WORKSHEET # 020 | Australian Curriculum Year 9 Mathematics | WM9T2U4

Simultaneous Equations, and Circles 9

Free and always will be!

Focus: A set of questions and solutions for Year 9 students focused on 'Simultaneous Equations' under the "Number and Algebra" strand, and 'Circles' under the "Measurement and Geometry", tailored to the Australian Curriculum:

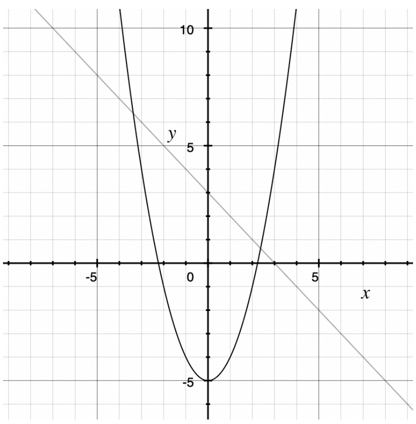
1. Understanding Simultaneous Equations:

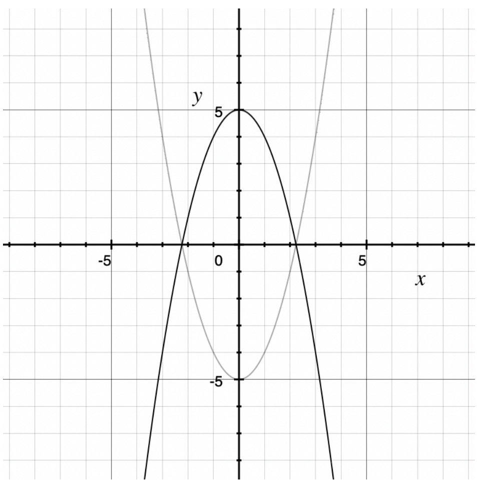
a) What are simultaneous equations?

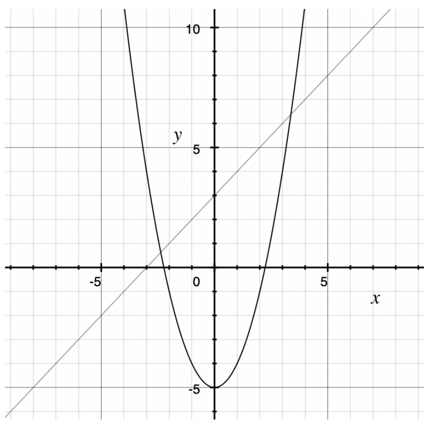
Solution:
Simultaneous equations are a set of equations with multiple variables that need to be solved at the same time. The solutions are the values of the variables that make all equations true simultaneously.

b) Why do we need more than one equation to solve for two variables?

Solution:
With one equation, you only have one relationship between the variables, which typically results in infinitely many solutions. Two or more equations provide enough constraints to find a unique solution (or no solution or infinitely many in some cases).







2. Solving by Substitution Method:

Solve using substitution: $\begin{cases} y = 2x + 1 \\ 3x + y = 10 \end{cases}$.

Solution:

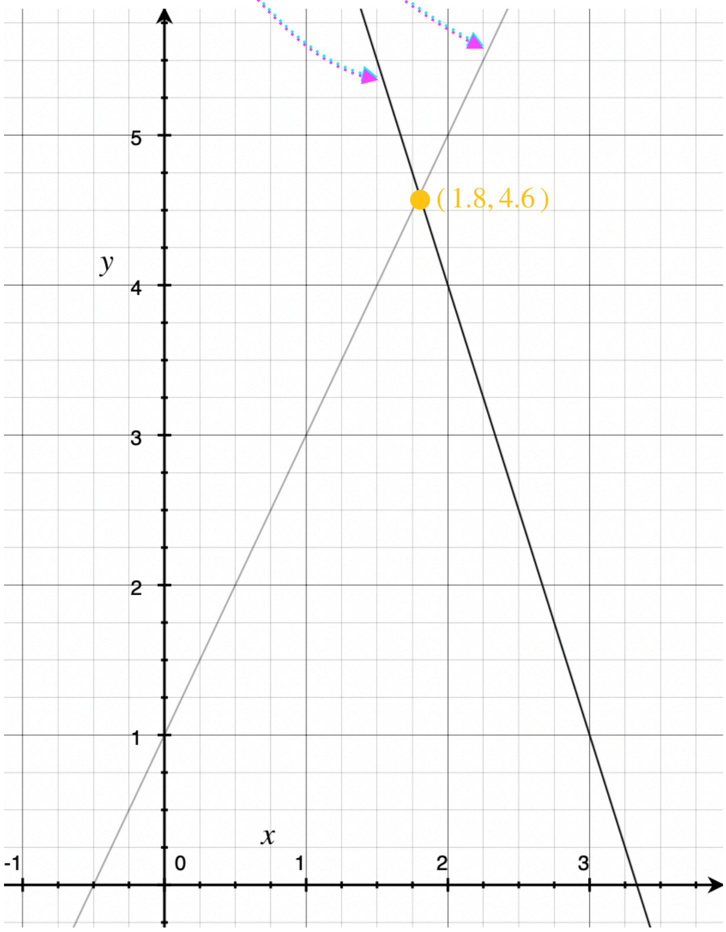
$$\begin{cases} y = 2x + 1 \\ 3x + y = 10 \end{cases}$$

Substitute $y = 2x + 1$ into the second equation:

$$3x + y = 10$$
$$3x + (2x + 1) = 10$$
$$5x + 1 = 10$$
$$5x = 9$$
$$x = \frac{9}{5}. (= 1.8)$$

Substitute $x = \frac{9}{5}$ back into $y = 2x + 1$:

$$y = 2 \times (\frac{9}{5}) + 1$$
$$y = (\frac{2 \times 9}{5}) + 1$$
$$y = (\frac{18}{5}) + 1$$
$$y = (\frac{18}{5}) + 1 \times \frac{5}{5}$$
$$y = \frac{18}{5} + \frac{5}{5}$$
$$y = \frac{23}{5}. (= 4.6)$$



Solution:

$$x = \frac{9}{5}, y = \frac{23}{5}.$$
$$x = 1.8, y = 4.6.$$

3. Solving by Elimination Method:

Solve using elimination: $\begin{cases} 2x + 3y = 11 \\ x - 3y = -4 \end{cases}$.

