WORKSHEET # 022 | Australian Curriculum Year 9 Mathematics | WM9T3U6

Trigonometry, and Transformations 9

Free and always will be!

Focus: A set of questions and solutions for Year 9 students focused on 'Trigonometry, and Transformations' under the "Measurement and Geometry" strand, tailored to the Australian Curriculum:

1. Understanding Trigonometric Ratios:

Remember, the Hypotenuse, is ALWAYS, ACROSS from the RIGHT ANGLE .

The Opposite, is always, opposite the angle, theta (θ) .

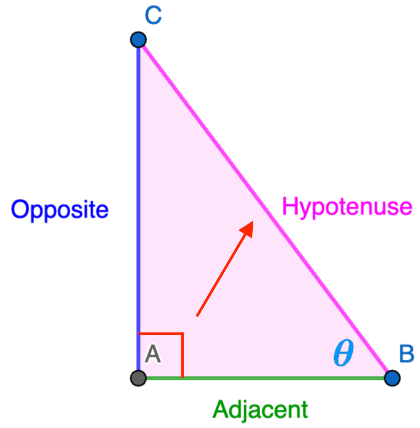
Adjacent, is the side left over.

a) Define sine, cosine, and tangent for an angle in a right-angled triangle.

Solution:

Sine (sin):
The ratio of the length of the opposite side to the hypotenuse.

$$\sin(\theta) = \frac{opposite}{hypotenuse} = \frac{O}{H}$$

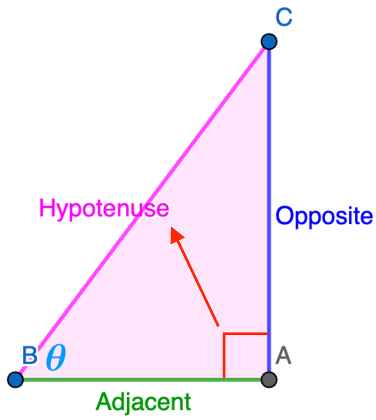


Cosine (cos):
The ratio of the length of the adjacent side to the hypotenuse.

$$\cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{A}{H}$$

Tangent (tan):
The ratio of the length of the opposite side to the adjacent side.

$$\tan(\theta) = \frac{opposite}{adjacent} = \frac{O}{A}$$



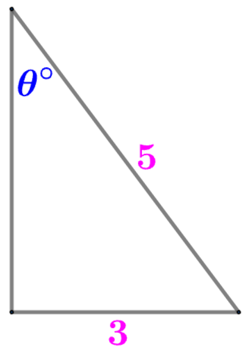
b) How are these ratios connected by the Pythagorean identity?

Solution:
The Pythagorean identity states that for any angle (θ) :

$$[\sin(\theta)]^2 + [\cos(\theta)]^2 = 1^2$$
$$\equiv \sin^2(\theta) + \cos^2(\theta) = 1$$

2. Calculating Trigonometric Ratios:

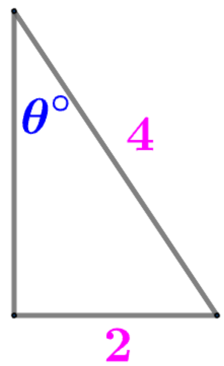
a) In a right-angled triangle, if the opposite side to an angle is $3\,cm$, and the hypotenuse is $5\,cm$, find $\sin(\theta)$.



Solution:

$$\sin(\theta) = \frac{Opposite}{Hypotenuse} = \frac{3}{5}$$
$$= \frac{3 \times 2}{5 \times 2} \text{ (Muliply by 2 so fraction / 10).}$$
$$= \frac{6}{10} \text{ (Six in the tenths column).}$$
$$= 0.6\,.$$

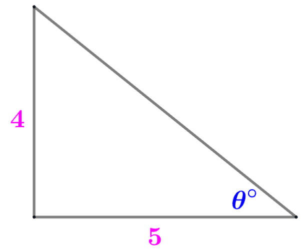
b) In a right-angled triangle, if the hypotenuse is $4\,cm$, and the adjacent is $2\,cm$, find $\cos(\theta)$.



Solution:

$$\cos(\theta) = \frac{Adjacent}{Hypotenuse} = \frac{2}{4}$$
$$= \frac{2 \times 25}{4 \times 25} \text{ (Muliply by 25 so fraction / 100).}$$
$$= \frac{50}{100} \text{ (50 in the hundredths column).}$$
$$= 0.5\,.$$

c) In a right-angled triangle, if the opposite side to an angle is $4\,cm$, and the adjacent is $5\,cm$, find $\tan(\theta)$.



