ACACIA TUTORING

Angles, Shapes, and Transformations 7

Free and always will be!

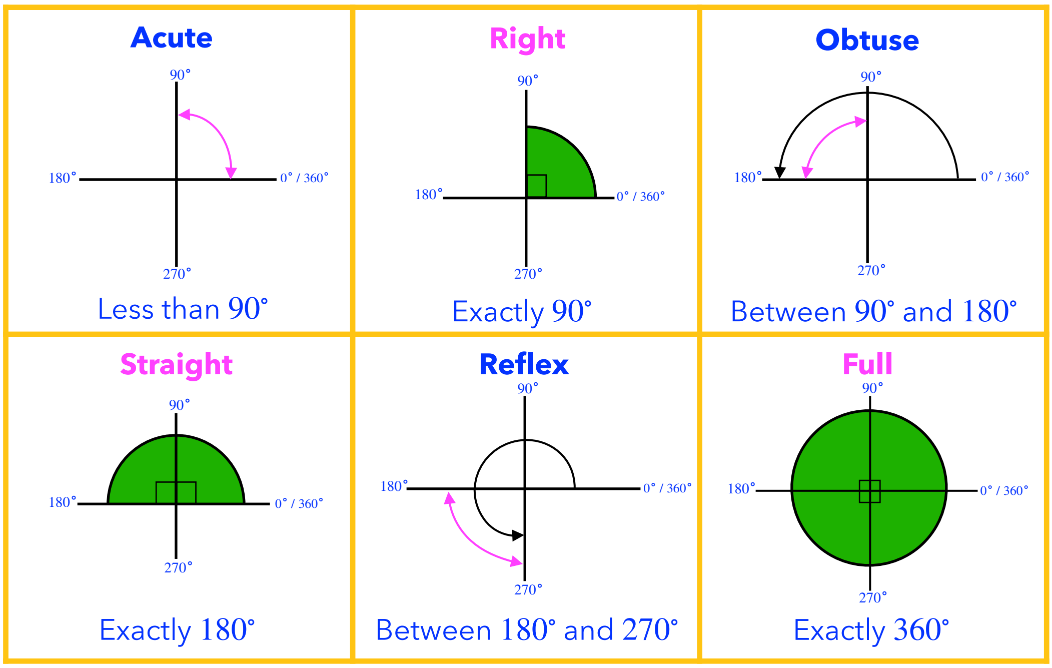
Focus: A set of questions and solutions for Year 7 students focused on 'Angles, Shapes, and Transformations' under the "Measurement and Space" strand, tailored to the Australian Curriculum:

1. Understanding Angles:

a) Define what an angle is. Describe the different types of angles (acute, right, obtuse, straight, reflex).

Solution:
An angle is formed by two rays sharing a common endpoint (vertex).
Types:

Acute: Less than 90°.
Right: Exactly 90°.
Obtuse: Between 90° and 180°.
Straight: Exactly 180°.
Reflex: More than 180° but less than 360°.



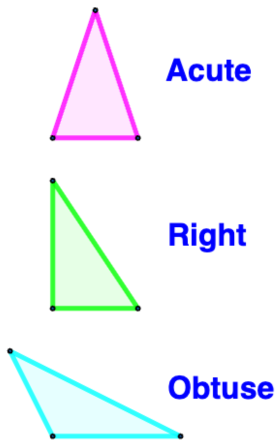
b) Identify the type of angle that measures 120°.

Solution:
120° is an obtuse angle.

2. Properties of Triangles:

a) Name and describe the three main types of triangles based on their angles.

Solution:
Acute Triangle: All angles are less than 90°.
Right Triangle: One angle is exactly 90°.
Obtuse Triangle: One angle is greater than 90°.