Solution:

$$\begin{cases} 2x + 3y = 11 \\ x - 3y \ \ = -4 \end{cases}$$

Add the two equations to eliminate y :

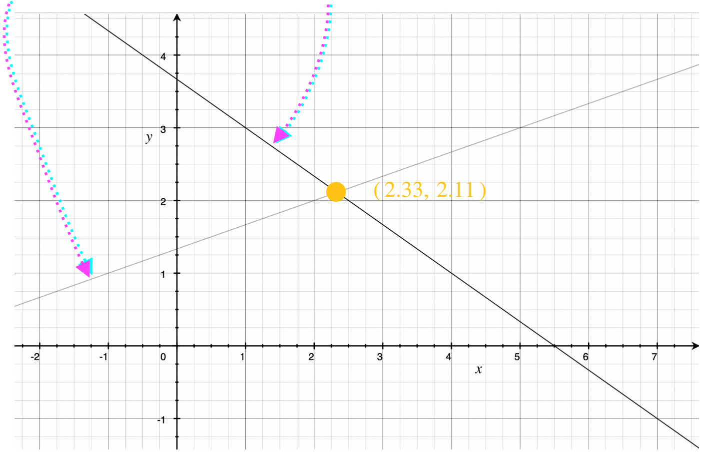
$$2x + 3y \ + (x - 3y) = 11 + (-4)$$
$$2x + x + \cancel{3y} \cancel{-3y} = 11 - 4$$
$$3x = 7$$
$$\frac{\cancel{3}x}{\cancel{3}} = \frac{7}{3}$$
$$x = \frac{7}{3}. \ (x \approx 2.33)$$

Substitute $x = \dfrac{7}{3}$ into one of the original equations, say $x - 3y = -4$:

$$\frac{7}{3} - 3y = -4$$

Multiply both sides by 3 to clear the fraction:

$$\begin{cases} 2x + 3y = 11 \\ x - 3y = -4 \end{cases}$$



$$3 \times \left[\frac{7}{3} - 3y\right] = 3 \times [-4]$$
$$\cancel{3} \times \frac{7}{\cancel{3}} - 3 \times 3y = 3 \times -4$$
$$7 - 9y = -12$$
$$\cancel{7} - 9y \cancel{-7} = -12 \ -7$$
$$-9y = -19$$
$$\frac{\cancel{-9}y}{\cancel{-9}} = \frac{-19}{-9}$$
$$y = \frac{19}{9}. \ (y \approx 2.11)$$

Solution: $x = \dfrac{7}{3}, \ y = \dfrac{19}{9}$.

$(x \approx 2.33, \ y \approx 2.11)$

4. Solving Word Problems with Simultaneous Equations:

Jack has 5 more apples than Mary. Together they have 23 apples. How many apples does each have?

Solution:
Let J be the number of apples Jack has and M the number Mary has.

Equations:

$$\begin{cases} J = M + 5\,, \\ J + M = 23\,. \end{cases}$$

Substitute J from the first equation into the second:

Using $J = M + 5$:

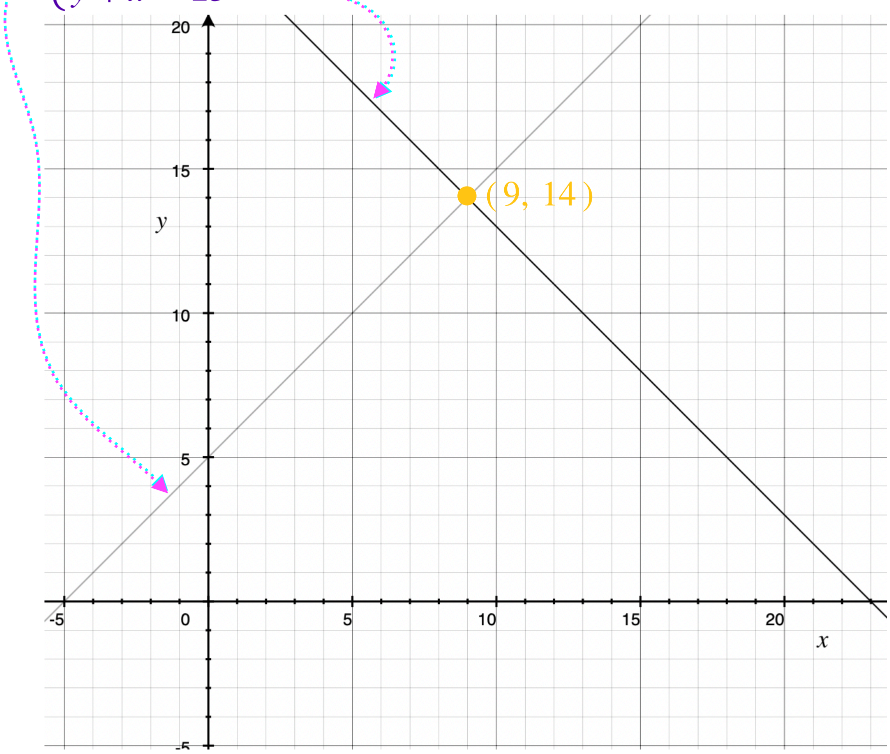
$$\begin{aligned} \rightarrow J + M &= 23 \\ (M+5) + M &= 23 \\ 2M + 5 &= 23 \\ 2M \cancel{+5} \cancel{-5} &= 23 - 5 \\ 2M &= 18 \\ \frac{\cancel{2}M}{\cancel{2}} &= \frac{18}{2} \\ M &= 9\,. \end{aligned}$$

