Probability, Tree Diagrams, and Venn Diagrams

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Focus: A set of questions and solutions for Year 8 students focused on 'Probability of Events' under the "Statistics and Probability" strand, tailored to the Australian Curriculum:

1. Understanding Probability:

a) Define probability. How is it measured?

Solution:

Probability is the measure of the likelihood that an event will occur. It is measured on a scale from 0 to 1, where 0 indicates impossibility, 1 indicates certainty, and values in between reflect increasing likelihood.

b) What does it mean for an event to have a probability of 0.75 ?

Solution:

An event with a probability of 0.75 has a $75\,\%$ chance of occurring, OR it is three times more likely to happen than not to happen.

P(likely to happen) $=\dfrac{3}{4}$, P(not likely to happen) $=\dfrac{1}{4}$.

2. Basic Probability Calculations:

a) If you roll a fair six-sided die, what is the probability of rolling a 4 ?

Solution:

There is 1 favourable outcome (rolling a 4) out of 6 possible outcomes, so:

$$P(\text{Event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}}$$

$$P(\text{rolling a } 4) = \frac{1}{6}, \approx 0.167\,, \approx 16.7\,\%\,.$$

b) What is the probability of drawing a spade from a standard deck of cards?

Solution:
There are 13 spades out of 52 cards, so:

$$P(\text{Event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}}$$

$$P(\text{drawing a spade}) = \frac{13 \div 13}{52 \div 13}$$
$$= \frac{1}{4}, = 0.25\,, = 25\,\%\,.$$

3. Complementary Events:

a) Explain what complementary events are in probability.

Solution:
Complementary events are two events that together cover all possible outcomes, so the sum of their probabilities must equal 1 .

For example, the probability of it raining or not raining on a given day. *See Question 1b. for example.*

b) If the probability that it will rain tomorrow is 0.4 , what is the probability that it will not rain?

Solution:
The probability of it not raining is the complement of raining, so:

$$P(not\ raining) = 1 - 0.4$$
$$= 0.6\,, = \frac{3}{5}\,, = 60\,\%\,.$$

4. Probability of Combined Events:

a) What is the probability of flipping two fair coins and getting at least one head?

Solution:
Possible outcomes: { HH, HT, TH, TT }. Three outcomes have at least one head { HH, HT, TH } , so :

$$P(\text{Event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}}$$

$$P(at\ least\ one\ head) = \frac{3}{4}, = 0.75, \ = 75\,\%\,.$$

b) If you draw two cards without replacement from a deck, what is the probability of getting two aces?

Solution:
First draw: $\frac{4}{52}$ (4 aces out of 52 cards).

Second draw: $\frac{3}{51}$ (3 aces left out of 51 cards).

Combined probability:

$$\begin{aligned} P(\text{one event AND another event}) &= P(1 \cap 2) \\ &= P(\text{event one}) \times P(\text{event two}) \\ &= \frac{4}{52} \times \frac{3}{51} \\ &= \frac{4 \times 3}{52 \times 51} \\ &= \frac{12 \div 12}{2652 \div 12} \\ &= \frac{1}{221}. \end{aligned}$$

5. Mutually Exclusive Events:

a) Define mutually exclusive events and give an example.

Solution:
Mutually exclusive events are events that cannot occur at the same time.

Example: Rolling a 1 and rolling a 6 on a *single* die roll; these events cannot happen simultaneously.

b) If $P(A) = 0.3$ and $P(B) = 0.5$, and A and B are mutually exclusive, what is $P(A\ or\ B)$?

Solution:
Since A and B are mutually exclusive:

$$\begin{aligned} P(A\ or\ B) &= P(A \cup B) \\ &= P(A) + P(B) \\ &= 0.3 + 0.5 \\ &= 0.8. \end{aligned}$$

6. Independent Events:

a) What are independent events?

Solution:
Events are independent if the occurrence of one does not affect the probability of the other.

Example: Flipping a coin twice; the result of the first flip does not influence the second.

b) If flipping a coin is independent, what is the probability of getting two tails in a row?

