ACACIA TUTORING

Volume, Surface Area, and Transformations 8

Free and always will be!

Focus: A set of questions and solutions for Year 8 students focused on 'Volume and Surface Area' under the "Measurement and Geometry" strand, tailored to the Australian Curriculum:

1. Understanding Volume and Surface Area:

a) Define volume and surface area. How are they different for a 3D shape?

Solution:
Volume: The amount of space enclosed by a three-dimensional object, measured in cubic units.

Surface Area: The total area of all the surfaces of a 3D object, measured in square units.

Difference: Volume deals with the inside space, while surface area deals with the total outer covering.

2. Volume of Prisms:

a) Write the formula for the volume of a rectangular prism.

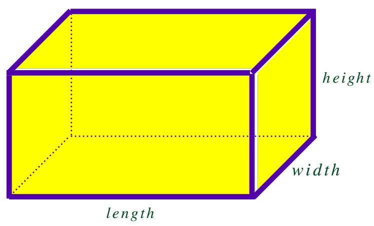
Solution:

V_{prism} = Area of Base $\times$ $height$

Volume = Length × Width × Height, or

$$V = l \times w \times h$$

$$V = lwh\,.$$



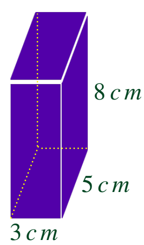
b) Calculate the volume of a rectangular prism with dimensions $5\,cm$, $3\,cm$, and $8\,cm$.

Solution:

V_{prism} = Area of Base $\times$ $height$

$$V = (3\,cm \times 5\,cm) \times 8\,cm \quad (cm \times cm \times cm = cm^3)$$

$$= 120\,cm^3\,.$$



3. Surface Area of Prisms:

a) What is the formula for the total surface area of a rectangular prism?

Solution:

Surface Area $= 2lw + 2lh + 2wh,$

where l is length, w is width, and h is height.

b) Find the surface area (SA) of the same rectangular prism from the question 2b.

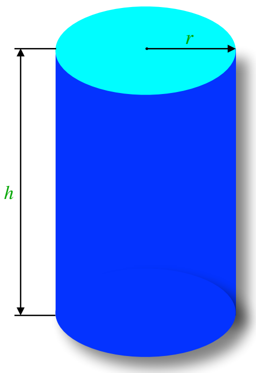
Solution:

$$SA = 2(5 \times 3) + 2(5 \times 8) + 2(3 \times 8)$$
$$= 2(15) + 2(40) + 2(24)$$
$$= 30\,cm^2 + 80\,cm^2 + 48\,cm^2$$
$$= 158\,cm^2\,.$$

4. Volume of Cylinders:

a) State the formula for the volume of a cylinder.

Solution:



Volume$_{prism}$ = Area of Base × Height , (Remember, a cylinder IS a prism)

Area of base $= \pi r^2 =$ Area of Circle .

→ Volume$_{cylinder}$ $= \pi r^2 h$,

V$_{cylinder}$ $= \pi r^2 h$, where r is the radius of the base and h is the height.

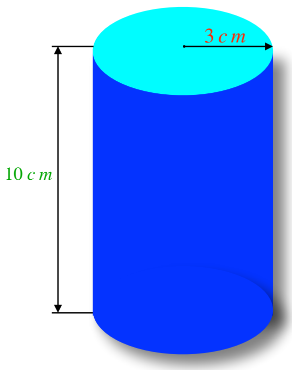
b) Calculate the volume of a cylinder with radius $3\,cm$ and height $10\,cm$.

Solution:

Volume$_{prism}$ = Area of Base × Height ,

V$_{cylinder}$ $= \pi r^2 h$:

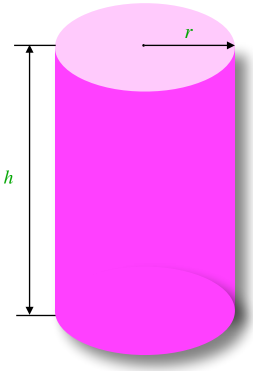
$$\rightarrow V = \pi \times (3\,cm)^2 \times 10\,cm$$
$$V = \pi \times 3^2\,cm^2 \times 10\,cm$$
$$V = \pi \times 9\,cm^2 \times 10\,cm$$
$$= 90\pi\,cm^3$$
$$\approx 282.74\,cm^3\,.$$



5. Surface Area of Cylinders:

a) Write the formula for the total surface area of a cylinder.