And $J = M + 5$

$$\begin{aligned} \rightarrow J &= 9 + 5 \\ J &= 14\,. \end{aligned}$$

$$\begin{cases} y = x + 5 \\ y + x = 23 \end{cases}$$

Jack has 14 apples, and Mary has 9 apples.



5. Graphical Interpretation:

Explain how to solve simultaneous equations graphically.

Solution: Plot the lines represented by each equation on the same coordinate plane. The point where the lines intersect represents the solution to the system of equations. If the lines are parallel, there is no solution; if they are the same line, there are infinitely many solutions.

6. Linear and Non-Linear Systems:

Solve this system where one equation is linear and one is quadratic: $\begin{cases} y = x^2 - 4 \\ y = 2x \end{cases}$**.**

Solution:

$$\begin{cases} y = x^2 - 4 \\ y = 2x \end{cases}$$

Set the equations equal to each other:

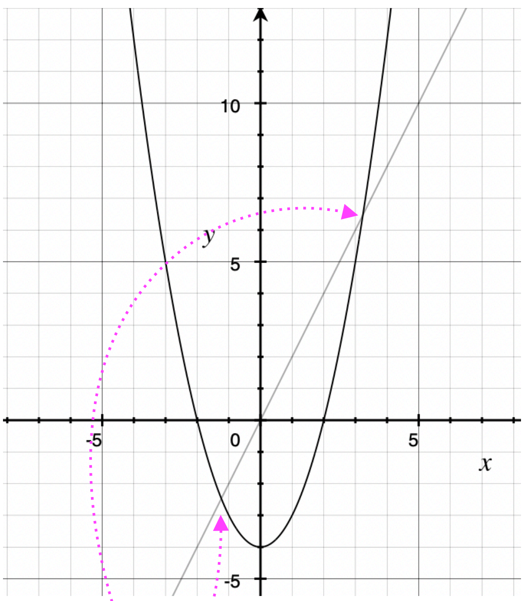
$$\cancel{2x}\cancel{-2x} = x^2 - 4 - 2x$$

$$x^2 - 2x - 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

Solve the quadratic equation:

Use the quadratic formula with $a = 1,\ b = -2,\ c = -4$:



$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times -4}}{2 \times 1}$$

$$x = \frac{2 \pm \sqrt{4 + 16}}{2}$$

$$= \frac{2 \pm \sqrt{20}}{2}$$

$$= \frac{2 \pm \sqrt{4 \times 5}}{2}$$

$$= \frac{2}{2} \pm \frac{2\sqrt{5}}{2}$$

$$x = 1 \pm 1\sqrt{5}$$

$$x = 1 \pm \sqrt{5}$$

Find corresponding y values:

For $x = 1 + \sqrt{5},\ y = 2(1 + \sqrt{5})$

For $x = 1 - \sqrt{5},\ y = 2(1 - \sqrt{5})$

Solutions: $(1 + \sqrt{5},\ 2 + 2\sqrt{5})$ and $(1 - \sqrt{5},\ 2 - 2\sqrt{5})$.

$(\approx 3.24,\ \approx 6.46)$ and $(\approx -1.24,\ \approx -2.47)$

7. Practical Application:

A shop sells small and large candles. Small candles cost $\$2$ each, and large candles cost $\$5$ each. If a total of 12 candles were sold for $\$44$, how many of each type were sold?

Solution:
Let s be the number of small candles and L be the number of large candles.

Equations:

$s + L = 12$ candles ,

$$s + L = 12\,.$$

Re-arrange to get $s = \ldots$,

$$s + \cancel{L} - \cancel{L} = 12 \; -L$$

$$\rightarrow s = 12 - L\,.$$

$$\$2s + \$5L = \$44\,,$$

$$2s + 5L = 44\,.$$

Solve using substitution (or elimination) :

Using $s = 12 - L$,

substitute into the second equation :

$$\rightarrow 2s + 5L = 44$$

$$2(12 - L) + 5L = 44$$

$$24 - 2L + 5L = 44$$

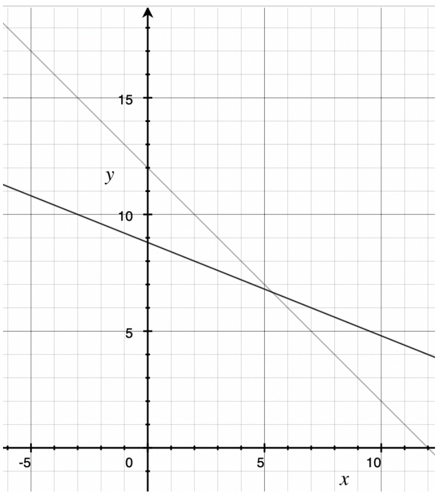
$$24 + 3L = 44$$

$$\cancel{24} + 3L - \cancel{24} = 44 \; -24$$

$$3L = 20$$

$$\frac{\cancel{3}L}{\cancel{3}} = \frac{20}{3}$$

$$L = \frac{20}{3}$$



$L \approx 6.67$ (we can't have a fraction of a candle,

so we round down to 6) :