Solution:
Each flip has a probability of 0.5 for tails, so:

$$\begin{aligned} P(\text{one tail AND then another tail}) &= P(1 \cap 2) \\ &= P(\text{one tail}) \times P(\text{another tail}) \end{aligned}$$

$$\begin{aligned} P(two\ tails) &= 0.5 \times 0.5 \\ &= 0.25\,. \end{aligned}$$

7. Practical Application:

In a bag, there are 5 red marbles, 3 blue marbles, and 2 green marbles. What is the probability of drawing a blue marble?

Solution:

$$\begin{aligned} \text{Total marbles} &= 5 + 3 + 2 \\ &= 10\,. \end{aligned}$$

$$P(\text{Event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}}$$

$$P(\text{blue}) = \frac{3}{10}, = 0.3, \ = 30\,\%\,.$$

8. Conditional Probability:

a) Define conditional probability. Give an example.

Solution:
Conditional probability is the probability of an event given that another event has occurred.

Example: The probability of drawing a red card from a deck, given that the card is a heart, is 1 since all hearts are red. $P(\text{red heart}) = 1$. (as all hearts are red)

b) If you roll two dice and know one die shows a 3 , what is the probability that the sum is 7 ?

Solution:

With one die showing 3 , the only way to get a sum of 7 is, if the other die shows 4 .

As 4 appears only once, and there are 6 possible outcomes for the second die:

$$P(\text{Event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}}$$

$$P(\text{sum} = 7 \,|\, \text{one die is } 3) = \frac{1}{6}, \approx 0.167, \approx 16.7\,\%.$$

In mathematics, this line means “given that”. So the above reads,
Probability that the sum = 7, given that one die is 3 .

9. Two-way Tables, Tree Diagrams and Venn diagrams:

a) A student is packing their lunch and has the following choices:

Main item: **Sandwich (S)** or **Wrap (W)**
Drink: **Juice (J)** or **Water (T)**

Draw a a two-way table and list all possible combinations of the two events:

Main item (Sandwich or Wrap) and Drink (Juice or Water).

Solution:

The possible combinations are:
Sandwich and Juice (S, J)
Sandwich and Water (S, T)
Wrap and Juice (W, J)
Wrap and Water (W, T)
Total combinations $= 4$.

Two-way table:

		Drink	
		Juice (J)	**Water (T)**
Main Item	**Sandwich (S)**	S, J	S, T
	Wrap (W)	W, J	W, T

Possible outcomes: The 4 combinations: (S, J), (S, T), (W, J), (W, T) .

b) Draw a tree diagram to represent all possible combinations, and check the possible outcomes are the same as calculated in question 1a). What is the probability of choosing sandwich and juice? $P(S\, and\, J)$.

Solution:
A tree diagram visually represents the combinations by branching out from the first event (Main item) to the second event (Drink).

Tree diagram:

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      Start
     /     \
    S       W
   / \     / \
  J   T   J   T
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Possible Outcomes: (S,J) (S,T) (W,J) (W,T)

The tree diagram confirms the 4 combinations: (S, J), (S, T), (W, J), (W, T) .

$$P(Sandwich\ and\ Juice) = \frac{Favourable\ Outcome}{Total\ Number\ of\ Outcomes}$$

$$= \frac{1}{4} = 0.25 = 25\,\%\,.$$

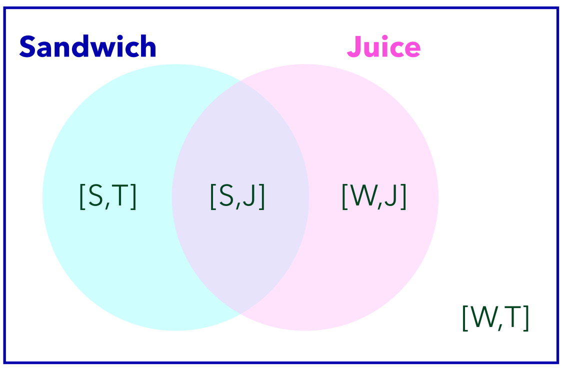
c) Use a Venn diagram to show the four possible outcomes, where Sandwich and Juice are considered "preferred" choices.

