Algebraic Techniques, and Scientific Notation

9

Free and always will be!

Focus: A comprehensive set of questions and solutions for Year 9 students focused on 'Algebraic Techniques' under the "Number and Algebra" strand, tailored to the Australian Curriculum:

1. Expanding and Simplifying:

Remember, mathematicians sometimes use the symbol $(\cdot)$ instead of $(\times)$ so we don't get $(\times)$ confused with (x)
E.g. $2 \times x \equiv 2 \cdot x$

a) Expand and simplify $(x+3)(x-2)$.

Solution:

$$(x+3)(x-2)$$ *Use Crab Claw*

$(+ \times - = -)$ If signs are:
opposite → change to −
same → change to +

Use the distributive property (FOIL method) , A.K.A. Crab Claw :

$$\begin{aligned}(x+3)(x-2) &= x \cdot x + x \cdot (-2) + 3 \cdot x + 3 \cdot (-2) \\ &= x^2 \boxed{-2x + 3x} - 6\,,\end{aligned}$$

Combine like terms:

$$= x^2 + x - 6\,.$$

b) Simplify $2x - 5 + 3x + 7$.

Solution:
Combine like terms:

$$\begin{aligned}\boxed{2x} - 5 \boxed{+3x} + 7 &= 2x + 3x - 5 + 7 \\ &= 5x + 2\,.\end{aligned}$$

c) Simplify $3x^2 + 2x - 5 - x^2 + 3x + 7$.

Solution:
Combine like terms:

$$\begin{aligned}\boxed{3x^2} + 2x - 5 \boxed{-x^2} + 3x + 7 &= 3x^2 - x^2 + 2x + 3x - 5 + 7 \\ &= 2x^2 + 5x + 2\,.\end{aligned}$$

2. Factorising:

a) Factorise $x^2 - 5x + 6$**.**

Solution:

$$x^2 - 5x + 6$$

$$\_ \times \_ = 6 \ and \ \_ + \_ = -5$$
$$\rightarrow -3 \times -2 = 6 \ and \ -3 + -2 = -5$$

$$= (x-3)(x-2).$$

OR

$$x^2 \ -5x \ + 6$$

$$\_ \times \_ = +6 \ and \ \_ + \_ = -5$$
$$\rightarrow -3 \times -2 = +6 \ and \ -3 + -2 = -5$$

$$= (x - 3)(x - 2)$$

$$= (x-3)(x-2).$$

b) Factorise $3x^2 + 15x$**.**

Solution:
Factor out the greatest common factor ($3x$) :

$$\begin{aligned} 3x^2 + 15x &= 3x \cdot x \ + \ 3x \cdot 5 \\ &= 3x \cdot (x+5) \\ &= 3x(x+5). \end{aligned}$$

3. Perfect Squares and Difference of Two Squares:

a) Expand $(x+4)^2$**.**

Solution:
Re-write, expand, then collect like terms:

$$(x+4)^2 = (x+4)(x+4) \quad \textit{Use Crab Claw}$$

$$\begin{aligned} &= x \cdot x \ + x \cdot 4 \ + 4 \cdot x + 4 \cdot 4 \\ &= x^2 + 4 \cdot x + 4 \cdot x + 16 \\ &= x^2 + 8 \cdot x + 16 \\ &= x^2 + 8x + 16. \end{aligned}$$

OR
Using rule for perfect squares:

$$\begin{aligned} (a+b)^2 &= a^2 + 2ab + b^2 \\ (x+4)^2 &= x^2 + 2 \cdot x \cdot 4 \ + 4^2 \\ &= x^2 + 8x + 16. \end{aligned}$$

Rules for perfect squares:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$

b) Factorise $x^2 - 16$**.**

Solution:
This is a difference of two squares:

$$\begin{aligned} x^2 - 16 &= x^2 - 4^2 \\ &= (x+4)(x-4). \end{aligned}$$

Rule for difference of two squares:

$$a^2 - b^2 = (a+b)(a-b)$$

4. Completing the Square:

Complete the square for $x^2 + 6x$.

Remember, $(+b - b = 0)$, so by adding and subtracting $b\ (= \frac{a}{2})$, we aren't changing the equation, just making it look different.