$$\rightarrow L \approx 6\,.$$

$$s = 12 - L$$

$$s \approx 12 - 6$$

$$\rightarrow s \approx 6\,.$$

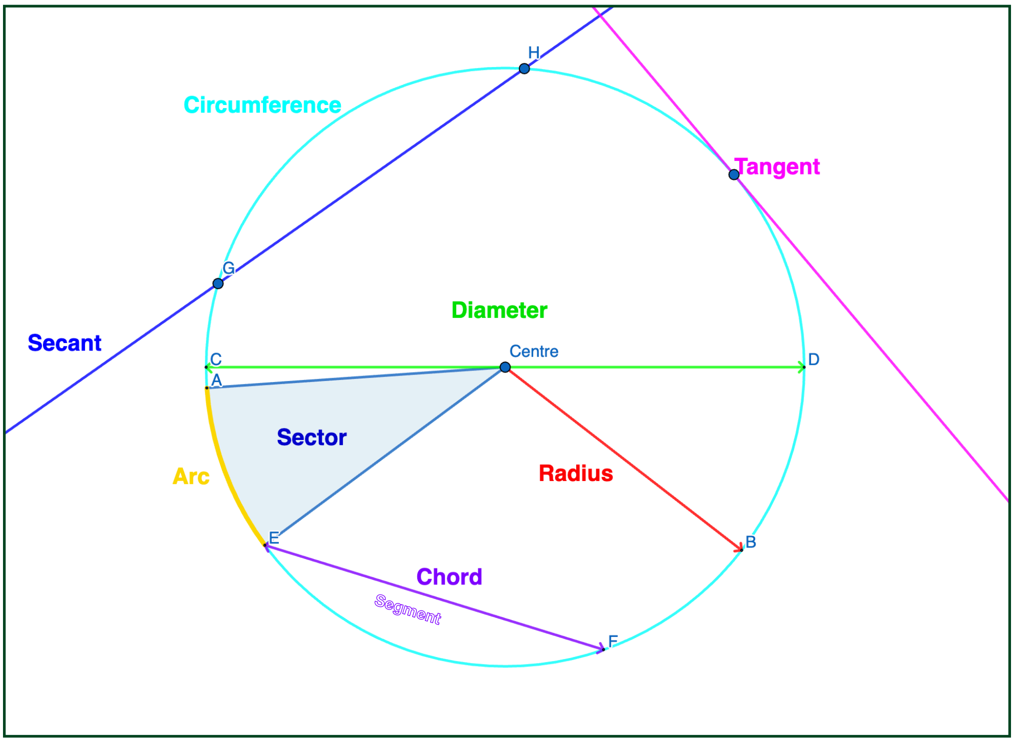
Solution:

6 large candles and 6 small candles were sold .

8. Understanding Circles:

a) Define a circle and identify its key components.

Solution:
A circle is a set of all points in a plane that are equidistant (the same distance) from a given point, called the centre. Key components include:
Centre: The fixed point from which all points on the circle are equidistant.
Radius (r) **:** The distance from the centre to any point on the circle.
Diameter (D) **:** The longest chord, passing through the centre, which is twice the radius $(D = 2r)$.
Circumference (C) **:** The perimeter of the circle.
Chord: A line segment whose endpoints lie on the circle.
Arc: A portion of the circumference.
Sector: A part of the circle enclosed by two radii and an arc.
Tangent: A line perpendicular to the circle, passing through one point on the outside of the circle.
Secant: A line passing through two points on the circle.
Segment: A part of the circle enclosed by a chord and the outside of the circle.



b) What is π (pi) and how is it used in relation to circles?

Solution:
π is an irrational number, approximately 3.14159,
Used to describe the ratio of a circle's circumference to its diameter.
It's used in formulas for the circumference and area of a circle:

$C = \pi d$ or
$C = 2\pi r$,
$A = \pi r^2$.

9. Calculating Circumference:

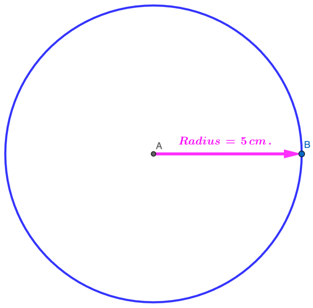
a) Find the circumference of a circle with a radius of $5\,cm$.

Solution:

$r = 5\,cm$.

$C = ?$.

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi \times 5 \\ &= 10\pi, \\ &\approx 31.42\,cm. \end{aligned}$$



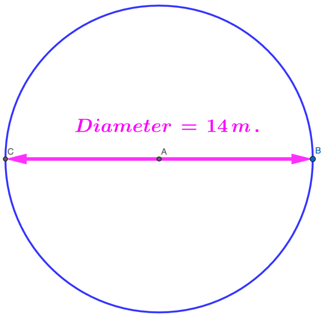
b) If the diameter of a circle is $14\,metres$, what is its circumference?

Solution:

$D = 14\,cm$.

$C = ?$.

$$\begin{aligned} C &= \pi d \\ &= \pi \times 14 \\ &= 14\pi, \\ &\approx 43.98\,metres. \end{aligned}$$



10. Area of a Circle:

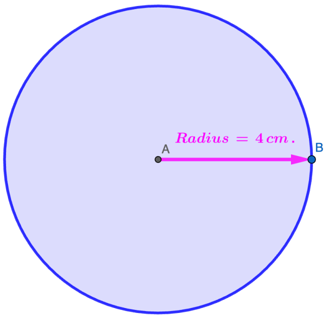
a) Calculate the area of a circle with a radius of $4\,cm$.

Solution:

$r = 4\,cm$.

