ACACIA TUTORING

Angle Relationships, Congruence, and Similarity

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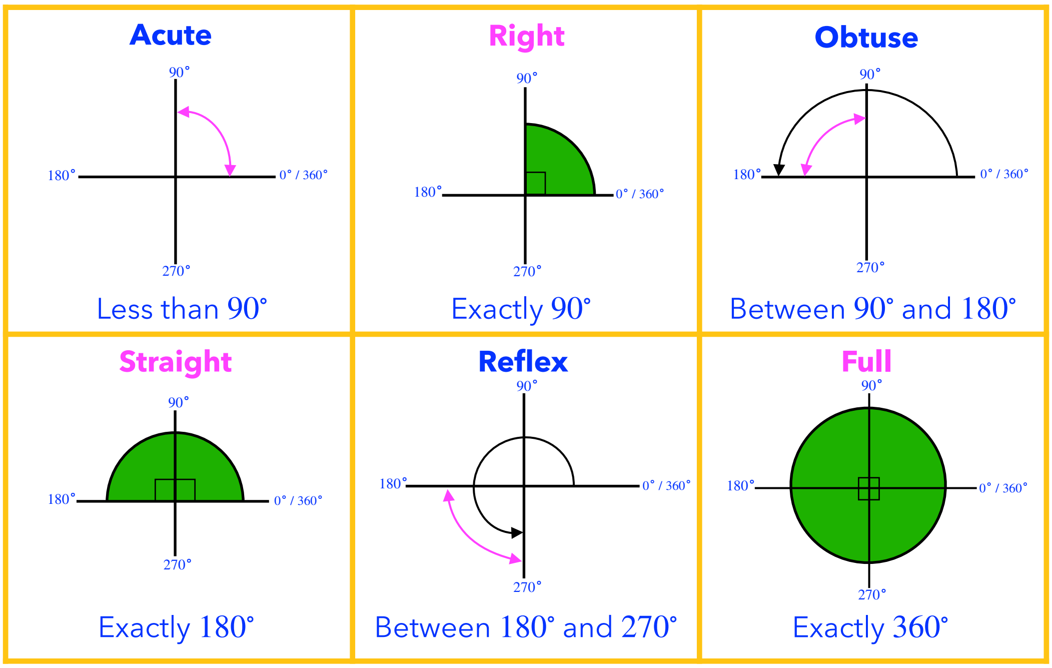
Focus: A set of questions and solutions for Year 8 students focused on 'Angle Relationships' under the "Measurement and Geometry" strand, tailored to the Australian Curriculum:

1. Understanding Basic Angle Concepts:

Define what an angle is, and list the types of angles based on their measure.

Solution:
An angle is formed when two lines or line segments meet at a point (vertex). Types include:

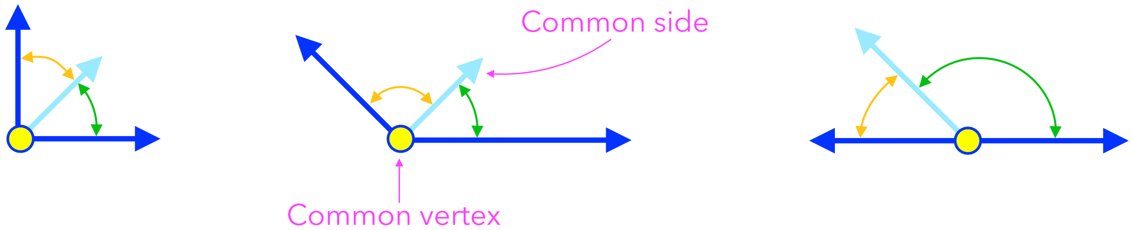
Acute: Less than 90° .
Right: Exactly 90° .
Obtuse: Between 90° and 180° .
Straight: Exactly 180° .
Reflex: More than 180° but less than 360° .
Full: Exactly 360° .



