WORKSHEET # 003 | Australian Curriculum Year 7 Mathematics | WM7T2U3

Formulas, and Finance 7

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Focus: A set of questions and solutions for Year 7 students focused on the topic of Formulas, and Finance, tailored to the Australian Curriculum:

1. Financial planning with percentages.

A student is saving for a new bicycle as part of a budgeting project. A bicycle costs $\$240$. A store offers a $15\,\%$ discount during a sale.

a) Calculate the discount amount and the sale price of the bicycle.

Solution:

Discount Amount and Sale Price

$$\begin{aligned} \text{Discount} &= 15\,\% \text{ of } \$240 \\ 15\,\% &= 0.15 \\ \text{Discount} &= 0.15 \times \$240 \\ &= \$36\,, \end{aligned}$$

$$\begin{aligned} \text{Sale price} &= \text{Original price} - \text{Discount} \\ \text{Sale price} &= \$240 - \$36 \\ &= \$204 \end{aligned}$$

b) The student has saved $\$180$. How much more do they need to save to buy the bicycle at the sale price?

Additional Savings Needed

$$\begin{aligned} \text{Student's savings} &= \$180 \\ \text{Sale price} &= \$204 \\ \text{Amount needed} &= \$204 - \$180 \\ &= \$24 \end{aligned}$$

The student needs to save an additional $\$24$.

c) Justify whether waiting for the sale is a good financial decision compared to buying the bicycle now at full price.

Solution: Justification of Waiting for the Sale
Buying now at full price costs $\$240$, while the sale price is $\$204$, saving $\$36$. Waiting for the sale is a good financial decision because it reduces the cost by $15\,\%$, allowing the student to afford the bicycle with less money. Since the student has $\$180$, they only need $\$24$ more for the sale price, compared to $\$60$ more for the full price. This smaller savings goal is more achievable and reduces financial strain. However, the student should ensure the sale is happening soon to avoid missing out or needing the bicycle urgently.

2. Using Formulas in a Financial Context

A student is organising a school fundraiser and needs to calculate costs using a formula. The cost of producing custom wristbands for a fundraiser is given by the formula $C = 50 + 2n$, where C is the total cost in dollars and n is the number of wristbands.

a) Calculate the cost of producing 75 wristbands.

Solution: Cost for 75 Wristbands
Formula :
$$C = 50 + 2n$$

Substitute $n = 75$:
$$\begin{aligned} C &= 50 + 2 \times 75 \\ C &= 50 + 150 \\ &= 200 \end{aligned}$$

Cost $= \$200$.

b) If the fundraiser has a budget of $\$200$, how many wristbands can they afford to produce?

Solution: Number of Wristbands for $\$200$ Budget
Budget: $C = 200$
Formula: $200 = 50 + 2n$

Solve for n :

Subtract 50 :
$$200 - 50 = \cancel{50} + 2n - \cancel{50}$$
$$150 = 2n$$

Divide by 2 :
$$\frac{150}{2} = \frac{\cancel{2}n}{\cancel{2}}$$
$$75 = n$$
$$n = 75$$
They can afford 75 wristbands.

Verification:
Substitute $n = 75$ into the formula :
$$\begin{aligned} C &= 50 + 2 \times 75 \\ &= 50 + 150 \\ &= 200 \text{, which matches the budget.} \end{aligned}$$

c) Explain why the formula $C = 50 + 2n$ is appropriate for this context.

Solution: Explanation of the Formula
The formula $C = 50 + 2n$ is appropriate because it models the total cost of producing wristbands. The fixed cost of $\$50$ likely represents a setup fee (e.g., design or machine setup), which is incurred regardless of the number of wristbands. The variable cost, $2n$, represents the cost per wristband of $\$2$ each, which depends on the number of wristbands n . This linear relationship accurately reflects costs in fundraising scenarios where there's an initial cost plus a per-item cost, making it suitable for budgeting and planning.

3. Financial Decision with Algebraic Modelling

A student is comparing two options for earning pocket money to save for a game console. A student can earn pocket money by mowing lawns. Option A pays $\$10$ per lawn. Option B pays a $\$20$ base fee plus $\$5$ per lawn.

a) Write algebraic expressions for the total earnings from each option, where n is the number of lawns mowed.