Solution:

$$\tan(\theta) = \frac{Opposite}{Adjacent} = \frac{4}{5}$$
$$= \frac{4 \times 2}{5 \times 2} \text{ (Muliply by 2 so fraction / 10).}$$
$$= \frac{8}{10} \text{ (Eight in the tenths column).}$$
$$= 0.8\,.$$

$$SOH \rightarrow \sin(\theta) = \frac{opposite}{hypotenuse} = \frac{O}{H}$$

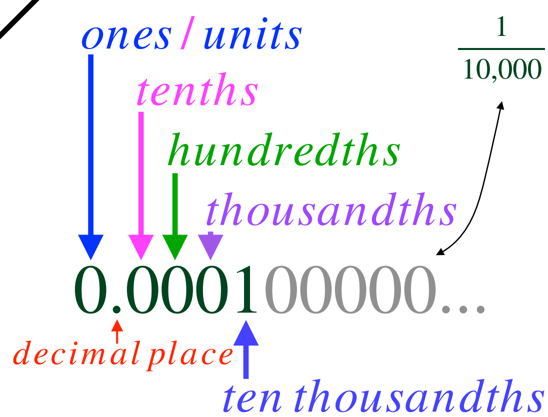
$$CAH \rightarrow \cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{A}{H}$$

$$TOA \rightarrow \tan(\theta) = \frac{opposite}{adjacent} = \frac{O}{A}$$

SOH CAH TOA

Some Old Hags, Can't Always Hide, Their Old Age.

SOH
CAH
TOA



d) In a right-angled triangle, if $\cos(\theta) = \frac{4}{5}$, what is $\tan(\theta)$, given the adjacent side is $4\,cm$?

Solution:

We want to find, $\tan\theta = \frac{Opposite}{Adjacent}$,

and we know the $Adjacent$ is 4 .

Therefore, we need to find the $Opposite$ before we can do this .

Firstly, find the $Hypotenuse$ using the rule for, $\cos\theta$,

SOH CAH TOA

$$\cos\theta = \frac{Adjacent}{Hypotenuse}$$

$$\cos\theta = \frac{4}{5}$$

$$\frac{4}{5} = \frac{Adjacent}{Hypotenuse}$$

$$\rightarrow Adjacent = 4\,,$$

$$\rightarrow Hypotenuse = 5\,.$$

Secondly, find the $Opposite$ using the Pythagorean theorem,

$$c^2 = a^2 + b^2$$

$$Hypotenuse^2 = Adjacent^2 + Opposite^2$$

$$5^2 = 4^2 + Opposite^2$$

$$5^2 - 4^2 = 4^2 - 4^2 + Opposite^2$$

$$25 - 16 = Opposite^2$$

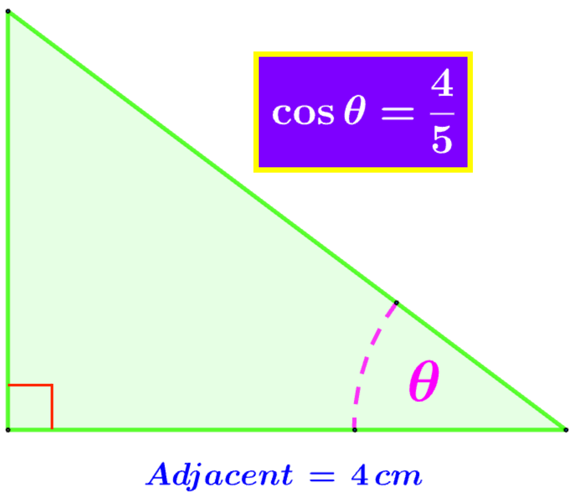
$$9 = Opposite^2$$

$$\sqrt{9} = \sqrt{Opposite^2}$$

$$\sqrt{9} = Opposite$$

$$Opposite = \sqrt{9}$$

$$Opposite = 3\,.$$

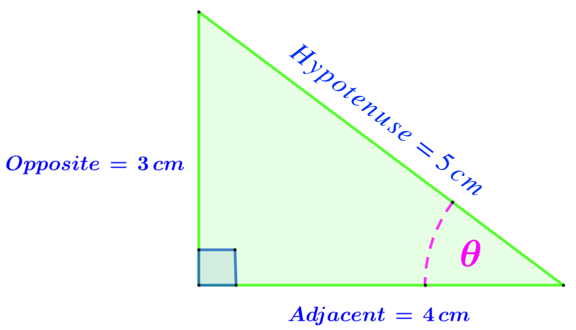


Lastly, $\tan(\theta) = \frac{Opposite}{Adjacent}$

$= \frac{3 \times 25}{4 \times 25}$ (Multiply by 25 so fraction / 100) .