Solution:
Take half of the coefficient of $x = (6/2)$, square it $= (9)$,

Then, add and subtract it:

$$\begin{aligned} &\to x^2 + 6x \\ &= x^2 + 6x + (\frac{6}{2})^2 - (\frac{6}{2})^2 \\ &= x^2 + 6x + 3^2 - 3^2 \\ &= [x^2 + 6x + 3^2] - 9\,, \end{aligned}$$

Rule for perfect squares:

$(a+b)^2 = a^2 + 2ab + b^2$

$a^2 + 2ab + b^2 = (a+b)^2$

Using rule for perfect squares:

$$\begin{aligned} [x^2 + 6x + 3^2] - 9 &= [x^2 + 2 \cdot x \cdot 3 + 3^2] - 9 \\ &= [(x+3)^2] - 9\,. \\ &= (x+3)^2 - 9\,. \end{aligned}$$

OR

Complete the square for $x^2 + 6x$.

Solution:
Using the rule for completing the square:

$$x^2 + ax = \left[\left(x + \left(\frac{a}{2}\right)\right)^2\right] - \left(\frac{a}{2}\right)^2$$

$$\begin{aligned} x^2 + ax &= [x + (\frac{6}{2})]^2 - (\frac{6}{2})^2 \\ &= [x+3]^2 - 3^2\,. \\ &= [x+3]^2 - 9\,. \end{aligned}$$

Rule for completing the square:

$$x^2 + ax = x^2 + ax \ + (\frac{a}{2})^2 - (\frac{a}{2})^2 \quad \left[= x^2 + ax + b^2 - b^2\,, \ \to \ b = \frac{a}{2}\,. \right]$$

$$\begin{aligned} &= [x^2 + 2 \cdot x \cdot (\frac{a}{2}) + (\frac{a}{2})^2] - (\frac{a}{2})^2 \\ &= [x^2 + 2ab + (\frac{a}{2})^2] - (\frac{a}{2})^2 \\ &= [\text{Perfect Square}] - \left(\frac{a}{2}\right)^2 \end{aligned}$$

Rule for perfect squares:

$(a+b)^2 = a^2 + 2ab + b^2$

$a^2 + 2ab + b^2 = (a+b)^2$

$$x^2 + ax = \left[\left(x + \left(\frac{a}{2}\right)\right)^2\right] - \left(\frac{a}{2}\right)^2$$

5. Simplifying Algebraic Fractions:

Simplify $\dfrac{2x+4}{x^2-4}$.

Solution:
Factorise both numerator and denominator:

$$\begin{aligned}\frac{2x+4}{x^2-4} &= \frac{2\cdot x+2\cdot 2}{x^2-2^2}\\ &= \frac{2(x+2)}{(x+2)(x-2)}\\ &= \frac{2\cancel{(x+2)}}{\cancel{(x+2)}(x-2)} \;\; \vert \; x \neq -2\\ &= \frac{2}{x-2}.\end{aligned}$$

Remember, " | " = "Such That"

(so there isn't a zero on the bottom)
(i.e. we aren't dividing by zero)

6. Solving Quadratic Equations:

a) Solve the equation $x^2 - 5x + 6 = 0$ by factorisation.

Solution:
Factorise:

$$x^2 - 5x + 6 = 0$$
$$\rightarrow (x-2)(x-3) = 0$$

$$(x-2)(x-3) = 0$$
$$\frac{(x-2)\cancel{(x-3)}}{\cancel{(x-3)}} = \frac{0}{(x-3)} \quad , \frac{0}{(x-3)} = 0$$
$$(x-2) = 0.$$

$$(x-2)(x-3) = 0$$
$$\frac{\cancel{(x-2)}(x-3)}{\cancel{(x-2)}} = \frac{0}{(x-2)} \quad , \frac{0}{(x-2)} = 0$$
$$(x-3) = 0.$$

$$\rightarrow x - 2 = 0 \;\; or \;\; x - 3 = 0$$

$$x - 2 = 0$$
$$x \cancel{-2} \cancel{+2} = 0 \; +2$$
$$x = 2.$$

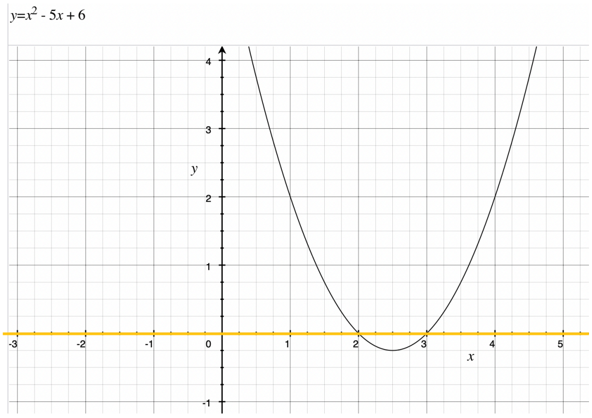
$$x - 3 = 0$$
$$x \cancel{-3} \cancel{+3} = 0 \; +3$$
$$x = 3.$$

