WORKSHEET # 010 | Australian Curriculum Year 8 Mathematics | WM8T1U2

ACACIA TUTORING

Expanding and Factorising 8

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Focus: A set of questions and solutions for Year 8 students focused on 'Expanding and Factoring' under the "Number and Algebra" strand, tailored to the Australian Curriculum:

1. Understanding Expansion:

a) What does it mean to expand an algebraic expression?

Solution:
Expanding an algebraic expression involves multiplying out the terms to remove brackets, using the *distributive property*.

b) Expand $2(x+3)$**.**

Solution:
Using the distributive property:

$$2(x+3)$$
$$= 2\times x + 2\times 3\cdot$$
$$= 2x + 6$$

$$a(b+c) = ab + ac$$

2. Expanding Binomials:

a) Expand $(x+4)(x-2)$**.** *(Use Crab Claw)*

$(+\times - = -)$ If signs are:
opposite → change to $-$
same → change to $+$

Solution:
Use the FOIL (First, Outer, Inner, Last) method (also known as Crab Claw):

$$\rightarrow (x+4)(x-2)$$
$$= x\cdot x + x\cdot(-2) + 4\cdot x + 4\cdot(-2)$$
$$= x^2 \boxed{-2x+4x} - 8$$

Combine like terms: $= x^2 + 2x - 8$.

Remember, mathematicians sometimes use the symbol $(\cdot)$ instead of $(\times)$ so we don't get $(\times)$ confused with (x)
E.g. $2\times x \equiv 2\cdot x$

b) Expand $(3y-1)(2y+5)$ **.** *(Use Crab Claw)*

Solution:

$\rightarrow 3y \cdot 2y + 3y \cdot 5 - 1 \cdot +2y - 1 \cdot +5$

$= 6y^2 \boxed{+ 15y - 2y} - 5$

(− × + = −) If signs are:
opposite → change to −
same → change to +

Combine like terms:

$= 6y^2 + 13y - 5$.

3. Factorising Quadratics:

a) What is factorising in algebra, and why is it useful?

Solution:

Factorising involves writing an expression as a product of its factors. It's useful for solving equations, simplifying expressions, and understanding the structure of polynomials.

b) Factorise $x^2 + 5x + 6$ **.**

Solution:

Look for two numbers that multiply to 6 (the constant term) and add to 5 (the coefficient of x):

Numbers are 2 and 3 , so:

$x^2 + 5x + 6$

$\_ \times \_ = 6 \; and \; \_ + \_ = 5$

$\rightarrow 3 \times 2 = 6 \; and \; 3 + 2 = 5$

$= (x+3)(x+2)$.

OR

$x^2 \; +5x \; + \; 6$

$\_ \times \_ = +6 \; and \; \_ + \_ = +5$

$\rightarrow 3 \times 2 = +6 \; and \; 3 + 2 = +5$

$= (x \; + \; 3)(x \; + \; 2)$

$= (x+3)(x+2)$.

c) Factorise $x^2 - 7x + 12$ **.**

Solution:

Find pairs of numbers that multiply to 12 and add to -7 :

Numbers are -3 and -4, so:

$x^2 - 7x + 12$

$\_ \times \_ = 12 \; and \; \_ + \_ = -7$

$\rightarrow -3 \times -4 = 12 \; and \; -3 + -4 = -7$

$= (x-3)(x-4)$.

OR

$x^2 \; -7x \; + \; 12$

$\_ \times \_ = +12 \; and \; \_ + \_ = -7$

$\rightarrow -3 \times -4 = +12 \; and \; -3 + -4 = -7$

$= (x \; - \; 3)(x \; - \; 4)$

$= (x-3)(x-4)$.

d) Factorise $2x^2 + 5x + 2$.