Solution:

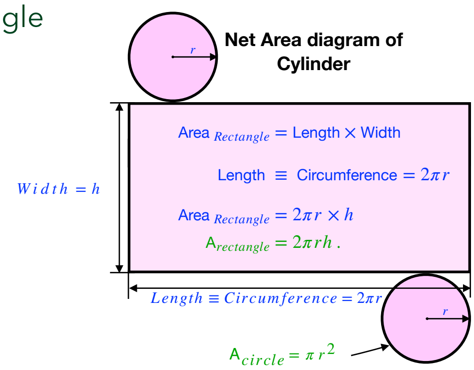


$$\text{Surface Area} = 2 \times \text{Area of Circle} + \text{Area of Rectangle}$$
$$= 2 \times \pi r^2 + \text{Lenght} \times \text{Width},$$
$$= 2\pi r^2 + 2\pi r \times h\ ,$$

$$SA_{cylinder} = 2\pi r^2 + 2\pi r h\ ,$$

where r is the radius and h is the height .

$$SA_{cylinder} = 2\pi r \times r\ + 2\pi r \times h\ ,$$
$$= 2\pi r \times (r + h),$$
$$SA_{cylinder} = 2\pi r (r + h)\ .$$

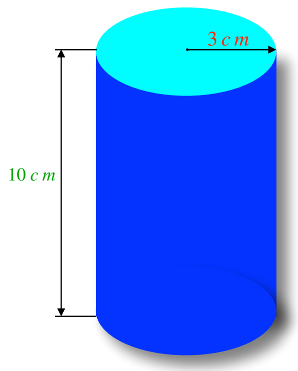


b) Find the surface area of the cylinder from question 4b.

Solution:

$$SA_{cylinder} = 2\pi r^2 + 2\pi r h\ ,$$

$$SA = 2\pi \times 3^2 + 2\pi \times 3 \times 10$$
$$= 2\pi \times 9 + 2\pi \times 30$$
$$= 18\pi\ cm^2 + 60\pi\ cm^2$$
$$= 78\pi\ cm^2$$
$$\approx 245.04\ cm^2\ .$$



6. Volume and Surface Area of Pyramids:

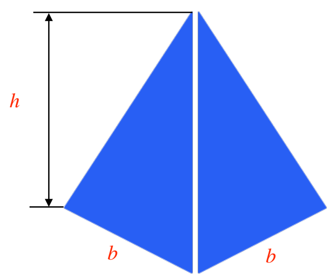
a) How do you calculate the volume of a pyramid?

Solution:

$$\text{Volume} = \frac{1}{3} \times \text{Base Area} \times \text{Height}$$

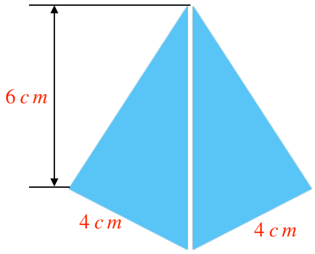
$$V = \frac{1}{3} \times b^2 \times h$$

$$V_{pyramid} = \frac{1}{3} b^2 h\ \ or\ \ V_{pyramid} = \frac{b^2 h}{3}$$



b) If a pyramid has a square base with side length $4\,cm$ and height $6\,cm$, what is its volume?

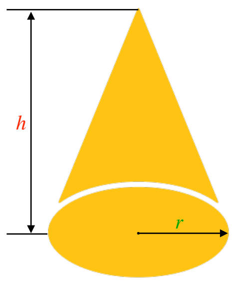
Solution:



$$\begin{aligned} \text{Base Area } &= 4 \times 4 \\ b^2 &= 16\,cm^2 \\ h &= 6\,cm \\ V &= \frac{1}{3}b^2h \\ \text{Volume } &= \frac{1}{3} \times 16\,cm^2 \times 6\,cm \\ &= \frac{1}{3} \times 6\,cm \times 16\,cm^2 \\ &= \frac{6\,cm}{3} \times 16\,cm^2 \\ &= 2\,cm \times 16\,cm^2 \\ &= 32\,cm^3. \end{aligned}$$

7. Volume and Surface Area of Cones:

a) What is the formula for the volume of a cone?