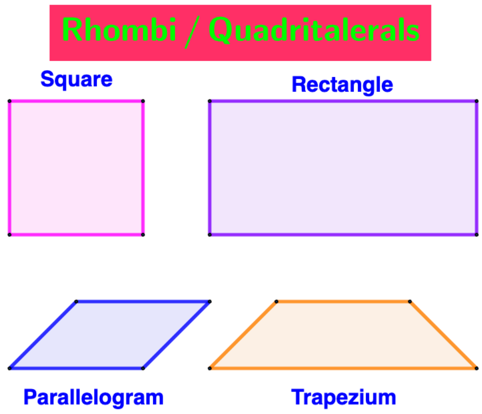
b) What is the sum of the interior angles of any triangle?

Solution:
The sum of the interior angles of a triangle is always 180°.

3. Quadrilaterals:

a) List four common types of quadrilaterals and one distinctive property for each.

Solution:
Square: All sides are equal, all angles are 90°.
Rectangle: Opposite sides are equal, all angles 90°.
Parallelogram: Opposite sides are equal and parallel.
Trapezium: At least one pair of parallel sides.



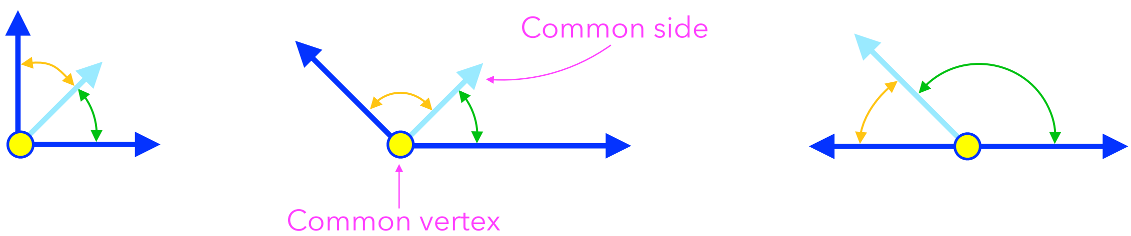
b) Calculate the sum of the interior angles of a quadrilateral.

Solution:
The sum of the interior angles of a quadrilateral is 360°.

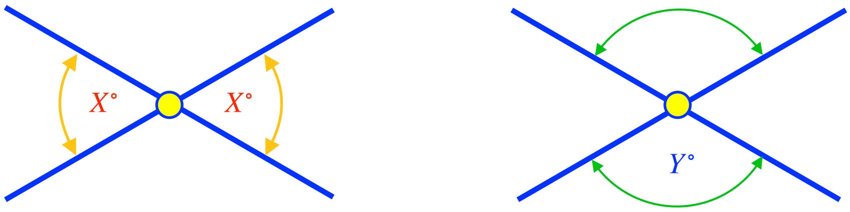
4. Angle Relationships:

a) Explain the difference between adjacent, vertically opposite, and supplementary angles.

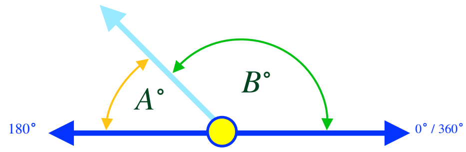
Solution:
Adjacent Angles: Share a common vertex and a common side but no interior points.



Vertically Opposite Angles: Formed by two intersecting lines, directly opposite each other and equal in measure.



Supplementary Angles: Two angles whose measures add up to 180°.



b) If one angle in a pair of supplementary angles is 55°, what is the other angle?

