Linear Equations, Inequalities, and Linear Relationships

ACACIA TUTORING

8

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Focus: A set of questions and solutions for Year 8 students focused on 'Linear Equations and Inequalities' under the "Number and Algebra" strand, tailored to the Australian Curriculum:

1. Understanding Linear Equations:

a) What is a linear equation? Give an example.

Solution:
A linear equation is an algebraic equation where each term is either a constant or the product of a constant and a single variable raised to the first power.
Example: $2x + 3 = 7 \ (2x^1 + 3 = 7)$.

b) How do you solve a simple linear equation like $x + 4 = 9$?

Solution:
Subtract 4 from both sides to isolate x:

$$x + 4 - 4 = 9 - 4$$
$$x = 5.$$

2. Solving Linear Equations:

a) Solve $3x - 5 = 10$.

Solution:
Add 5 to both sides, then divide by 3:

$$3x - 5 + 5 = 10 + 5$$
$$3x = 15$$
$$\frac{3x}{3} = \frac{15}{3}$$
$$x = 5.$$

b) Solve $2(4 - y) = 6$ **.**

> **Solution:**
> Distribute, then solve:

$$2(4 - y) = 6$$
$$2 \cdot 4 - 2 \cdot y = 6$$
$$8 - 2y = 6$$

Subtract 8 from both sides:

OR

$$\cancel{8} \; \cancel{-8} - 2y = 6 \; -8$$
$$-2y = -2$$

Divide by -2

$$\frac{\cancel{-2}y}{\cancel{-2}} = \frac{-2}{-2}$$
$$y = 1 \,.$$

$$2(4 - y) = 6$$

Remove $2 \times$ from left side of equation:

$$\frac{\cancel{2}(4 - y)}{\cancel{2}} = \frac{6}{2}$$

Subtract 4 from both sides of equation:

$$4 - y = 6$$
$$\cancel{4} \; \cancel{-4} \; -y = 3 \; -4$$

Divide by -1 on both sides of equation:

$$-y = -1$$
$$-1 \times y = -1$$
$$\frac{\cancel{-1} \times y}{\cancel{-1}} = \frac{-1}{-1}$$
$$y = 1 \,.$$

3. Understanding Inequalities:

a) Explain the difference between an equation and an inequality.

> **Solution:**
> An equation states that two expressions are equal, while an inequality states that one expression is less than, greater than, less than or equal to, or greater than or equal to another expression.

b) How is solving an inequality different from solving an equation?

> **Solution:**
> When multiplying or dividing both sides of an inequality by a negative number, you must reverse the inequality sign.

4. Solving Linear Inequalities:

a) Solve the inequality $x + 3 < 7$ **.**

> **Solution:**
> Subtract 3 from both sides:

$$x \; \cancel{+3} \cancel{-3} < 7 \; -3$$
$$x < 4 \,.$$

b) Solve $2x - 1 \geq 5$ **.**

Solution:
Add 1 to both sides, then divide by 2:

$$2x - 1 + 1 \geq 5 + 1$$
$$2x \geq 6$$
$$\frac{2x}{2} \geq \frac{6}{2}$$
$$x \geq 3.$$

c) Solve $-3x > 6$ **.**

Solution:
Divide both sides by -3, reversing the inequality:

$$-3x > 6$$
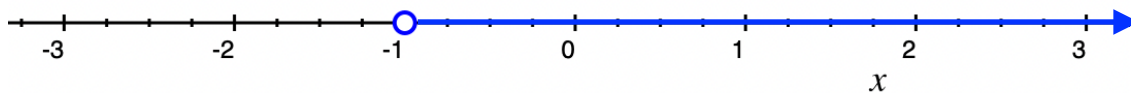
$$\frac{-3x}{-3} < \frac{6}{-3}$$
$$x < -2.$$

5. Graphing Solutions on a Number Line:

a) Graph the solution to $x > -1$ **on a number line.**

Solution:
Draw an open circle at -1 and shade all points to the right:

b) Graph $x \leq 3$ **on a number line.**

Solution:
Draw a closed circle at 3 and shade all points to the left of and including 3.

6. Practical Application:

You have $\$20$ to spend on books, each costing $\$4$. Write and solve an inequality to find how many books you can buy.

Solution:
Let b be the number of books
The inequality is: $4b \leq 20$
Solve:

$$\frac{\cancel{4}b}{\cancel{4}} \leq \frac{20}{4}$$
$$b \leq 5 .$$

You can buy up to 5 books.

7. Combining Equations and Inequalities:

If x satisfies $2x - 3 = 5$, is x also a solution to $x + 2 < 8$?

Solution:
Solve the equation:

$$2x \cancel{-3} \cancel{+3} = 5 + 3$$
$$\frac{\cancel{2}x}{\cancel{2}} = \frac{8}{2}$$
$$x = 4 .$$

Now check if $x = 4$, satisfies the inequality:

$$x + 2 < 8$$
$$\rightarrow 4 + 2 < 8$$
$$6 < 8 .$$

And the inequality $6 < 8$, is true.