Solution:

$$\text{Volume } = \frac{1}{3} \times \text{ Area of Base} \times \text{Height}$$

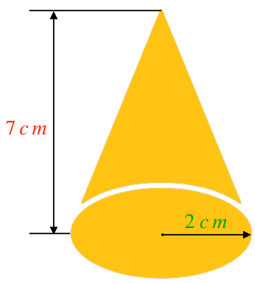
$$V_{cone} = \frac{1}{3}\pi r^2 h,$$

where r is the radius of the base and h is the height.

b) Calculate the volume of a cone with radius $2\,cm$ and height $7\,cm$.

Solution:

$$V_{cone} = \frac{1}{3}\pi r^2 h,$$



$$\begin{aligned} \rightarrow V &= \frac{1}{3}\pi \times (2cm)^2 \times 7\,cm \\ &= \frac{1}{3}\pi \times 2^2\,cm^2 \times 7\,cm \\ &= \frac{1}{3}\pi \times 4\,cm^2 \times 7\,cm \\ &= \frac{28\pi}{3}\,cm^3 \\ &\approx 29.32\,cm^3. \end{aligned}$$

8. Practical Application:

a) You need to paint a rectangular prism shaped box with dimensions $2\,m \times 1\,m \times 1\,m$ **. If one can of paint covers** 5 **square** $metres$ **, how many cans do you need?**

Solution:

$$\begin{aligned} \text{Surface Area} &= 2(2\times 1) + 2(2\times 1) + 2(1\times 1) \\ &= 10\,m^2. \\ \text{Coverage} &= 5\,m^2/can \\ \text{Cans needed} &= \frac{10\,\cancel{m^2}}{5\,\cancel{m^2}\,/\,can} \\ &= 2\,cans. \end{aligned}$$

b) A water tank in the shape of a cylinder has a radius of $1.5\,metres$ **and a height of** $3\,metres$ **. How many** $litres$ **of water can it hold?** ($1\,m^3 = 1{,}000\,L$) **.**

Solution:

$$\begin{aligned} V_{cylinder} &= \pi r^2 h \;: \\ \text{Volume in } m^3 &= \pi \times 1.5^2 \times 3 \\ &= 21.20575\,m^3. \end{aligned}$$

$$\begin{aligned} &\text{Volume in } L : (\,1m^3 = 1000\,L\,) \\ 21.20575\,\cancel{m^3} \times 1{,}000\,L/\cancel{m^3} &= 21{,}205.75\,L \\ &\approx 21{,}206\,litres. \end{aligned}$$

9. Comparing Shapes:

a) Compare the volumes of a cube with side length $4\,cm$ **and a cylinder with radius** $2\,cm$ **and height** $4\,cm$ **.**

Solution:

Cube volume:

$$\begin{aligned} s^3 &= 4^3 \\ &= 64\,cm^3. \end{aligned}$$

Cylinder volume

$$\begin{aligned} &pi \times 2^2 \times 4 \\ &= 16\pi\,cm^3 \\ &\approx 50.27\,cm^3. \end{aligned}$$

So: $V_{cube} > V_{cylinder,}$ therefore the cube has a greater volume.

10. Transformations and Angle Relationships.

10-1. Translation and Angle Relationships.

A triangle ABC with vertices $A(2, 3)$, $B(4, 1)$, and $C(1, 1)$ is translated 3 units to the right and 2 units up to form triangle $A'B'C'$.

a) Determine the coordinates of the vertices of triangle $A'B'C'$.

b) Confirm the $\angle CAB$ is the same as $\angle C'A'B'$, by

I) finding the slope of each line then,

II) finding the angles between each.

Solution:

a) Coordinates of $A'B'C'$:

A translation 3 units right adds 3 to the $x-coordinate$, and 2 units up adds 2 to the $y-coordinate$.

For $A(2, 3)$:
$x' = 2 + 3 = 5$,
$y' = 3 + 2 = 5$.

