WORKSHEET # 004 | Australian Curriculum Year 7 Mathematics | WM7T2U4

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Rates, Ratios, and Algebraic Expressions 7

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Focus: A set of questions and solutions for Year 7 students focused on 'Rates, Ratios, and Algebraic Expressions', tailored to the Australian Curriculum:

1. Understanding Ratios:

a) Explain what the ratio $3:2$ means.

Solution:
The ratio $3:2$ means for every 3 parts of one quantity, there are 2 parts of another quantity. For example, if you have 3 apples for every 2 oranges, the ratio of apples to oranges is $3:2$.

b) Simplify the ratio $8:12$.

Solution:
Simplify by dividing both numbers by their greatest common divisor (4)

$$8 \div 4 : 12 \div 4$$
$$= 2 : 3.$$

2. Comparing Ratios:

Is $2:3$ the same ratio as $4:6$?

Solution:
$$\frac{4 \div 2}{6 \div 2} = \frac{2}{3} \; so \; \frac{4}{6} = \frac{2}{3}$$

Yes, both ratios simplify to $2:3$.
Therefore, they are equivalent.

3. Ratios in Real Life:

If a recipe calls for flour and sugar in the ratio $5:3$, how much sugar is needed if you use 20 cups of flour?

Solution:

Since the ratio is $5:3$, for every 5 parts flour, there are 3 parts sugar.

This can be written: 3 out of 5 parts are sugar, or $\frac{3}{5}$.

$$\text{Sugar Needed} = \frac{3\ cups\ of\ sugar}{5\ \cancel{cups\ of\ flour}} \times 20\ \cancel{cups\ of\ flour}$$

$$= 12 \text{ cups of sugar}.$$

OR

Flour : Sugar

$$\times ? \left(\begin{matrix} 5:3 \\ 20:x \end{matrix} \right) \times ?$$

So $? = 4\ (20 \div 5)$

$$\times 4 \left(\begin{matrix} 5:3 \\ 20:12 \end{matrix} \right) \times 4$$

$\rightarrow$ So 12 cups of sugar are needed.

4. Rates:

a) If a car travels $150\,km$ in $3\,hours$, what is its speed in km/h?

Solution:

Speed = Distance ÷ Time

$$= \frac{150\,km}{3\,hours}\ (150 \div 3)$$

$$= 50\,km/h.$$

b) If you earn $\$45$ for working $5\ hours$, what is your hourly wage?

Solution:

Hourly wage = Earnings ÷ Hours

$$= \frac{\$45}{5\,hours} (45 \div 5)$$

$$= \$9 \text{ per } hour.$$

5. Unit Rates:

a) A $2-litre$ **bottle of juice costs** $\$3$ **. What is the cost per** $litre$ **?**

Solution:
Cost per $litre$ = Total cost ÷ Total litres

$$= \frac{\$3}{2\,L}$$

$$= \$1.5 \text{ per } litre\,.$$

b) How many $kilometres$ **can you travel on one litre of fuel if your car uses** $30\ litres$ **to go** $450\ km$ **?**

Solution:
$Kilometers$ per $litre$ = Total $kilometers$ ÷ Total $litres$

$$= \frac{450\,km}{30\,L}$$

$$= 15\,km \text{ per } litre\,.$$

6. Word Problems Involving Rates:

a) If 5 **workers can paint a fence in** $4\ hours$ **, how long will it take for** 10 **workers to paint the same fence, assuming they work at the same rate?**

Solution:
Since 5 workers take $4\ hours$, 10 workers (twice as many, or 2 times as many) would take half the time:

$$\frac{4}{2} = 2\ hours\,.$$

b) A tap fills a tank at a rate of $8\ litres$ **per** $minute$**. How long will it take to fill a** $120-litre$ **tank?**

Solution:
Time = Volume ÷ Rate

$$= \frac{120\,\mathcal{L}}{8\,\mathcal{L}/m} \quad (120 \div 8)$$

$$= 15\ minutes\,.$$

7. Ratio Conversion:

Convert the ratio $4:7$ into a percentage of the total.

Solution:

The total parts are $4+7=11$.

The percentage of the first part (4 parts) is given by:

$$\frac{4}{11}\times 100\,\%$$
$$\approx 0.3636\times 100\,\%$$
$$\approx 36.36\,\%.$$

8. Basics of Algebraic Expressions:

a) What is an algebraic expression? Give an example.