This gives:

$$x = 2 \;\; or \;\; x = 3.$$

All we are doing here is finding where the equation: $y = x^2 - 5x + 6$, passes through the $x - axis$, or where it passes through $y = 0$.

Graphical representation of:
Setting $y = 0 \;\; in \;\; y = x^2 - 5x + 6$



$y = 0$

b) Use completing the square to solve $x^2 + 6x - 7 = 0$.

Solution:
Complete the square:

$$x^2 + 6x + 9 - 9 - 7 = 0$$
$$(x+3)^2 - 16 = 0$$
$$(x+3)^2 = 16$$
$$x + 3 = \pm 4$$

Solutions:

$$x = +4 - 3$$
$$= 1,$$

AND

$$x = -4 - 3$$
$$= -7.$$

OR

$$x^2 + 6x - 7 = 0$$

Complete the square:

$$x^2 + 6x - 7 + 9 - 9 = 0$$
$$[x^2 + 6x + 9] - 9 - 7 = 0$$
$$[x^2 + 6x + 3^2] - 9 - 7 = 0$$
$$[x^2 + 2 \cdot x \cdot 3 + 3^2] - 9 - 7 = 0$$
$$(x+3)^2 - 16 = 0$$
$$(x+3)^2 = 16$$
$$\cancel{\sqrt{(x+3)^{\cancel{2}}}} = \sqrt{16}$$
$$x + 3 = \pm\sqrt{16}$$
$$x + 3 = \pm 4.$$

Solutions:

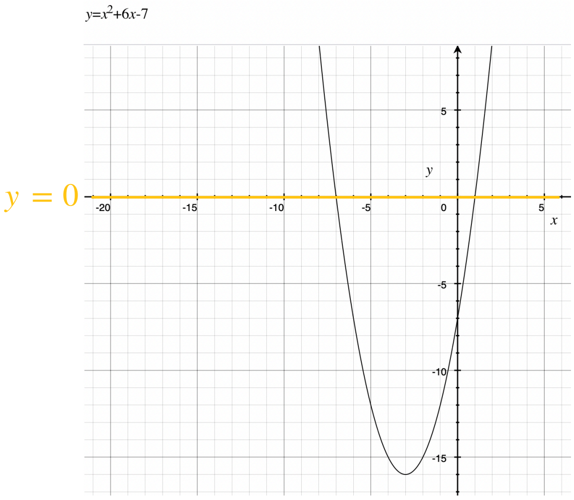
$$x + 3 = 4$$
$$x \cancel{+3} \cancel{-3} = +4 - 3$$
$$x = 1,$$

AND

$$x + 3 = -4$$
$$x \cancel{+3} \cancel{-3} = -4 - 3$$
$$x = -7.$$

All we are doing here is finding where the equation: $y = x^2 + 6x - 7$, passes through the $x - axis$, or where it passes through $y = 0$.

Graphical representation of:
Setting $y = 0 \ in \ y = x^2 + 6x - 7$



7. Practical Application:

The area of a rectangle is given by $x^2 + 5x + 6 \, square \;\; metres$ **. What are the possible dimensions of the rectangle in terms of** x **?**

Solution:
Factorise the area to find dimensions:

$$\begin{aligned} & x^2 + 5x + 6 \\ &= (x+2)(x+3)\,, \\ &= (x+2) \times (x+3)\,. \end{aligned}$$

$$\text{Area} = \text{Length} \times \text{Width}$$

Possible dimensions:

$$\text{Length} = (x+2)\,m\,,$$
$$\text{Width} = (x+3)\,m\,.$$

8. Manipulating Algebraic Fractions:

Simplify $\dfrac{x}{x^2-1} - \dfrac{1}{x+1}$ **.**

Solution:

$$\begin{aligned} \frac{x}{x^2-1} - \frac{1}{x+1} &= \frac{x}{x^2-1^2} - \frac{1}{x+1} \times \frac{(x-1)}{(x-1)} \\ &= \frac{x}{(x+1)(x-1)} - \frac{1(x-1)}{(x+1)(x-1)} \\ &= \frac{x \times 1}{(x+1)(x-1)} - \frac{(x-1) \times 1}{(x+1)(x-1)} \\ &= \left[x - (x-1)\right] \times \left[\frac{1}{(x+1)(x-1)}\right] \\ &= \frac{x-(x-1)}{(x+1)(x-1)} \\ & \frac{x-x+1}{(x+1)(x-1)} \\ &= \frac{1}{(x+1)(x-1)}\,, \end{aligned}$$