2. Adjacent Angles:

a) Explain what adjacent angles are.

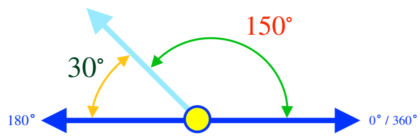
Solution:
Adjacent angles are two angles that share a common vertex and a common side but do not overlap.



b) If one *adjacent angle* measures 30° , and together they form a *straight line*, what is the measure of the other angle?

Solution:
Since they form a straight line (180°) , the other angle is:

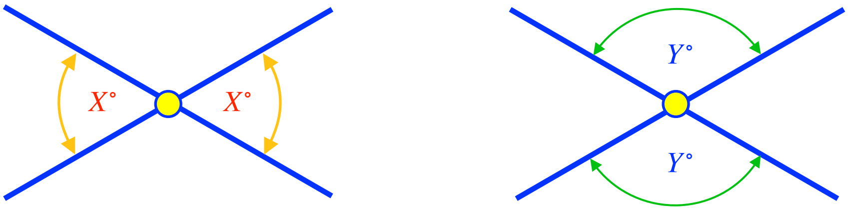
$180^\circ - 30^\circ = 150^\circ$.



3. Vertically Opposite Angles:

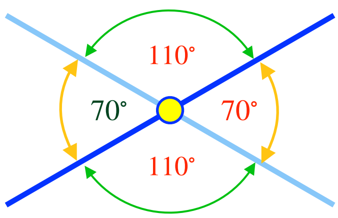
a) What are vertically opposite angles?

Solution:
Vertically opposite angles are formed when two lines intersect. They are opposite each other at the intersection and are equal in measure.



b) If one of the angles at the intersection of two lines is 70°, what are the measures of the other three angles?

Solution:
The vertically opposite angle to the 70° angle is also 70° . The other two angles are supplementary (add to 180°), each being $180^\circ - 70^\circ = 110^\circ$.
See Question 4 Supplementary Angles.

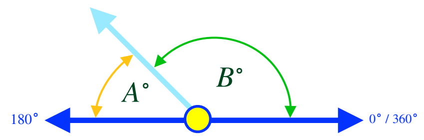


4. Supplementary Angles:

a) Define supplementary angles.

Solution:
Two angles are supplementary if their measures add up to 180°.

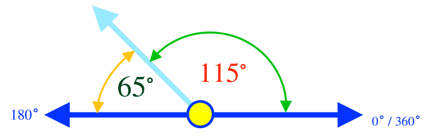
E.g. $A^\circ + B^\circ = 180^\circ$.



b) One angle is 65°. What must the measure of its supplementary angle be?

Solution:
The supplementary angle would be:

$180^\circ - 65^\circ = 115^\circ$.

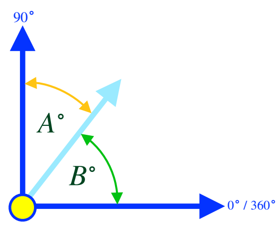


5. Complementary Angles:

a) What are complementary angles?

Solution:
Two angles are complementary if their measures add up to 90°.

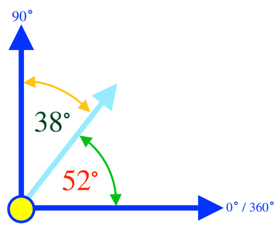
E.g. $A^\circ + B^\circ = 90^\circ$.



b) Find the complement of an angle measuring 38°.

Solution:
The complementary angle is:

$90^\circ - 38^\circ = 52^\circ$.

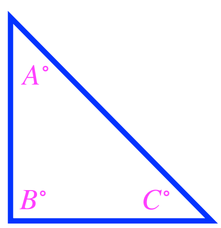


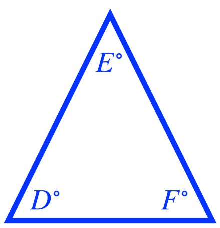
6. Angles in Triangles:

a) State the sum of the interior angles in a triangle.