$= \frac{75}{100}$, (75 in the hundredths place) .

$\tan\theta = 0.75\,.$



3. Using Trigonometry to Find Unknown Sides:

a) Find the length of the side opposite to an angle of 30° **if the hypotenuse is** $10\,cm$ **.**

Solution:

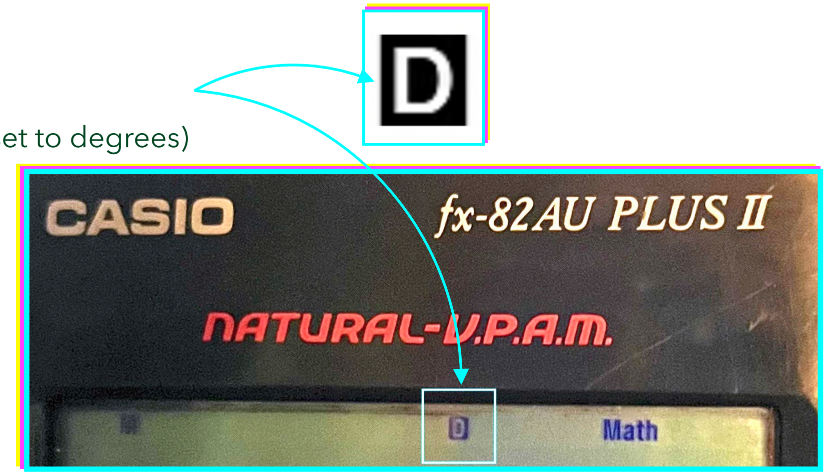
$$\theta = 30^\circ$$
$$Hypotenuse = 10\,.$$
$$Opposite = ?$$

$SOH\ CAH\ TOA$

$$\sin\theta = \frac{Opposite}{Hypotenuse}$$

$$\sin(30^\circ) = \frac{Opposite}{10}$$

$$\sin(30^\circ) = \frac{1}{2} \text{ (Using calculator, set to degrees)}$$



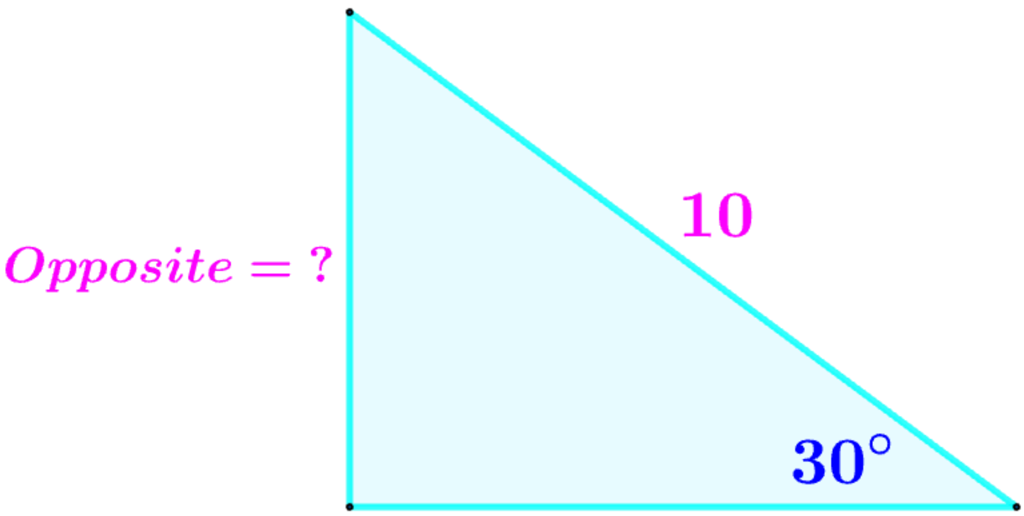
$$\frac{1}{2} = \frac{Opposite}{10}$$

$$\frac{1}{2} \times 10 = \frac{Opposite}{\cancel{10}} \times \cancel{10}$$

$$\frac{10}{2} = Opposite$$

$$5 = Opposite$$

$$\rightarrow Opposite = 5\,cm\,.$$



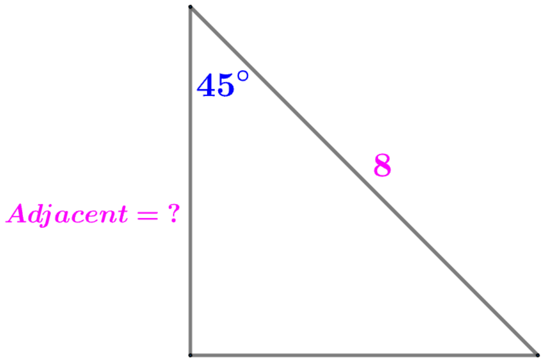
b) Calculate the length of the adjacent side to an angle of 45° if the hypotenuse is $8\,cm$.