Therefore, $x = 4$ is also a solution to the inequality.

8. Systems of Linear Equations or Inequalities:

a) Describe what a system of linear equations or inequalities is.

Solution:
A system of linear equations or inequalities involves two or more linear equations or inequalities that must be solved simultaneously.

b) Solve the system of equations: $\begin{cases} x + y = 6 \\ 2x - y = 3 \end{cases}$ **, to find values for** x **and** y.

Solution:

Add the equations to eliminate y:

$x + y + 2x - y = 6 + 3$

$\boxed{x} + y + \boxed{2x} - y = \boxed{6 + 3}$

$$\frac{3x}{3} = \frac{9}{3}$$

$$\rightarrow x = 3.$$

Substitute $x = 3$ into the first equation to find y (or second equation):

$3 + y - 3 = 6 - 3$

$$\rightarrow y = 3.$$

This gives:

$x = 3$, $y = 3$.

Graphical representation of question 8b.
(Becomes relevant in years 9, 10, 11 and 12)
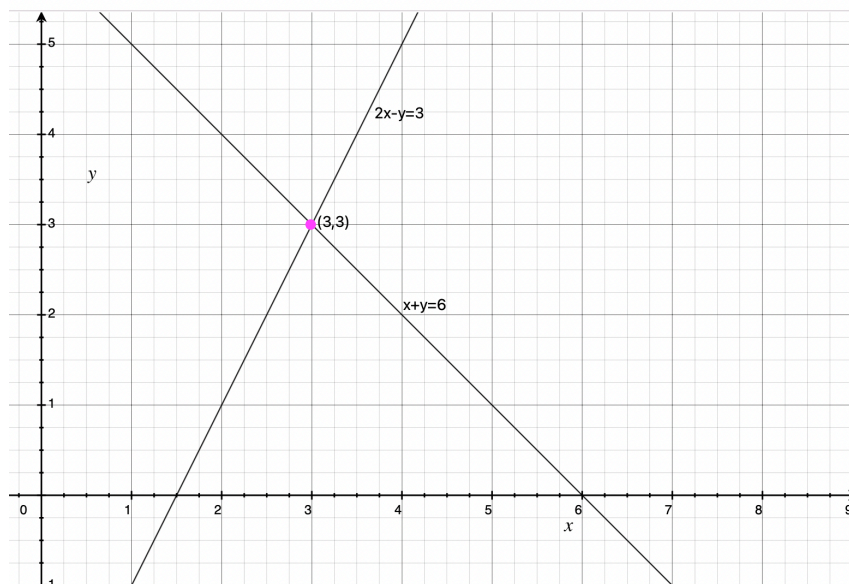
9. Linear Relationships:

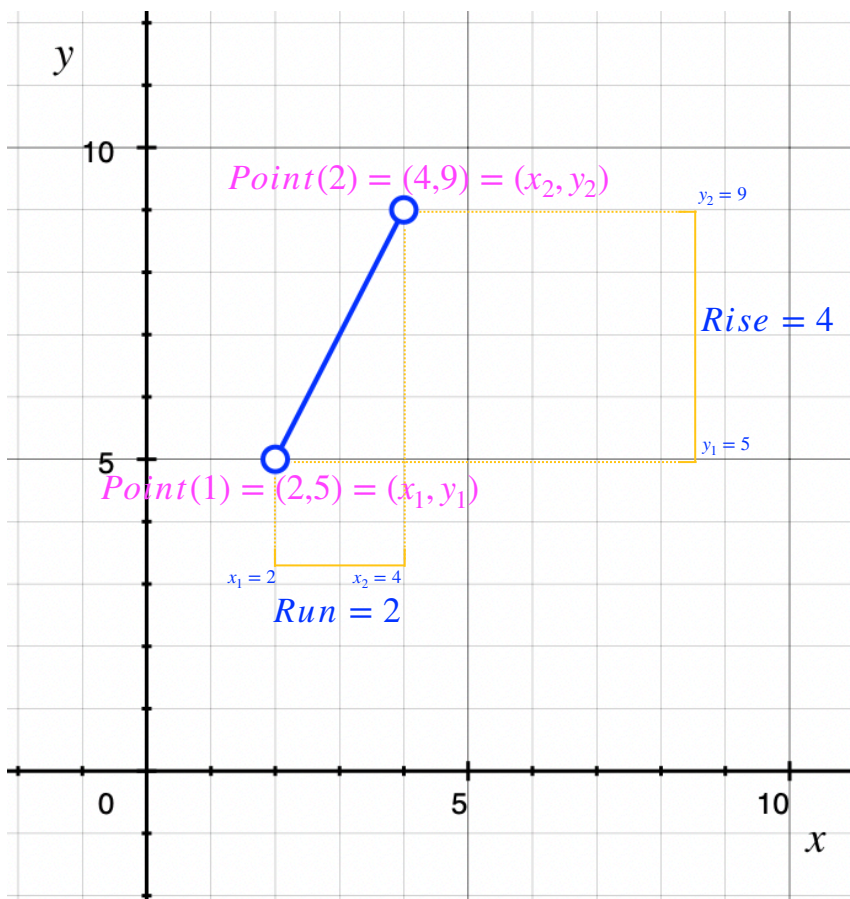**a) Find the slope of a line passing through the points** $(2,5)$ **and** $(4,9)$**:**