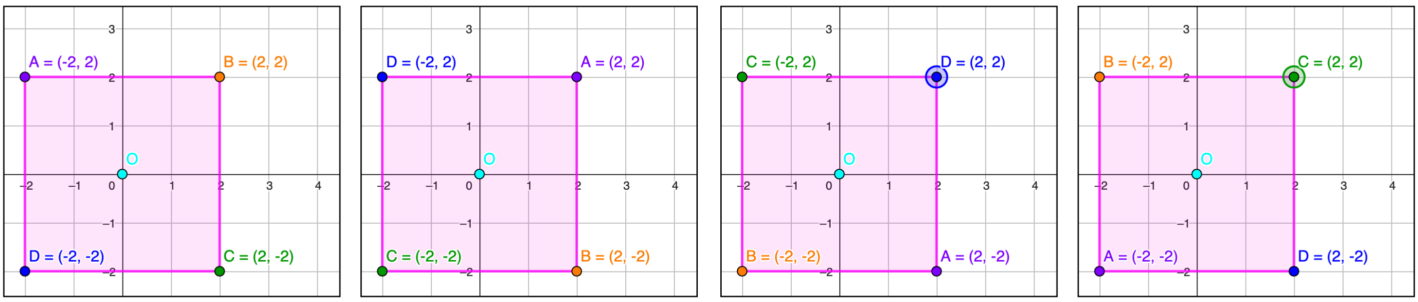
Solution:
The other angle is $180^\circ - 55^\circ = 125^\circ$.

5. Symmetry in Shapes:

a) Define line symmetry and rotational symmetry.

Solution:
Line Symmetry: A shape has line symmetry if it can be folded along a line so that one half matches the other half exactly.
Rotational Symmetry: A shape has rotational symmetry if it looks the same after some rotation (less than 360°).

Initial Image / 360° | 90° | 180° | 270°



b) How many lines of symmetry does an equilateral triangle have?

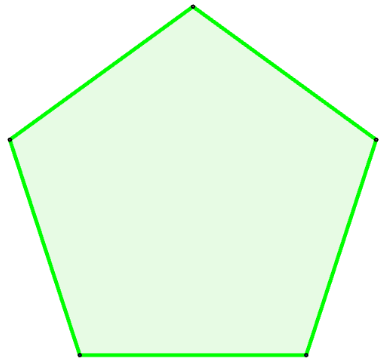
Solution:
An equilateral triangle has **3 lines of symmetry**.

6. Practical Application:

Design a floor tile pattern using only squares and equilateral triangles. Describe the symmetry in your design.

Solution:
(This can be varied; an example might be alternating squares and triangles in a grid pattern.) The pattern could have both line symmetry (along the lines where squares meet triangles) and rotational symmetry (for instance, every 120° if centred on a triangle).

7. Angles in Polygons:



Calculate the sum of the interior angles of a pentagon.

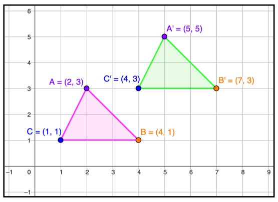
Solution:
For a polygon with n sides, the sum of interior angles is $(n-2) \times 180^\circ$.
For a pentagon ($n=5$), it's $(5-2) \times 180^\circ = 540^\circ$.

8. Understanding Transformations:

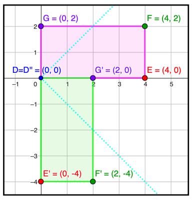
a) Define what transformations in geometry are. List four types of basic transformations.

Solution:
Transformations in geometry involve changing the position, size, or orientation of a shape. The four basic types are:

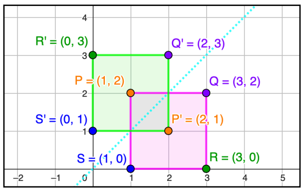
Translation (slide)



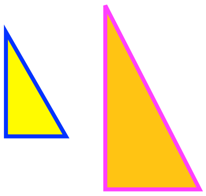
Rotation (turn)



Reflection (flip)



Dilation (resize)



b) How does each transformation affect a shape's size, shape, and orientation?

Solution:
Translation: Size and shape unchanged, orientation unchanged.
Rotation: Size and shape unchanged, orientation changed,
Reflection: Size and shape unchanged, orientation reversed,
Dilation: Size changed (enlarged or reduced), shape unchanged if uniform, orientation unchanged.

9. Translations:

a) Describe a translation by 3 units right and 2 units down.

Solution:
Every point of the shape moves 3 units to the right along the x-axis and 2 units down along the y-axis.

b) If point $A(2,4)$ is translated by $(3,-2)$, what are the coordinates of A' ?