Solution: Algebraic Expressions

$$\text{Option A}:$$
$$\$10 \text{ per lawn, so earnings } = 10n\ .$$
$$\text{Option B}:$$
$$\$20 \text{ base fee } + \$5 \text{ per lawn, so earnings } = 20 + 5n\ .$$

b) If the student mows 6 lawns, which option pays more, and by how much?

Solution: Earnings for 6 Lawns

$$\text{Option A}:$$
$$\begin{aligned} 10n &= 10 \times 6 \\ &= 60 \\ \text{Earnings } &= \$60 \end{aligned}$$

$$\text{Option B}:$$
$$\begin{aligned} 20 + 5n &= 20 + 5 \times 6 \\ &= 20 + 30 \\ &= 50 \\ \text{Earnings } &= \$50 \end{aligned}$$

$$\text{Comparison}:$$
$$\$60 - \$50 = \$10$$

Option A pays $\$10$ more for 6 lawns.

c) Determine the number of lawns where both options pay the same amount, and justify which option is better for mowing more than this number.

Solution: Number of Lawns for Equal Pay and Justification
Set the expressions equal :

$$10n = 20 + 5n$$

Solve for n ,

Subtract $5n$:

$$10n \; -5n = 20 + \cancel{5n} \; -\cancel{5n}$$
$$5n = 20$$

Divide by 5 :

$$\frac{\cancel{5}n}{\cancel{5}} = \frac{20}{5}$$
$$n = 20 \div 5$$
$$= 4$$

Verification , for $n = 4$

Option A :

$$10 \times 4 = 40$$

Option B :

$$20 + 5 \times 4 = 20 + 20$$
$$= 40$$

Both pay $\$40$ for 4 lawns.

Justification:
For more than 4 lawns ($n > 4$), test $n = 5$,

Option A :

$$10 \times 5 = 50$$

Option B :

$$20 + 5 \times 5 = 20 + 25$$
$$= 45 .$$

Option A pays more $\$50$ vs. $\$45$.

Option A is better for more than 4 lawns because its rate $\$10$ per lawn, is higher than Option B's $\$5$ per lawn, and the $\$20$ base fee in Option B becomes less significant as n increases. For fewer than 4 lawns, Option B may pay more due to the base fee (e.g., for $n = 1$, Option A: $\$10$, Option B: $\$25$).

4. Financial Modelling with Percentages and Formulas

A student is planning a school event and needs to manage ticket sales. A school event charges $\$8$ per ticket, but groups of 10 or more get a $10\,\%$ discount per ticket.

a) Write a formula for the total cost T of n tickets, considering the discount for groups of 10 or more, use a piecewise formula to display the final two equations.

Solution: Formula for Total Cost

For $n < 10$:

$$\text{Cost } = \$8 \text{ per ticket, so } T = 8n\,,$$

$$n = \text{ Number of Tickets Sold}$$
$$T = \text{ Total Sales in } \$'s\,.$$

For $n \geq 10$:

$$\begin{aligned}\text{Discount } &= 10\,\% \text{ of } \$8\\ &= \frac{10}{100} \times 8\\ &= \frac{1\cancel{0}}{10\cancel{0}} \times 8\\ &= \frac{1}{10} \times 8\\ &= 0.1 \times 8\\ &= \$0.80 \text{ per ticket.}\end{aligned}$$

$$\begin{aligned}\text{Discounted price per ticket } &= \$8 - \$0.80\\ &= \$7.20\end{aligned}$$

So, total cost :

$$T = 7.2n$$

Piecewise formula :

$$T = \begin{cases} 8n & \text{if } n < 10 \\ 7.2n & \text{if } n \geq 10 \end{cases}$$

b) Calculate the cost for a group of 12 students and verify the discount was applied correctly.

Solution: Cost for 12 Students
Since , $n = 12 \geq 10$, use $T = 7.2n$:

$$T = 7.2 \times 12$$
$$= 86.4$$
Cost $= \$86.40$

Verification ,
Without discount :
$$8 \times 12 = 96$$
Discount amount :
$$10\,\%\ of\ \$96 = 0.1 \times 96$$
$$= \$9.60$$
Discounted cost :
$\$96 - \$9.60 = \$86.40$, which matches.

c) If a class has $\$90$, how many tickets can they buy, and justify whether they should buy as a group or individually.