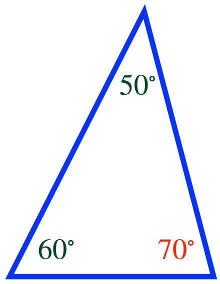
Solution:
The sum of the interior angles of ANY triangle is 180°.

E.g. $A^\circ + B^\circ + C^\circ = 180^\circ$.
E.g. $D^\circ + E^\circ + F^\circ = 180^\circ$.





b) If two angles of a triangle are 50° and 60°, what is the third angle?



Solution:

The third angle is:

$$180^\circ - (50^\circ + 60^\circ)$$
$$= 70^\circ.$$

7. Angles in a Quadrilateral:

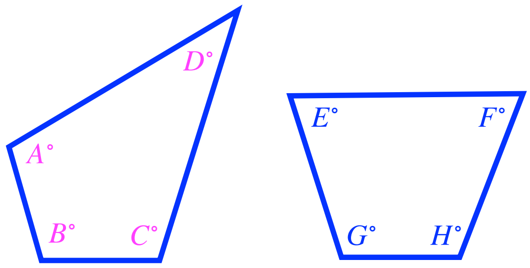
a) What is the sum of the interior angles in a quadrilateral (any four sided figure)?

Solution:

The sum of the interior angles in a quadrilateral is 360°.

E.g. $A^\circ + B^\circ + C^\circ + D^\circ = 360^\circ$.

E.g. $E^\circ + F^\circ + G^\circ + H^\circ = 360^\circ$.

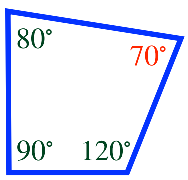


b) If three angles of a quadrilateral measure $90^\circ, 120^\circ,$ and $80^\circ,$ what is the measure of the fourth angle?

Solution:

The fourth angle is:

$$360^\circ - (90^\circ + 120^\circ + 80^\circ)$$
$$= 70^\circ.$$



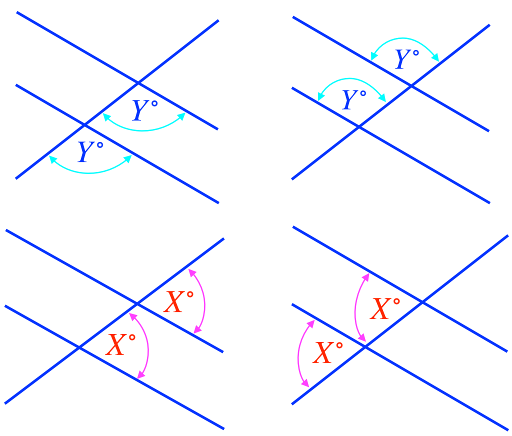
8. Angles Formed by Parallel Lines and Transversals:

a) Describe the four types of angles formed when a transversal intersects two parallel lines.

Solution:

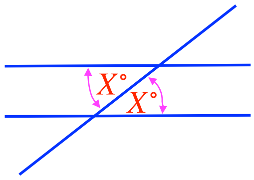
Corresponding angles:

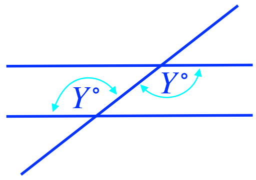
Angles in matching corners (same relative position).



Alternate interior angles:

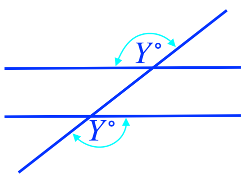
Angles on opposite sides of the transversal inside the parallel lines.