Solution:
$$A' = (2+3, 4-2)$$
$$= (5,2)\,.$$

Rotaion Coordinate Rules (around the origin)

90° counterclockwise or 270° clockwise
$(x, y) \to (-y, x)$
180° clockwise or 180° counterclockwise
$(x, y) \to (-x, -y)$
90° clockwise or 270° counterclockwise
$(x, y) \to (y, -x)$

10. Rotations:

a) Explain how to rotate a point 90° clockwise around the origin.

Solution:
For a 90° clockwise rotation around the origin $(0,0)$, swap the x and y coordinates and then multiply the new x-coordinate by -1. So, (x, y) becomes $(-y, x)$.

b) What are the new coordinates of point $B(-3,1)$ after a 90° clockwise rotation around the origin?

Solution:
$$(x, y) \to (-y, x)$$
$$B = (-3, 1)$$
$$\to B' = (-1, -3)\,.$$

11. Reflections:

a) Describe the reflection of a shape over the x-axis.

Solution:
Each point (x, y) of the shape will move to $(x, -y)$, mirroring across the x-axis.

b) Reflect the point $C(4,-2)$ over the y-axis. What are the coordinates of C' ?

Solution:
Remember for a reflection in the y-axis,the x -coordinates change sign, but the y-coordinates remain the same:

$$C = (4, -2)$$
$$\to C' = (-4, -2)$$

12. Dilation:

a) What does it mean to dilate a shape by a scale factor of 2, centred at the origin?

Solution:
Each point (x, y) of the shape moves to $(2x, 2y)$, effectively doubling the size of the shape with respect to the origin.

b) If point $D(1, 3)$ is dilated by a scale factor of 2 from the origin, what are the new coordinates?

Solution:
Remember for a dilation, multiply each coordinate by the scale factor:

$$\begin{aligned} D &= (1, 3) \\ \rightarrow D' &= (2, 6)\,. \end{aligned}$$

13. Combining Transformations:

Describe the effect of translating a triangle 4 units right, then rotating it 180° around the origin.

Solution:
First, each point moves 4 units to the right. Then, after the rotation, each point (x, y) becomes $(-x, -y)$, but adjusted for the initial translation. E.g., if a vertex was at $(1, 2)$, it moves to $(5, 2)$ (translation), then to $(-5, -2)$ (rotation).

14. Practical Application:

Design a simple logo using only transformations on a basic shape (like a square or triangle). Describe how you used transformations.

Solution:
(This can vary; an example might be) Starting with a square, translate it to the right, reflect part of it over a vertical line to create a mirrored effect, and rotate one section by 45° to suggest movement. This design uses translation, reflection, and rotation for a dynamic look.

Additional Notes for Teachers:

Learning Outcomes: Students should understand angle properties, recognise different types of angles and shapes, and apply knowledge of symmetry and angle relationships. Students should understand how to perform and describe various transformations and recognise their effects on shapes.

Teaching Strategies: Use physical manipulatives like protractors, angle rulers, or shape cutouts for hands-on learning. Encourage drawing and sketching to help students visualise angles and shapes. Incorporate art projects or design tasks that utilise geometric concepts. Use grid paper for students to plot and transform shapes manually. Incorporate digital tools for interactive learning where students can see transformations in real-time. Encourage students to create art or patterns using transformations to deepen understanding.

Assessment: Assess through practical tasks where students identify, measure, or create angles and shapes, demonstrating understanding of symmetry and angle properties. Evaluate students through tasks where they must transform shapes, predict outcomes of transformations, or reverse-engineer transformations to return shapes to their original form.

Resources: Use geometric drawing apps, interactive geometry software, or traditional tools like compasses and rulers. Use geometry software like GeoGebra, apps for dynamic geometry, or even simple paper cutouts for physical transformation exercises.

This question set aligns with the Australian Curriculum for Year 7, emphasising the proficiencies of understanding, fluency, problem-solving, and reasoning in the context of angles, shapes, and geometric transformations.