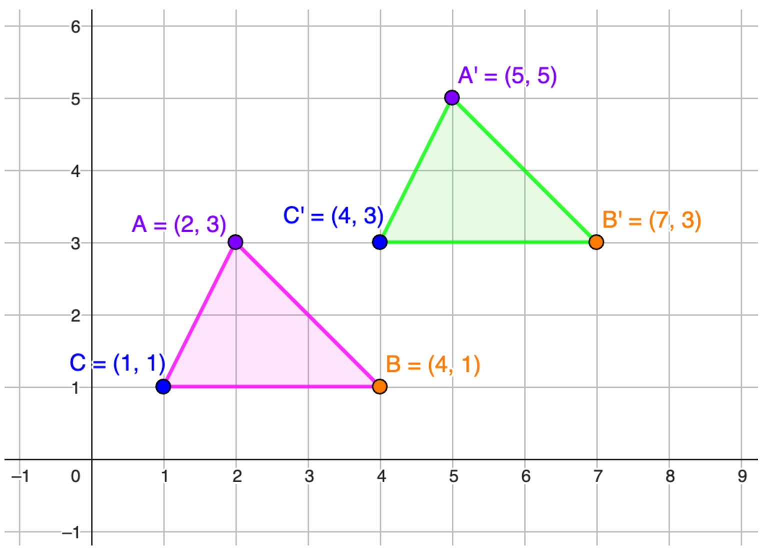
So, $A'(5, 5)$.

For $B(4, 1)$:
$x' = 4 + 3 = 7$,
$y' = 1 + 2 = 3$.

So, $B'(7, 3)$.

For $C(1, 1)$:
$x' = 1 + 3 = 4$,
$y' = 1 + 2 = 3$.

So, $C'(4, 3)$.



The vertices of triangle $A'B'C'$ are $A'(5, 5)$, $B'(7, 3)$, and $C'(4, 3)$.

b) Corresponding Angle:

I)

Slope of AB and $A'B'$:

From $A(2, 3)$ to $B(4, 1)$,

$$\text{Slope } = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{4-2} = \frac{-2}{2} = -1.$$

From $A'(5, 5)$ to $B'(7, 3)$,

$$\text{Slope } = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-5}{7-5} = \frac{-2}{2} = -1.$$

Slope of AC and $A'C'$:

From $A(2, 3)$ to $C(1, 1)$,

$$\text{Slope } = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{1-2} = \frac{-2}{-1} = 2.$$

From $A'(5, 5)$ to $C'(4, 3)$,

$$\text{Slope } = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-5}{4-5} = \frac{-2}{-1} = 2.$$

II)

To find the $\angle CAB$, use the slopes of :
$AB(slope = -1)$ and
$AC(slope = 2)$.

The angle θ between two lines with slopes m_1 and m_2 is given by :

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{2-(-1)}{1+(2)(-1)}\right| = \left|\frac{2+1}{1-2}\right| = \left|\frac{3}{-1}\right| = 3$$

$$\theta = \tan^{-1}(3) \approx 71.6^\circ$$

To find the $\angle C'A'B'$, use the slopes of :
$A'B'(slope = -1)$ and
$A'C'(slope = 2)$.

The angle θ between two lines with slopes m_1 and m_2 is given by :

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{2-(-1)}{1+(2)(-1)}\right| = \left|\frac{2+1}{1-2}\right| = \left|\frac{3}{-1}\right| = 3$$

$$\theta = \tan^{-1}(3) \approx 71.6^\circ.$$

Therefore $\angle CAB = \angle C'A'B'$.

Translations preserve angles and parallelism.

The corresponding angle is approximately 71.6°.

10-2. Reflection and Parallel Lines.

A quadrilateral $PQRS$ with vertices $P(1, 2)$, $Q(3, 2)$, $R(3, 0)$, and $S(1, 0)$ is reflected over the line $y = x$ to form quadrilateral $P'Q'R'S'$.

a) Find the coordinates of $P'Q'R'S'$.

b) **I)** Check $PQ \parallel SR$,

II) Prove that $P'Q' \parallel S'R'$ after the reflection, and,

III) Identify the co-interior angle formed with a transversal line $P'S'$.

Solution:

a) Coordinates of $P'Q'R'S'$:

Reflection over $y = x$ swaps the $x-$ and $y-coordinates$: $(x, y) \rightarrow (y, x)$.