Solution:
For the Venn diagram, we consider "preferred" choices: Sandwich (S) for the main item and Juice (J) for the drink. We'll use two circles: one for Sandwich and one for Juice, with overlap for combinations involving both.

Possible outcomes: The 4 combinations: (S, J), (S, T), (W, J), (W, T).

Left circle (Sandwich): Includes (S, J) and (S, T)
Right circle (Juice): Includes (S, J) and (W, J)
Overlap: (S, J) (both Sandwich and Juice)
Outside both circles: (W, T) (neither Sandwich nor Juice)

Venn diagram:

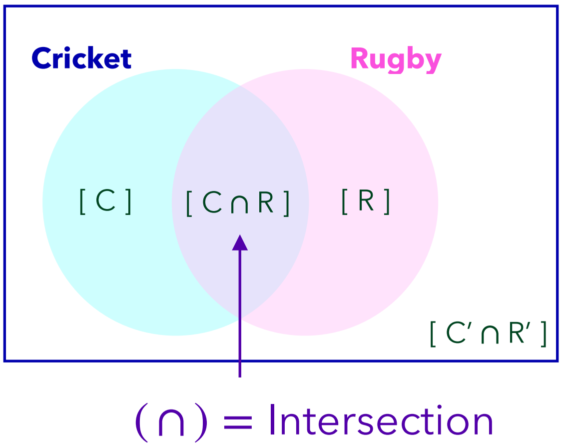


d) In a Venn diagram where set C represents students who play cricket, and set R represents students who play rugby, what does the region inside both circles represent and what does the region outside both circles represent?

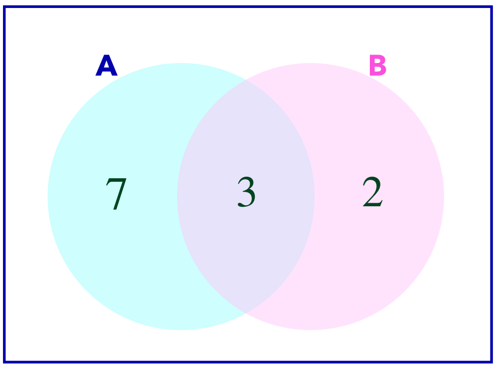
Solution:

C - Cricket   C′ - Not Cricket   R - Rugby   R′ - Not Rugby

Inside: Intersection of students who play both cricket and rugby. [C ∩ R]
Outside: Intersection of students who do not play cricket nor rugby. [C′ ∩ R′]



e) Draw a Venn diagram for two sets, A and B, where:
Set A has 10 elements
Set B has 5 elements
3 elements are in both A and B



Solution:
Description for drawing: Two overlapping circles, labelled A and B,

Circle A contains 10 elements, Circle B contains 5 elements, and there is an overlap of 3 elements,

Therefore, 3 elements are in A ∩ B, 7 elements are only in A ($10 - 3$), and 2 elements are only in B ($5 - 3$).

Additional Notes for Teachers:

Learning Outcomes: Students should understand how to calculate simple and compound probabilities, recognise different types of events, and apply probability concepts to real-life scenarios.

Teaching Strategies: Use simulations or physical activities like coin flipping, dice rolling, or drawing cards to demonstrate probability. Encourage students to predict outcomes before calculating probabilities to test their understanding. Discuss real-world scenarios where probability plays a role, like weather forecasts or game strategies.

Assessment: Assess through problems involving calculating probabilities, determining if events are independent or mutually exclusive, and solving conditional probability problems.

Resources: Dice, coins, decks of cards, or probability games/apps for interactive learning.

This question set aligns with the Australian Curriculum for Year 8, focusing on the proficiencies of understanding, fluency, problem-solving, and reasoning in statistics and probability, specifically in the context of the probability of events.