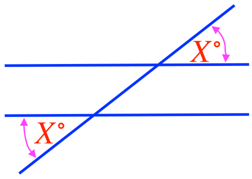




Alternate exterior angles:

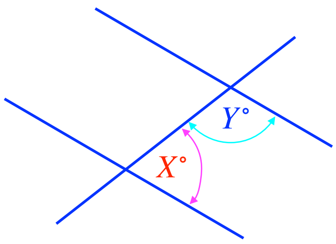
Angles on opposite sides of the transversal outside the parallel lines.

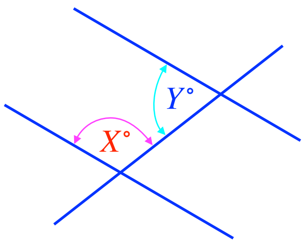




Consecutive interior angles (or Co-interior angles):

Angles on the same side of the transversal inside the parallel lines, which are supplementary.





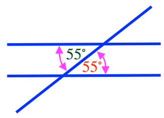
b) If one of the alternate interior angles is 55° , what are the measures of the other alternate interior angle and the two corresponding angles?

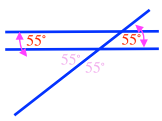
Solution:

The other alternate interior angle is also 55° .

The corresponding angles to the given 55°

angles are also 55° and 55° .





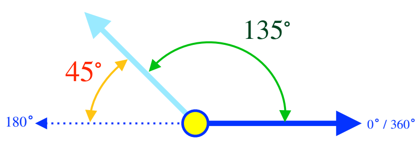
9. Practical Application:

You are designing a room layout where two walls meet at a 135° angle. What is the measure of the other supplementary angle?

Solution:

Since the other angle is supplementary:

$$180^\circ - 135^\circ$$
$$= 45^\circ .$$



10. Understanding Congruence:

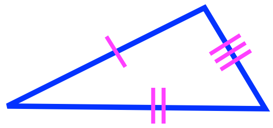
a) Define what it means for two shapes to be congruent. *See Question 2a. Identifying congruent shapes for examples.*

Solution:
Two shapes are congruent if they are identical in shape and size, meaning every corresponding angle is equal, and every corresponding side length is equal. They can be moved (translated, rotated, or reflected) but **not** resized.

b) List the conditions under which triangles are congruent. *See Question 2b. Identifying congruent shapes for examples.*

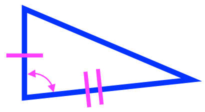
Solution:

SSS (Side-Side-Side): All three sides are equal.



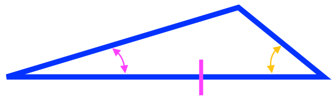
SSS

SAS (Side-Angle-Side): Two sides and the included angle are equal.



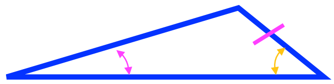
SAS

ASA (Angle-Side-Angle): Two angles and the included side are equal.



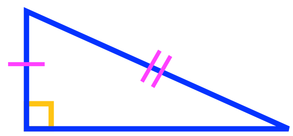
ASA

AAS (Angle-Angle-Side): Two angles and a *non*-included side are equal.



AAS

RHS (Right Angle - Hypotenuse -Side). The right angle, side and hypotenuse are equal.



RHS

11. Identifying Congruent Shapes:

a) If two triangles each have sides of $5\,cm, 6\,cm,$ and $7\,cm$, are they congruent?

Solution:
Yes, these two triangles are identical. If all sides are equal, the triangles are congruent by the SSS criterion.

b) If two seperate triangles each have an angle of 60° and one side of $8\,cm$, are they necessarily congruent?

Solution: No, because the conditions given aren't enough to ensure congruence (only two are given, three are needed). They would need another condition like, another angle, to ensure congruence (by the AAS rule).

12. Understanding Similarity:

a) Define what it means for two shapes to be similar.

Solution:
Two shapes are similar if they have the same shape but **not necessarily** the same size. This means all corresponding angles are equal, and all corresponding sides are in proportion. I.e. They are the same shape but magnified (or reduced) and/or rotated.