Solution:

$Point(2) = (4,9) = (x_2, y_2)$

$Point(1) = (2,5) = (x_1, y_1)$

$$m = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 5}{4 - 2}$$

$$= \frac{4}{2}$$

$$= 2.$$

$$= \frac{2}{1} = \frac{\text{Rise}}{\text{Run}}$$

b) Construct a table of values for the equation $y = 2x + 1$ **for** $x = (-1, 0, 1)$.

Solution:

Substitute the values for $x = (-1, 0, 1)$, into the equation $y = 2x + 1$:

$x = -1 \rightarrow 2 \times (-1) + 1$
$\qquad = -1,$
$x = 0 \rightarrow 2 \times (0) + 1$
$\qquad = 1,$
$x = 1 \rightarrow 2 \times (1) + 1$
$\qquad = 3.$

x	y
-1	-1
0	1
1	3

c) A linear relationship is given by the equation $y = -2x + 5$. **Find the** $x - intercept$ **and the** $y - intercept$ **of the line. Draw the graph of this equation on the coordinate plane.**

Solution:

To find the y-intercept, set x to 0:

$$y = -2 \times (0) + 5,$$
$$y = 0 + 5,$$
$$y - intercept = 5.$$

The $y - intercept$ is at $(0, 5)$.

To find the x-intercept, set y to 0:

$$0 + 2x = \cancel{-2x} \cancel{+2x} + 5,$$
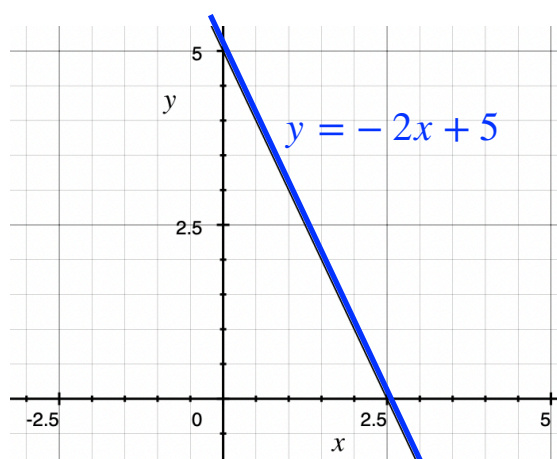$$\rightarrow 2x = 5,$$
$$\frac{\cancel{2}x}{\cancel{2}} = \frac{5}{2},$$
$$x - intercept = \frac{5}{2} = 2.5.$$

The x-intercept is at $\left(\frac{5}{2}, 0 \right)$.

Draw a set of axes, with $x - values$ between 0 and 5, and $y - values$ between 0 and 5. Then, draw in the two points you have calculated (the $x - intercept$ and the $y - intercept$). After this, use a ruler to draw a straight line between the two points:

d) A taxi company charges $2 **for starting the trip and** $0.50 $per\ kilometre$ **traveled. Write the linear equation that represents the cost (** C **) of a trip based on the distance (** d **) in** $kilometres$**. How much would a** $10\,km$ **trip cost?**

Solution:

The equation is $C = 0.50d + 2$.

For a $10\,km$ trip: $C = 0.50\,\$/\cancel{km} \times (10\,\cancel{km}) + 2$

$$= \frac{5}{\cancel{10}} \times \cancel{10} + 2$$
$$= 5 + 2$$
$$= \$7.$$

The cost would be $7.

Additional Notes for Teachers:

Learning Outcomes: Students should be adept at solving linear equations and inequalities, understanding how to graph their solutions, and applying these concepts to real-world problems.

Teaching Strategies: Use balance scales or diagrams to visually explain equation solving. Incorporate graphing on number lines for inequalities to help visualise solutions. Encourage students to check their solutions by substitution or graphical verification.

Assessment: Evaluate through problem sets that require both solving and interpreting linear equations and inequalities, including word problems that relate to everyday scenarios.

Resources: Graph paper for number line representations, algebra tiles, or interactive math software for solving and graphing.

This question set aligns with the Australian Curriculum for Year 8, focusing on the proficiencies of understanding, fluency, problem-solving, and reasoning in number and algebra, specifically in the context of linear equations and inequalities.

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