$A = ?$.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 4^2 \\ &= 16\pi, \\ &\approx 50.27\,cm^2. \end{aligned}$$

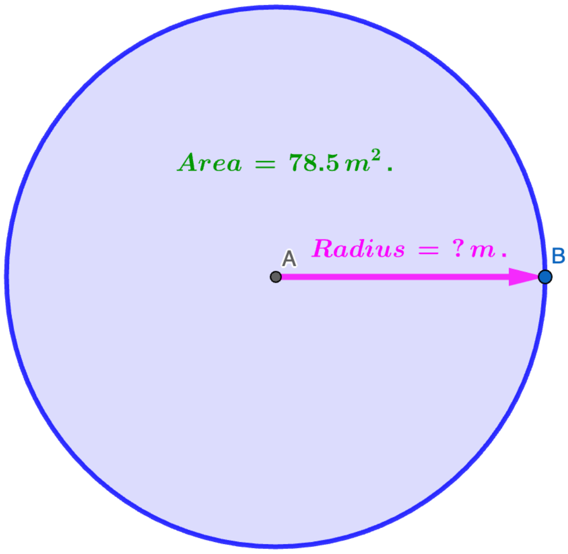


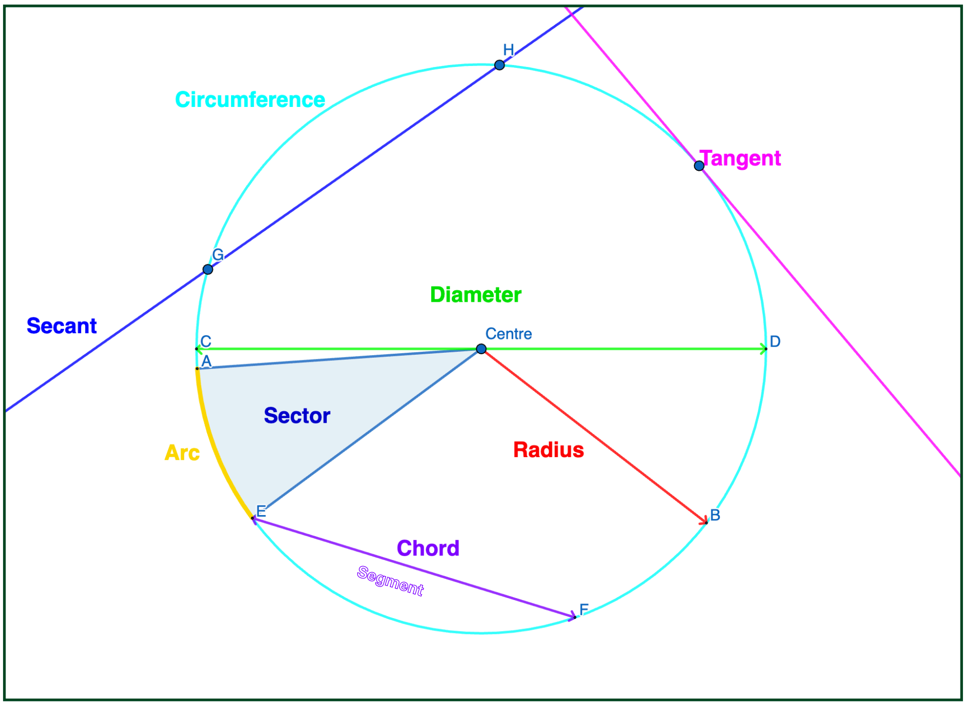
b) A circle has an area of $78.5\, square\ metres$ **. What is its radius?**

Solution:

$$A = 78.5\, m^2 .$$
$$r = ?\, m .$$

$$A = \pi r^2$$
$$\rightarrow 78.5 = \pi r^2$$
$$\pi r^2 = 78.5$$
$$\frac{\cancel{\pi} r^2}{\cancel{\pi}} = \frac{78.5}{\pi}$$
$$r^2 = \frac{78.5}{\pi}$$
$$\cancel{\sqrt{r^2}} = \sqrt{\frac{78.5}{\pi}}$$
$$r = \sqrt{\frac{78.5\, m^2}{\pi}}$$
$$r = \sqrt{\frac{78.5}{\pi}}\cancel{\sqrt{m^2}}$$
$$r = \sqrt{\frac{78.5}{\pi}}\, m ,$$
$$\approx 5\, metres .$$





11. Arc Length and Sector Area:

a) Find the length of an arc if the radius is $6\,cm$ and the angle is $60\,degrees$.

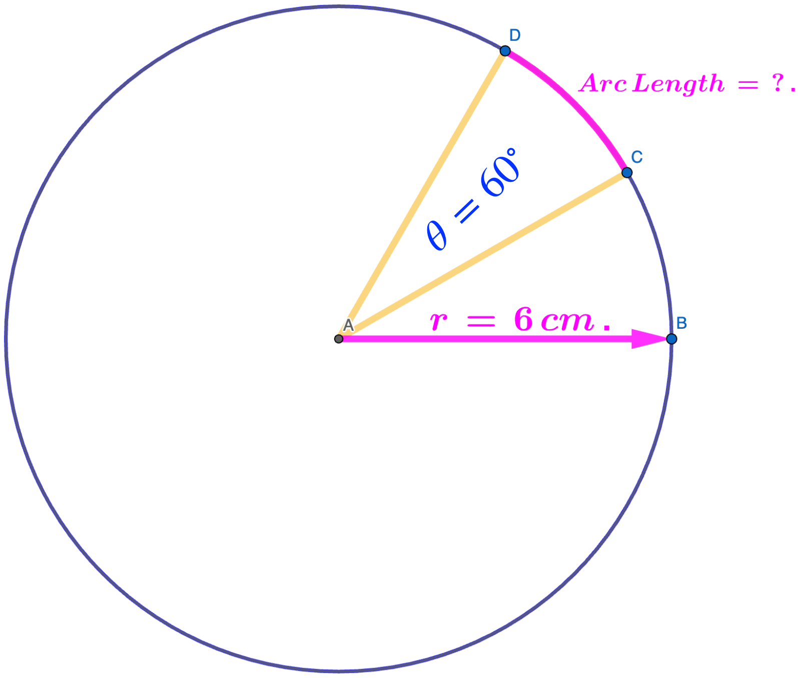
Solution:

$$r = 6\,cm\,.$$
$$\theta = 60^\circ\,.$$

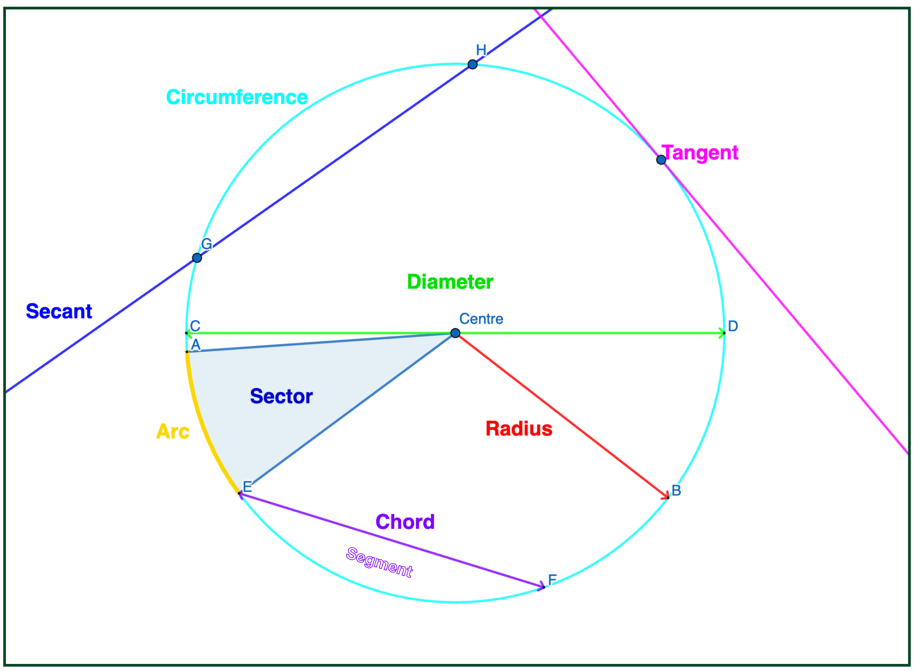
Arc Length $= ?\,.$

Arc Length $=$ Fraction of Circle $\times$ Circ

$$= \frac{\theta}{360} \times \text{Circumference}\,.$$



$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r$$
$$= \frac{60}{360} \times 2\pi \times 6$$
$$= \frac{60 \div 60}{360 \div 60} \times 2\pi \times 6$$
$$= \frac{1}{6} \times 2\pi \times 6$$
$$= \frac{2\pi}{6} \times 6$$
$$= \frac{2\pi \times 6}{3}$$
$$= \frac{12\pi}{3}$$
$$= 4\pi\,,$$
$$\approx 12.57\,cm\,.$$

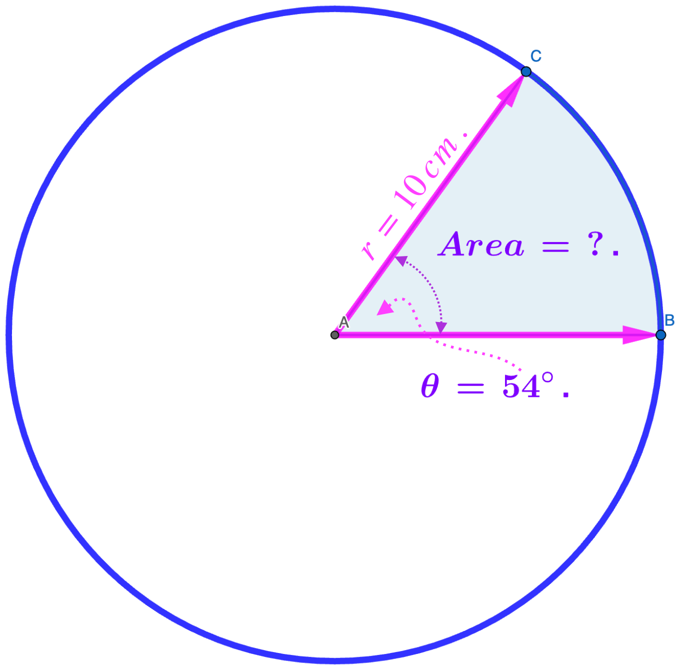


b) Calculate the area of a sector with a radius of $10\,cm$ and an angle of $54\,degrees$.