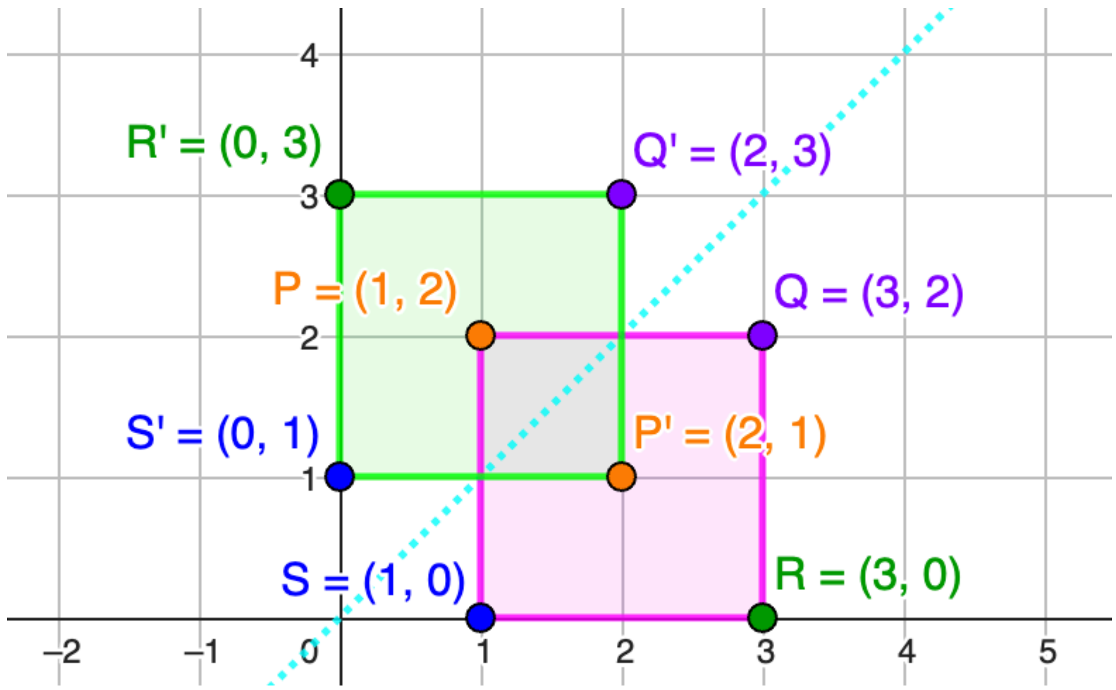
For $P(1, 2)$: $P'(2, 1)$.

For $Q(3, 2)$: $Q'(2, 3)$.

For $R(3, 0)$: $R'(0, 3)$.

For $S(1, 0)$: $S'(0, 1)$.

The vertices are $P'(2, 1)$, $Q'(2, 3)$, $R'(0, 3)$, and $S'(0, 1)$.



b) Parallel Lines and Co-interior Angle:

I)

Check if $PQ \parallel SR$:

Slope of PQ :

From $P(1, 2) = (x_1, y_1)$ to

$Q(3, 2) = (x_2, y_2)$

$$\text{Slope} = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-2}{3-1} = 0 \text{ (horizontal line).}$$

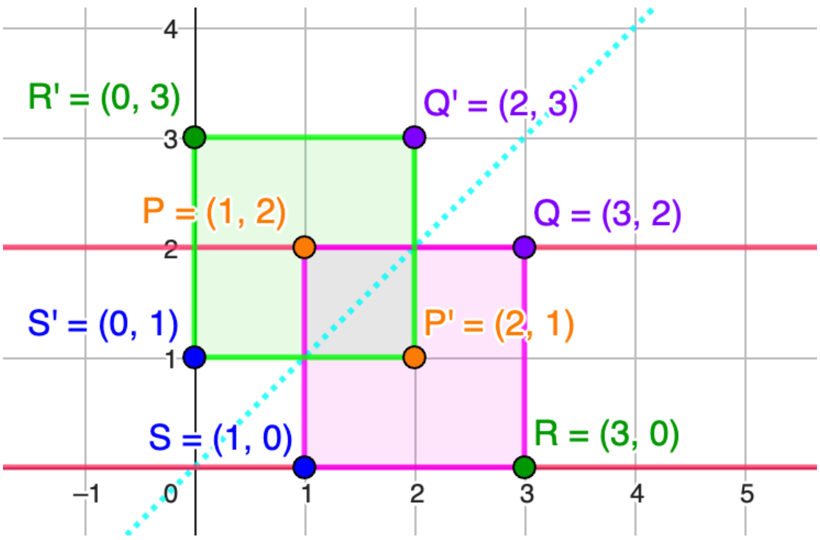
Slope of SR :

From $S(1, 0) = (x_1, y_1)$ to

$R(3, 0) = (x_2, y_2)$

$$\text{Slope} = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-0}{3-1} = 0 \text{ (horizontal line).}$$

Since slopes are equal, $PQ \parallel SR$.