Solution: Number of Tickets for $\$90$ and Justification
Since they're a class, assume they buy as a group $n \geq 10$, so use $T = 7.2n$.

Budget $\$90$.

Solve :
$$7.2n \leq 90$$

Divide by 7.2 :
$$\frac{\cancel{7.2}n}{\cancel{7.2}} \leq \frac{90}{7.2}$$
$$n \leq 90 \div 7.2$$
$$= 12.5 .$$

Since n must be a whole number, try $n = 12$:

Cost :
$7.2 \times 12 = 86.4$, which is $\leq \$90$.

Try $n = 13$
$7.2 \times 13 = 93.6$, which exceeds $\$90$.

Maximum tickets $= 12$.

Justification: Buying as a group is better because the discounted price $\$7.20$ per ticket, allows them to buy more tickets than the full price, $\$8$.

For 12 tickets,
Group cost: $\$86.40$
Individual cost: $8 \times 12 = 96$, which exceeds $\$90$.

With $\$90$, they can only buy $90 \div 8 = 11.25$, or 11 tickets individually. The group discount maximises the number of tickets (12 vs. 11) and saves money.

5. Simple Interest

a) A savings account earns simple interest using the formula $I = PRT$, where I is the interest, P is the principal, R is the annual interest rate (as a decimal), and T is the time in years. Calculate the interest earned on $\$100$ at $3\,\%$ per year for $2\,years$.

Solution:
Simple interest :

$$I = PRT$$

Given :

$$P = 100\,,$$
$$R = 3\,\% = 0.03\,,$$
$$T = 2\,.$$

$$\begin{aligned} I &= 100 \times 0.03 \times 2 \\ &= \$6\,. \end{aligned}$$

b) A savings account earns simple interest using the formula $I = PRT$. Calculate the interest earned on $\$100$ at $3\,\%$ per year for $18\,months$, then calculate the total amount in the account using the formula $A = P + I$.

Solution:
Simple interest :

$$I = PRT$$

Given :

$$P = 100\,,$$
$$R = 3\,\% = 0.03\,,\ per\ \ year$$
$$T = 18\,months\ = 1.5\,years\,.$$ Ensure Time and Rate are in the same units i.e. years.

$$\begin{aligned} I &= 100 \times 0.03 \times 1.5 \\ &= \$4.50\,. \end{aligned}$$

$$\begin{aligned} A &= P + I \\ &= \$100 + \$4.50 \\ &= \$104.50\,. \end{aligned}$$

c) You wish to open a savings account and you have two options:

Option A: Earns simple interest at rate of $0.27\,\%$ per month for $2\,years$.
Option B: Earns simple interest at rate of $3\,\%$ per year for $2\,years$.

Calculate the interest earned on a $\$100$ deposit and decide which account you will invest your money in, ensuring you justify your choice.

Solution:

$$I = PRT$$

Option A :

Given :

$P = 100$,

$R = 0.27\% \div 100 = 0.0027$, $per\ month$

$T = 2\,years = 24\,months$. Ensure Time and Rate are in the same units i.e. months.

$$I = 100 \times 0.0027 \times 24$$
$$= \$6.48.$$

Option B :

Given :

$P = 100$,

$R = 3\% \div 100 = 0.03$, $per\ year$

$T = 2\,years$. Ensure Time and Rate are in the same units i.e. years.

$$I = 100 \times 0.03 \times 2$$
$$= \$6.00.$$

Option A is the best choice as it earns more interest, $\$6.48$ compared to $\$6.00$.

Additional Notes for Teachers:

Learning Outcomes: Students should be able to solve simple financial equations, understand and solve formulas in financial situations, and apply these concepts to real-world problems.
Teaching Strategies: Use digital tools to visualise formulas and financial concepts. Encourage students to compare and estimate costs to check the reasonableness of their calculations. Encourage students to check their solutions by substituting back into the original equation or inequality.
Assessment:
Assess students on accurate calculations, correct use of formulas, and clear justifications, aligning with QCAA's five-point scale (A–E). Emphasise reasoning in justifications to meet the curriculum's focus on interpreting solutions in context.

These questions and solutions provide a comprehensive approach to teaching and assessing Year 7 Number and Algebra topics related to finance, formulas, and justification, ensuring alignment with the Australian Curriculum V9.0 and QCAA guidelines.