Solution:

An algebraic expression is a combination of numbers, variables, and operations without an equals sign or inequality sign.

Example: $3x+4$.

b) Identify the terms, coefficient, and constant in the expression $5y-7$.

Solution:

Terms: $5y$ ($+5y$) and -7

Coefficient: $+5$ (of y)

Constant: -7

9. Simplifying Expressions:

a) Simplify $2x+3x+5-2$.

Solution:

Combine like terms:

$$2x+3x=5x,$$
$$5-2=3,$$

Result: $5x+3$.

b) Simplify $4a - 2b + 3a - b$.

Solution:
Combine like terms:

$$4a + 3a = 7a,$$
$$-2b - b = -3b,$$

Result: $7a - 3b$.

10. Expanding Expressions:

a) Expand $2(x + 3)$.

Solution:
Use the distributive property:

$$a(b + c) = ab + ac$$

So, $2(x + 3)$
$\rightarrow 2 \cdot x + 2 \cdot 3$
$= 2x + 6.$

In mathematics we sometimes use ($\cdot$) instead of ($\times$) so we don't get multiplied mixed up with the letter x.

b) Expand $3(a - 2)$.

Solution:
Use the distributive property:

$$a(b + c) = ab + ac$$

So, $3(a - 2)$
$\rightarrow 3 \cdot a + 3 \cdot -2 \qquad (+ \times - = -)$
$= 3 \cdot a - 3 \cdot 2$
$= 3a - 6.$

11. Substituting Values:

a) If $x = 2$**, find the value of** $3x + 1$.

Solution:
Substitute $x = 2$ into the expression:

$3x + 1 = 3(2) + 1$
$= 6 + 1$
$= 7.$

b) Evaluate $y^2 - 4$ when $y = 3$.

Solution:

Substitue $y = 3$ into the expression:

$$y^2 - 4$$
$$\rightarrow 3^2 - 4$$
$$= 9 - 4$$
$$= 5.$$

12. Writing Expressions from Words:

a) Write an expression for "five more than twice a number".

Solution:

Let the number be x, the expression is $2x + 5$.

b) Write an algebraic expression for "the difference between a number and seven".

Solution:

Let the number be x, the expression is $x - 7$.

13. Real-Life Application:

a) If tickets cost $\$15$ each, write an expression for the total cost of buying n tickets.

Solution:

The expression is $15n$.

b) A shop sells cookies at $\$2$ per cookie. If you buy c cookies, write an expression for the total cost.

Solution:

The expression would be $2c$.

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14. Recognising Equivalent Expressions:

Are $3x + 2x - 1$ and $5x - 1$ equivalent?

Solution:

Yes, because $3x + 2x - 1$ simplifies to $5x - 1$.

Additional Notes for Teachers:

Learning Outcomes: Students should understand how to express, simplify, and use ratios in various contexts, calculate rates, and solve problems involving unit rates. Students should be able to write, simplify, expand, and evaluate algebraic expressions, recognising how they relate to real-world contexts.
Teaching Strategies: Use real-life scenarios like cooking recipes, speed calculations, or financial rates to make the learning tangible. Engage students with hands-on activities like scaling recipes or comparing different rates of usage (like water or electricity). Promote the use of diagrams or tables to visually represent ratios and rates. Use visual aids like algebra tiles or balance scales to show how expressions are simplified or expanded. Employ real-life scenarios to practice writing expressions from word problems. Encourage students to discuss and explore how changing one part of an expression affects the whole.

Assessment: Assess students' ability to apply ratios and rates in problem-solving, their understanding of equivalency, and their accuracy in calculations involving rates. Evaluate students on their ability to manipulate expressions, understand the role of variables, and apply algebraic concepts to practical situations. Monitor students' ability to solve equations and inequalities, interpret solutions in context, and represent inequalities graphically.

Resources: Utilise kitchen scales for ratio activities, speedometers or stopwatches for rate experiments, or digital tools for quick calculations. Use algebra games, online interactive tools for expression manipulation, or physical models to aid in understanding abstract concepts.

This question set aligns with the Australian Curriculum for Year 7, emphasising the proficiencies of understanding, fluency, problem-solving, and reasoning in the context of algebraic expressions, equations and inequalities.

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