II)

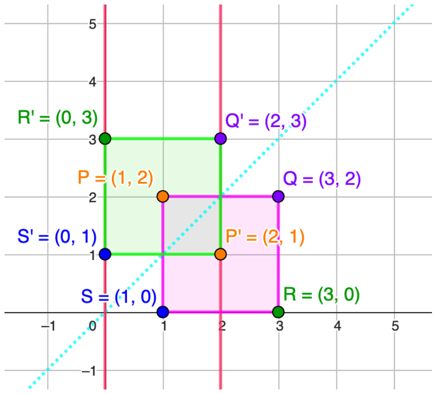
After reflection, check $P'Q' \parallel S'R'$:

Slope of $P'Q'$:

From $P'(2,1)$ to $Q'(2,3)$,

Slope $= \dfrac{3-1}{2-2}$

$= \dfrac{2}{0}$ (undefined, vertical line).



Slope of $S'R'$:

From $S'(0,1)$ to $R'(0,3)$,

Slope $= \dfrac{3-1}{0-0}$

$= \dfrac{2}{0}$ (undefined, vertical line).

Since both are vertical, $P'Q' \parallel S'R'$.

Therefore reflections preserve parallelism.

Since $PQ \parallel SR$, the image lines $P'Q' \parallel S'R'$.

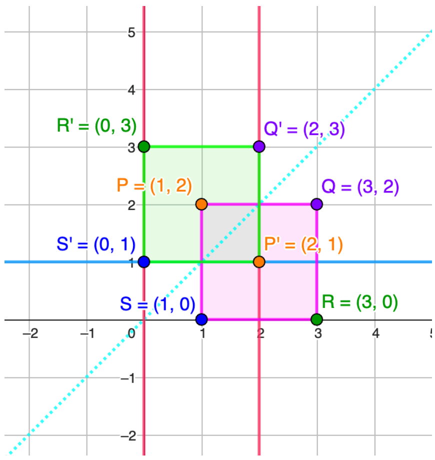
III)

Transversal $P'S'$:

From $P'(2,1)$ to $S'(0,1)$,

Slope $= \dfrac{1-1}{0-2}$

$= 0$ (horizontal line).



Co-interior angles :

$P'Q'$ and $S'R'$ are vertical (parallel), and $P'S'$ is horizontal.

The angle between $P'Q'$ (vertical) and $P'S'$ (horizontal) is 90° .

Similarly, the angle between $S'R'$ and $P'S'$ is 90° .

Co-interior angles sum to 180° :

$\rightarrow 90^\circ + 90^\circ$

$= 180^\circ$.

$P'Q' \parallel S'R'$, and the co-interior angles are each 90° , summing to 180° .

10-3. Rotation

A point $M(2,\ 4)$ is rotated 90° clockwise about the origin to form point M'.
Find the coordinates of M'.

Solution:

Coordinates of M':

A 90° clockwise rotation about the origin maps $(x, y) \rightarrow (y, -x)$.

$M(2,\ 4) \rightarrow M'(4, -2)$.

Rotaion Coordinate Rules (around the origin)

90° counterclockwise or 270° clockwise
$(x, y) \rightarrow (-y, x)$

180° clockwise or 180° counterclockwise
$(x, y) \rightarrow (-x, -y)$

90° clockwise or 270° counterclockwise
$(x, y) \rightarrow (y, -x)$

Additional Notes for Teachers:

Learning Outcomes: Students should understand how to calculate volume and surface area for basic 3D shapes and apply these calculations in practical contexts.

Teaching Strategies: Use physical models or digital tools to visualise how volume and surface area relate to each shape. Encourage students to explore how changing one dimension affects volume and surface area. Relate concepts to real-life scenarios like packaging, construction, or art projects.

Assessment: Assess through problems requiring calculation of volume and surface area, comparisons between shapes, and practical applications.

Resources: Use 3D models, volume and surface area apps, or real-world objects for hands-on learning.

This question set aligns with the Australian Curriculum for Year 8, focusing on the proficiencies of understanding, fluency, problem-solving, and reasoning in measurement and geometry, specifically in the context of volume and surface area.