Solution:

$$r = 10\,cm.$$
$$\theta = 54^\circ.$$
$$\text{Sector Area} = ?.$$

$$\text{Sector Area} = \text{Fraction of Circle} \times \text{Area of Circle}$$
$$= \frac{\theta}{360} \times \text{Area of Circle}$$



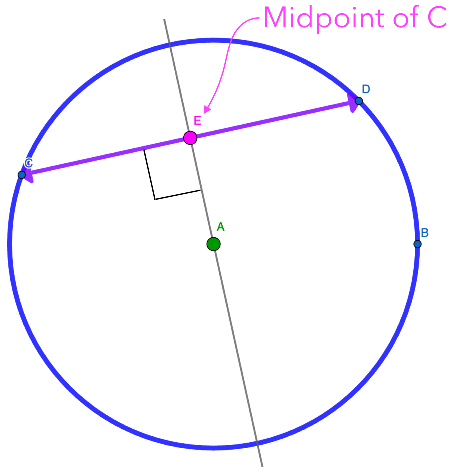
$$\begin{aligned}\text{Sector Area} &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{54}{360} \times \pi \times 10^2 \\ &= \frac{54 \div 6}{360 \div 6} \times \pi \times 10 \times 10 \\ &= \frac{9 \div 3}{60 \div 3} \times 100\pi \\ &= \frac{3 \times 100\pi}{20} \\ &= \frac{30\cancel{0}\pi}{2\cancel{0}} \\ &= \frac{30\pi}{2} \\ &= 15\pi, \\ &\approx 47.12\,cm^2.\end{aligned}$$

12. Chords and Tangents:

a) Describe the relationship between a chord and the centre of a circle.

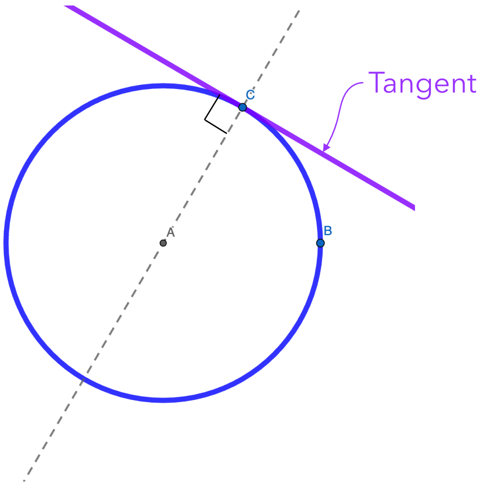
Solution:

A chord is always equidistant from the centre at its midpoint. The perpendicular from the centre to the chord bisects it.



b) Explain what a tangent to a circle is.

Solution:
A tangent is a straight line that touches the circle at exactly one point, known as the point of tangency. It's **perpendicular** to the radius at the point of tangency.

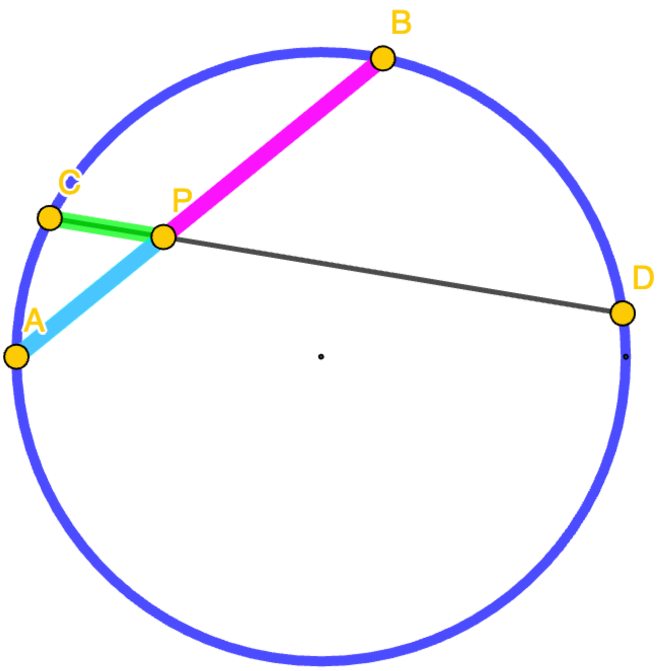


c) In a circle, two chords AB and CD intersect at point P . If $AP = 8\,cm$ **,** $PB = 12\,cm$ **, and** $CP = 6\,cm$ **, what is the length of PD ?**

Solution:
Using the Intersecting Chords Theorem:

$$\begin{aligned} AP &= 8\,cm\,. \\ PB &= 12\,cm\,. \\ CP &= 6\,cm\,. \\ PD &= ?\,cm\,. \end{aligned}$$

$$\begin{aligned} AP \times PB &= CP \times PD \\ 8 \times 12 &= 6 \times PD \\ 96 &= 6 \times PD \\ \frac{96}{6} &= \frac{\cancel{6} \times PD}{\cancel{6}} \\ \frac{96}{6} &= PD \\ PD &= \frac{96}{6} \\ &= 16\,cm\,. \end{aligned}$$



13. Practical Application:

A circular garden has a path around it that is $1\,metre$ wide. If the garden's radius is $5\,metres$, what is the area of the path?

Solution:

$$r = 5\,m\,.$$
$$\text{Path Width } = 1\,m\,.$$
$$\text{Path Area } = ?\,.$$

Original garden area:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 5^2 \\ &= 25\pi\ m^2\,. \end{aligned}$$

