WORKSHEET # 009 | Australian Curriculum Year 8 Mathematics | WM8T1U1

Indices, Scientific Notation, Rational Numbers, and Surds

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Focus: A set of questions and solutions for Year 8 students focused on 'Indices, Scientific Notation, Surds, and Rational Numbers' under the "Number and Algebra" strand, tailored to the Australian Curriculum:

1. Understanding Indices:

a) Define what indices (or exponents) are. How do they affect numbers?

Solution:
Indices (or exponents) indicate how many times a number (the base) is multiplied by itself. For example, 2^3 means 2 is multiplied by itself 3 times: $2 \times 2 \times 2 = 8$.

They affect numbers by either increasing or decreasing their size exponentially based on the exponent.

b) Simplify $3^2 \times 3^3$.

Solution:
Using the law of indices for multiplication:

$$\begin{aligned} & 3^2 \times 3^3 \\ = \ & 3^{2+3} \\ = \ & 3^5 \\ = \ & 243. \end{aligned}$$

2. Laws of Indices:

a) State the three basic laws of indices for multiplication, division, and raising a power to a power.

Solution:

Multiplication: $a^m \times a^n = a^{m+n}$.

Division: $\frac{a^m}{a^n} = a^m \div a^n = a^{m-n}$.

Power of a power: $(a^m)^n = a^{m \times n}$.

b) Simplify $\frac{5^6}{5^3}$.

Solution:

Using the law of indices for division:

$$\begin{aligned} & \frac{5^6}{5^3} \\ &= 5^{6-3} \\ &= 5^3 \\ &= 125 . \end{aligned}$$

3. Negative and Zero Indices:

a) Explain what negative indices mean and how to interpret them.

Solution:

A negative index means the reciprocal of the positive power of the base.

For example,

$$\begin{aligned} & 2^{-3} \\ &= \frac{1}{2^3} \\ &= \frac{1}{8} . \end{aligned}$$

b) Simplify 4^0.

Solution:

Any number (except 0) raised to the power of 0 , is 1 .

Thus, $4^0 = 1$.

4. Scientific Notation:

a) Define scientific notation and explain its purpose.

Solution:
Scientific notation is a way of writing very large or very small numbers in a compact form as a product of a number between 1 and 10, and a power of 10. It's used to simplify dealing with numbers that are either too large or too small to express conveniently in standard form.

b) Convert $3{,}000{,}000$ to scientific notation.

Solution:
$3{,}000{,}000$ in scientific notation is:

$$3{,}000{,}000$$

(6 5 4 3 2 1)

3×10^6.

5. Operations with Scientific Notation:

a) Multiply $(2 \times 10^3) \times (3 \times 10^2)$.

Solution:
Multiply the numbers and add the exponents:

$$\begin{aligned} 2 \times 3 &= 6 \\ 10^3 \times 10^2 &= 10^{3+2} \\ &= 10^5 \end{aligned}$$

Result: 6×10^5.

b) Divide $\dfrac{8 \times 10^9}{4 \times 10^6}$.

Solution:
Divide the numbers and subtract the exponents:

$$\frac{8}{4} = 2$$

$$\begin{aligned} 10^9 \div 10^6 &= 10^{9-6} \\ &= 10^3 \end{aligned}$$

Result: 2×10^3.

6. Practical Application:

The distance from Earth to the sun is approximately $150{,}000{,}000\,kilometres$. Express this distance in scientific notation.

Solution:
$150{,}000{,}000\,km$ in scientific notation is:
$1.5 \times 10^8\,km$.

7. Complex Indices Problems:

Simplify $(2^3)^2$.

Solution:
Using the law of indices for raising a power to a power:

$$\begin{aligned} & (2^3)^2 \\ &= 2^{3\times2} \\ &= 2^6 \\ &= 64\,. \end{aligned}$$

8. Converting Between Standard and Scientific Notation:

Convert 7.2×10^{-4} to standard form.

Solution:
$$\begin{aligned} & 7.2 \times 10^{-4} \\ &= 0.00072\,. \end{aligned}$$

9. Negative Numbers:

a) What is the result of $-3 + 5$?

Solution:
$+2$.

b) Put the following set of numbers from least to greatest: $\{\,3,\ -3, -1, 1, 0\,\}$

Solution:
$-3, -1, 0, 1, 3$.
Negative numbers are less than zero, and the numbers increase in value from left to right.

c) Solve the following multiplication: $-2 \times (-3)$.

Solution:
-2 times -3 gives a positive result because multiplying two negatives results in a positive:

($-$ × $-$ = +) If signs are:
opposite → change to –
same → change to +

$$-2 \times (-3) = 2 \times 3$$
$$= 6.$$

d) A temperature drops from $-5^{\circ}C$ to $-8^{\circ}C$. By how many degrees did the temperature decrease?

Solution:
$-13^{\circ}C$.

10. Rational Numbers:

a) Perform the division: $\frac{-15}{-5}$.

Solution:
Dividing two negative numbers gives a positive result:

$$\frac{-15}{-5} = \frac{15}{5}$$
$$= 3.$$

b) Calculate the following expression involving rational numbers: $\frac{-7}{2} + \frac{3}{4}$.

Solution:
First, find a common denominator (4 in this case):

$$\frac{-7 \times 2}{2 \times 2} = \frac{-10}{4}$$

$$\rightarrow \frac{-14}{2} + \frac{3}{4}$$
$$= \frac{-14}{4} + \frac{3}{4}$$
$$= \frac{-14 + 3}{4}$$
$$= \frac{-11}{4}.$$

10. Surds :

a) Simplify: $\sqrt{48}$ as much as possible without using a calculator.

Solution:

Factorise 48 into prime factors: $48 = 16 \times 3$

$$48 = 2^4 \times 3$$

$$\rightarrow \sqrt{48} = \sqrt{2^4 \times 3}$$

$$= \sqrt{2^4} \times \sqrt{3}$$

$$= (2^4)^{\frac{1}{2}} \times \sqrt{3}$$

$$= 2^{4 \times \frac{1}{2}} \times \sqrt{3}$$

$$= 2^{\frac{4}{2}} \times \sqrt{3}$$

$$= 2^2\sqrt{3}$$

$$= 4\sqrt{3}\,.$$

Remember, $\sqrt{x} = x^{\frac{1}{2}}$.

b) Simplify: $\sqrt{64}$.

Solution:

Turn 64 into a power of 2:

$$\sqrt{64} = \sqrt{2^6}$$

$$= (2^6)^{\frac{1}{2}}$$

$$= 2^{6 \times \frac{1}{2}}$$

$$= 2^{\frac{6}{2}}$$

$$= 2^3\,.$$

Additional Notes for Teachers:

Learning Outcomes: Students should master the manipulation of indices, understand and use scientific notation, and apply these concepts in practical contexts.

Teaching Strategies: Use real-life examples like distances in space, sizes of atoms, or large numbers in population statistics to teach scientific notation. Employ hands-on activities where students can practice converting between forms or simplifying expressions with indices. Encourage the use of calculators for verification but stress understanding the process.

Assessment: Assess through problems that require simplification of expressions, conversion between notation forms, and application in practical scenarios.

Resources: Calculators with scientific notation capabilities, real-world data sets, or interactive math software can aid in teaching these concepts.

This question set aligns with the Australian Curriculum for Year 8, focusing on the proficiencies of understanding, fluency, problem-solving, and reasoning in number and algebra, specifically in the areas of indices and scientific notation.