expression* for the nth term.

Solution:

I. First 5 terms: $2, 5, 8, 11, 14$.

II. Algebraic expression: The starting number is 2, and we add 3 for each step, so the nth term can be expressed as:

$$T_n = 2 + (n - 1) \times 3$$
$$= 3n - 1 .$$

Additional Notes for Teachers:

Learning Outcomes: Students should demonstrate an understanding of integers, including their place on the number line, comparisons, operations, and how they apply to real-life scenarios.

Teaching Strategies: Use number lines for visual representation of integers and operations. Engage students with practical examples like temperatures, financial gains/losses, or game scores. Encourage problem-solving by asking students to create their own integer questions.

Assessment: Observe students' ability to order, compare, and perform operations with integers. Look for conceptual understanding through their explanations of patterns and properties. Use these questions to assess understanding of number properties, fluency in calculations, and ability to apply concepts in context. Refer to QCAA's "Sequence of content descriptions: Years 7–10 – Mathematics" () and ACARA's Australian Curriculum V9.0 () for detailed content and achievement standards. Use the QCAA P–10 Planning app for creating aligned assessment tasks ().

Resources: Utilise digital tools or physical number lines, and consider games or interactive activities that involve moving along a number line to add or subtract integers.

This question set aligns with the Australian Curriculum for Year 7, focusing on the key proficiencies of understanding, fluency, problem-solving, and reasoning in the context of integers.