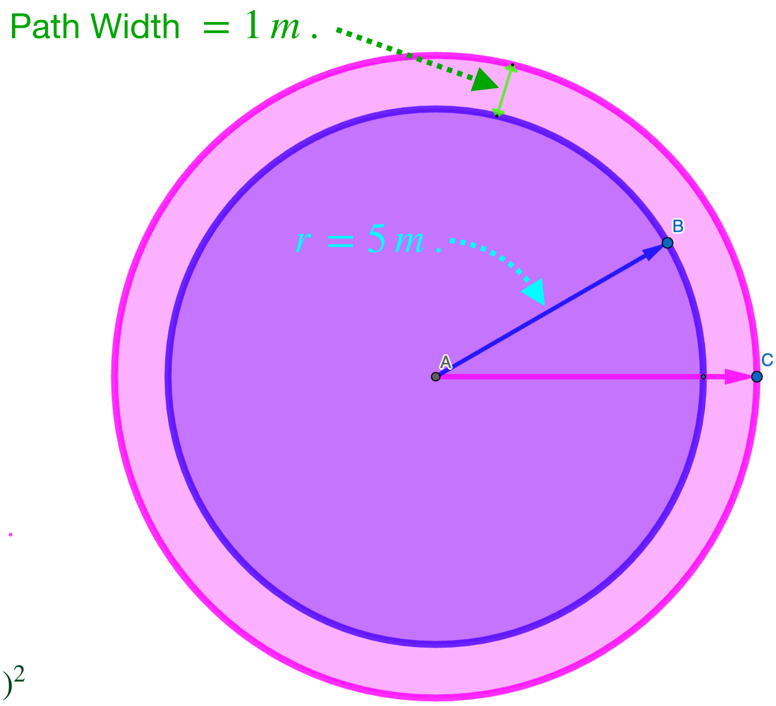
New radius (garden + path):

$$\begin{aligned} &\rightarrow 5 + 1 \\ &= 6\,metres\,. \end{aligned}$$

New area:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times (6\,m)^2 \\ &= 36\pi\ m^2\,. \end{aligned}$$

$$\begin{aligned} \text{Path area } &= \text{ New area } - \text{ Original area}\,, \\ &= 36\pi - 25\pi \\ &= 11\pi\,, \\ &\approx 34.56\,m^2\,. \end{aligned}$$



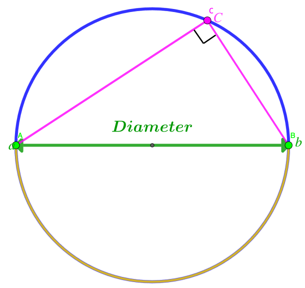
14. Circle Theorems:

a) State the angle in a semicircle theorem.

Solution:

The angle subtended by the diameter in a semicircle is always $90\,degrees$ (a right angle).

I.e. If you draw in a point (C) on a semi-circle, and then connect this point up with each end of the diameter (points a and b), the angle at (C) will always be $90\,degrees$.



b) If an angle at the circumference of a circle, subtends an arc of $120\,degrees$, what is the angle at the centre, subtending the same arc?

Solution:

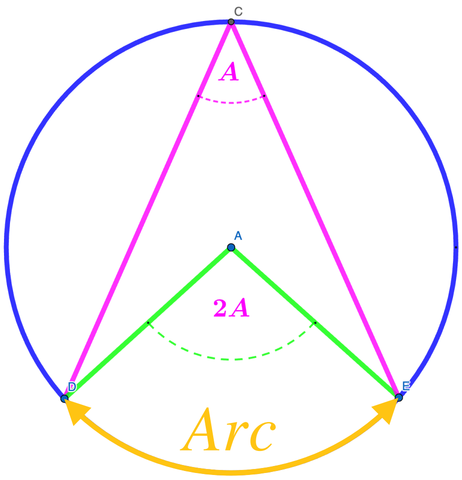
The angle at the centre is always twice the angle at the circumference, for the same arc ,

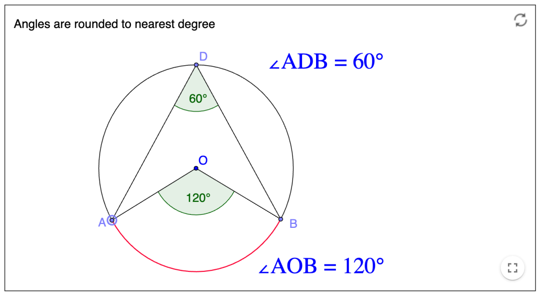
Angle at circumference : $A = 120^{\circ}$.

Angle at centre : $B = 2A = ?$.

So, angle at centre is :

$$2A = 2 \times 120$$
$$= 240\,degrees\,.$$





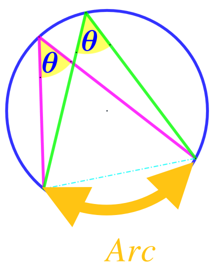
GeoGebra Interactive, *Available 10 March 2025, from* **: https://www.geogebra.org/m/ekqyj9dx**

c) If an angle subtended by an arc at the circumference of a circle is $54\,degrees$, what is the measure of another angle subtended by the same arc?

The angles (θ) subtended by the same arc at the circumference, are equal ,

So, the other angle is :

$$\theta = 54\,degrees\,.$$



Additional Notes for Teachers:

Learning Outcomes: Students should be able to solve systems of equations using various methods, understand the graphical interpretation, and apply these skills to practical problems. Students should understand the properties of circles, calculate various aspects like circumference, area, arc length, and sector area, and apply circle theorems.
Teaching Strategies: Use visual aids to demonstrate substitution, elimination, and graphical methods. Encourage students to check solutions by substituting back into original equations. Use physical circles or digital tools to illustrate concepts like radius, circumference, and angles.
Engage students with activities where they measure circles or use compasses to draw them.
Discuss and prove circle theorems to deepen understanding of geometric properties.
Relate problems to real-life scenarios where simultaneous equations could be used.
Assessment: Evaluate through exercises where students solve systems, interpret solutions graphically, and apply concepts to word problems. Assess through problems that require calculation, application of theorems, and real-world problems involving circles.
Resources: Graph paper for plotting, graphing calculators or software for dynamic visualisations, and real-life datasets for application. Compasses, protractors, string for measuring circumference, or geometry software for dynamic exploration.

This question set aligns with the Australian Curriculum for Year 9, focusing on the proficiencies of understanding, fluency, problem-solving, and reasoning in number and algebra, measurement and geometry, specifically in the context of simultaneous equations, and circles.