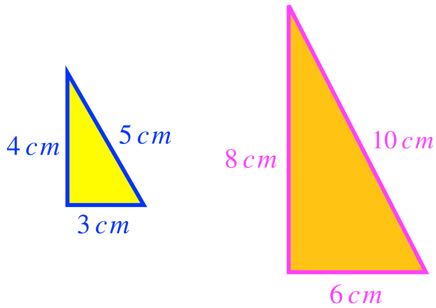
b) What is the ratio of similarity?

Solution:
The ratio of similarity between two similar shapes is the scale factor by which one shape can be enlarged or reduced to match the other. It's the **ratio of the lengths of any corresponding sides**.

13. Identifying Similar Shapes:

a) If one triangle has sides of $3\,cm$, $4\,cm$, and $5\,cm$, and another has sides of $6\,cm$, $8\,cm$, and $10\,cm$, are they similar? What is their ratio of similarity?

Solution:
Yes, they are similar because each side of the second triangle is twice as long as the corresponding side of the first triangle. Choose any pair of corresponding sides, simplify the ratio, then, big : small = $2:1$.



E.g. $\frac{10}{5} : \frac{5}{5} = 2 : 1$.

So the ratio of similarity is $2:1 \; or \; 1:2$.

b) Two rectangles have side lengths of $2\,cm$ by $3\,cm$ and $4\,cm$ by $6\,cm$. Using the similarity ratio, are they similar?

Solution:
Yes, because the ratio of corresponding sides:
$2:4 = 1:2 \;\; or \;\; 3:6 = 1:2$, are the same, making the rectangles similar.

14. Practical Applications:

a) A map is drawn to a scale where $1\,cm$ represents $5\,km$. If a road measures $3\,cm$ on the map, how long is it in reality?

Solution:
Real length $=$ Map length $\times$ Scale factor

$$= 3\cancel{cm} \times 5\,km/\cancel{cm}$$
$$= 15\,km.$$

b) If you have a photo that is width: $10\,cm$ by length: $15\,cm$, and you want to enlarge it so that the new photo's width is: $20\,cm$, using the ratio of similarity, what will be the new height?

Solution:
The ratio of the new width to the original width is (ratio of similarity): $\rightarrow 20 : 10$,
Turn this into a scale factor:
$20\,\cancel{cm} \div 10\,\cancel{cm} = 2$.

Therefore, the new height is:
$15\,cm \times 2 = 30\,cm$.

15. Combining Congruence and Similarity:

Can two shapes be both congruent and similar? Explain.

Solution:
Yes, if two shapes are congruent, they are also similar because they have the same shape and size (the scale factor is 1). Congruence implies similarity, with a scale factor of $1 : 1$.

16. Proving Congruence and Similarity:

a) Prove that two triangles are congruent if they have two sides and the included angle equal.

Solution:
This is the SAS (Side-Angle-Side) criterion for congruence. If two sides and the angle between them in one triangle are equal to two sides and the included angle of another, the triangles are congruent.

b) Determine if two triangles are similar if they have corresponding angles of 30° , 60° , and 90° .

Solution:
Yes, because if all corresponding angles are equal, the triangles are similar (AAA similarity criterion).

Additional Notes for Teachers:

Learning Outcomes: Students should understand different types of angles, their relationships, and how to use these concepts in geometric constructions and proofs.

Teaching Strategies: Use interactive geometry software or physical models to demonstrate angle relationships visually. Encourage sketching and labelling angles to help students visualise these concepts. Apply angle relationships in real-life scenarios like room planning or art projects.

Assessment: Assess through exercises where students identify, measure, or calculate angles based on given relationships, including problems involving triangles, quadrilaterals, and parallel lines.

Resources: Protractors, angle rulers, or digital tools for dynamic angle exploration.

This question set aligns with the Australian Curriculum for Year 8, focusing on the proficiencies of understanding, fluency, problem-solving, and reasoning in measurement and geometry, particularly in the context of angle relationships.