Simplify further:

$$\begin{aligned} &\rightarrow \frac{1}{(x+1)(x-1)} \\ &= \frac{1}{x^2-1^2} \\ &= \frac{1}{x^2-1}\,. \end{aligned}$$

9. Scientific Notation:

a) Convert $3{,}000{,}000$ **to scientific notation.**

Solution:
$3{,}000{,}000$ in scientific notation is:

6 5 4 3 2 1

$3{,}000{,}000$

3×10^6.

b) Convert $3{,}141{,}000$ **to scientific notation.**

Solution:
$3{,}141{,}000$ in scientific notation is:

6 5 4 3 2 1

$3{,}141{,}000$

3.141×10^6.

c) Convert 0.00108 **to scientific notation.**

Solution:
0.00108 in scientific notation is:

1 2 3

0.00108

1.08×10^{-3}.

d) Convert $h = 0.000000000000000000000000000000000006626 \dfrac{J}{Hz}$ $(Planck's\ Constant)$ **to scientific notation.**

Solution:
$0.000000000000000000000000000000000006626$ in scientific notation is:

$h = 6.626 \times 10^{-34} \dfrac{J}{Hz}$.

e) Convert $2{,}997{,}925$ **to scientific notation with three significant figures.**

Solution:
First, round $2{,}997{,}925$ to three significant figures:

$= 2{,}990{,}000$

6 5 4 3 2 1

$= 2{,}990{,}000$

$= 2.99 \times 10^6$.

10. Operations with Scientific Notation:

a) Multiply $(2 \times 10^3) \times (3 \times 10^2)$**.**

Solution:
Multiply the numbers and add the exponents:

$$
\begin{aligned}
2 \times 3 &= 6 \\
10^3 \times 10^2 &= 10^{3+2} \\
&= 10^5
\end{aligned}
$$

Result: 6×10^5.

b) Divide $\dfrac{8 \times 10^9}{4 \times 10^6}$**.**

Solution:
Divide the numbers and subtract the exponents:

$$\frac{8}{4} = 2$$

$$
\begin{aligned}
10^9 \div 10^6 &= 10^{9-6} \\
&= 10^3
\end{aligned}
$$

Result: 2×10^3.

c) Simplify $(2 \times 10^3)^4$**.**

Solution:
Apply the power to each term:

$$
\begin{aligned}
(2 \times 10^3)^4 &= 2^4 \times 10^{3 \times 4} \\
&= 16 \times 10^{12}.
\end{aligned}
$$

d) Evaluate $(2.161 \times 10^3)^0$**.**

Solution:
Anything to the power of zero equals one. $(2.161 \times 10^3)^0 = 1$.

11. Practical Application:

The distance from Earth to the Sun is approximately $149{,}600{,}000\ kilometres$**. Express this distance in scientific notation with two significant figures.**

Solution:
Firstly round $149{,}600{,}000\ km$ to two significant figures:
$150{,}000{,}000\ km$, then convert to scientific notation:
$1.5 \times 10^8\ km$.

Additional Notes for Teachers:

Learning Outcomes: Students should master techniques like expanding, factorising, completing the square, and solving quadratic equations, applying these in various contexts.

Teaching Strategies:
Use visual aids like algebra tiles to demonstrate expansion and factorisation.
Employ a step-by-step approach to solving equations to build confidence and understanding.
Relate algebraic techniques to real-world problems like area, volume, or optimisation.

Assessment: Assess through tasks that require students to simplify, solve, factorise, or expand expressions, and apply these techniques to solve problems.

Resources: Algebra tiles, graphing software to visualise quadratic functions, or interactive algebra apps for practice.

This question set aligns with the Australian Curriculum for Year 9, focusing on the proficiencies of understanding, fluency, problem-solving, and reasoning in number and algebra, specifically in the context of algebraic techniques.