Solution:
$2x^2+5x+2$
Here, consider pairs for 2 times 2, (4), that add to $+5$:
Numbers are 1 and 4, but we need to account for the leading coefficient:

$$\begin{aligned} &\rightarrow 2x^2 + 5x + 2 \\ &= (2x + 1)(x + 2)\,. \end{aligned}$$

OR

$$2x^2+5x+2$$

We need numbers that multiply to give $(2 \times 2) = 4$ and add to $+5$:
These numbers are 1 and 4,
so we break $5x$ into $x + 4x$,

$$\begin{aligned} &\rightarrow 2x^2 + 5x + 2 \\ &= 2x^2 + x + 4x + 2 \\ &= x \times 2x + x \times 1 + 2 \times 2x + 2 \times 1 \\ &= x \times 2x + x \times 1 + 2 \times 2x + 2 \times 1 \\ &= x(2x + 1) + 2(2x + 1) \\ &= (2x + 1)(x + 2)\,. \end{aligned}$$

4. Factorising Common Factors:

a) Factorise $3x + 12$.

Solution:
The common factor is 3 :

$$\begin{aligned} &\rightarrow 3x + 12 \\ &= 3 \cdot x + 3 \cdot 4 \\ &= 3 \cdot (x + 4) \\ &= 3(x + 4)\,. \end{aligned}$$

b) Factorise $6a^2 + 9a$.

Solution:
The common factor is 3a:

$$\begin{aligned} &\rightarrow 6a^2 + 9a \\ &= 3a \cdot 2a + 3a \cdot 3 \\ &= 3a \cdot (2a + 3) \\ &= 3a(2a + 3)\,. \end{aligned}$$

5. Practical Application:

If the area of a rectangle is given by $x^2 + 5x + 6$ square units, what could be the dimensions of the rectangle?

Solution:
Area $= x^2 + 5x + 6$

$$\_ \times \_ = 6 \ and \ \_ + \_ = 5$$
$$\rightarrow 3 \times 2 = 6 \ and \ 3 + 2 = 5$$

$$= (x + 3)(x + 2).$$

So factorising gives:

Area $= (x + 3) \times (x + 2)$
$=$ Length $\times$ Width , which means the dimensions could be:
$\rightarrow$ $(x + 3)$ units by $(x + 2)$ units.

6. Combining Expansion and Factorisation:

Expand $(x - 1)(x - 1)$ and then factorise the result back to check your work.

Solution:

Expand: $(x - 1)(x - 1)$ $(Use\ Crab\ Claw)$

$$= x^2 \boxed{- x - x} + 1$$
$$= x^2 - 2x + 1.$$

Factorise:
$x^2 - 2x + 1 = (x - 1)(x - 1)$.
This equals the original expression, confirming the expansion.

7. Factorising Special Cases:

Difference of two squares:
$x^2 - a^2$
$= (x - a)(x + a)$.

a) Factorise $x^2 - 9$.

Solution:
This is a difference of two squares:
$x^2 - 9 = x^2 - 3^2$
$= (x - 3)(x + 3)$.

b) Factorise $x^2 - 25$.

Solution:
This is a difference of two squares:
$x^2 - 25 = x^2 - 5^2$
$= (x - 5)(x + 5)$.

Additional Notes for Teachers:

Learning Outcomes: Students should be able to expand and factorise algebraic expressions, understand the relationship between these processes, and apply them in solving problems.

Teaching Strategies: Use visual aids like algebra tiles to demonstrate expansion and factorisation. Encourage students to check their factorisation by expanding back to the original expression. Provide varied examples, including real-world applications like area calculations or optimisation problems.

Assessment: Assess through exercises that require both expansion and factorisation, emphasising understanding of the process rather than just memorisation.

Resources: Algebra tiles, online algebra tools, or worksheets with a mix of problems to practice these skills.

This question set aligns with the Australian Curriculum for Year 8, focusing on the proficiencies of understanding, fluency, problem-solving, and reasoning in number and algebra, specifically in expanding and factorising expressions.