Solution:

$$\theta = 45^\circ$$
$$Hypotenuse = 8\,cm.$$
$$Adjacent = ?$$
$$\cos\theta = \frac{Adjacent}{Hypotenuse}$$

SOH CAH TOA

$$\cos(45^\circ) = \frac{Adjacent}{8}$$
$$0.707 \approx \frac{Adjacent}{8}$$
$$0.707 \times 8 \approx \frac{Adjacent}{\cancel{8}} \times \cancel{8}$$
$$8 \times 0.707 \approx Adjacent$$
$$\rightarrow Adjacent \approx 8 \times 0.707$$
$$\approx 5.66\,cm.$$

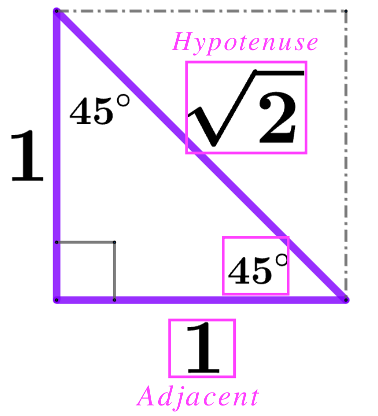


OR

Using Exact Values:

$$\theta = 45^\circ$$
$$Hypotenuse = 8\,cm.$$
$$Adjacent = ?$$
$$\cos\theta = \frac{Adjacent}{Hypotenuse}$$
$$\cos(45^\circ) = \frac{Adjacent}{8}$$
$$\approx 0.707 = \frac{\sqrt{2}}{2} = \frac{Adjacent}{8}$$
$$\frac{\sqrt{2}}{2} \times 8 = \frac{Adjacent}{\cancel{8}} \times \cancel{8}$$
$$8 \times \frac{\sqrt{2}}{2} = Adjacent$$
$$\rightarrow Adjacent = 8 \times \frac{\sqrt{2}}{2}$$
$$= \frac{8}{2}\sqrt{2},$$
$$= 4\sqrt{2},$$
$$\approx 5.66\,cm.$$

$$\cos(\theta) = \frac{Adj.}{Hyp.}$$
$$\cos(45^\circ) = \frac{Adj.}{Hyp.}$$
$$\frac{Adj.}{Hyp.} = \cos(45^\circ)$$
$$\approx 0.707 \approx \frac{1}{\sqrt{2}} = \cos(45^\circ)$$



4. Using Trigonometry to Find Angles:

a) If the $tangent$ **of an angle is** 1 **, what is the angle?** *(See fig. 1 below)*

Solution:

$\tan(\theta) = \dfrac{Opposite}{Adjacent}$ $SOH\ CAH\ \boxed{TOA}$

$\tan(\theta) = 1$ corresponds to , θ

$\tan(\theta) = \dfrac{1}{1} = \dfrac{Opp.}{Adj.}$ (This corresponds to a square, with $side = 1$) .

$\therefore \theta = 45^\circ$.

b) An angle has a $cosine$ **of** 0.5 **. What is the angle?** *(Tricky! See fig. 2 below)*

Solution:

$\cos(\theta) = \dfrac{Adjacent}{Hypotenuse}$ $SOH\ \boxed{CAH}\ TOA$

$\cos(\theta) = 0.5$ corresponds to , θ (This corresponds to a right-angled triangle,

$\cos(\theta) = \dfrac{1}{2} = \dfrac{Adj.}{Hyp.}$ with $base = 1, hypotenuse = 2, (height = \sqrt{3})$) .

$\therefore \theta = 60^\circ$.

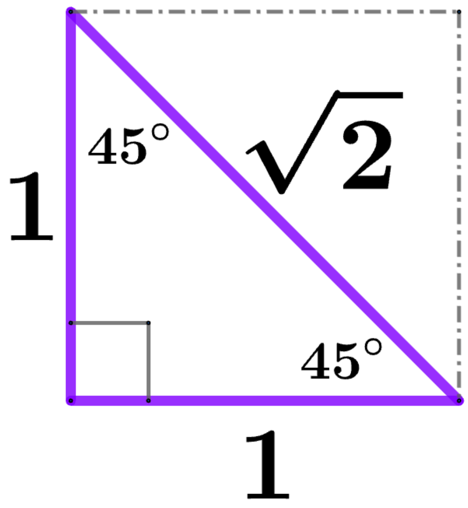


fig. 1

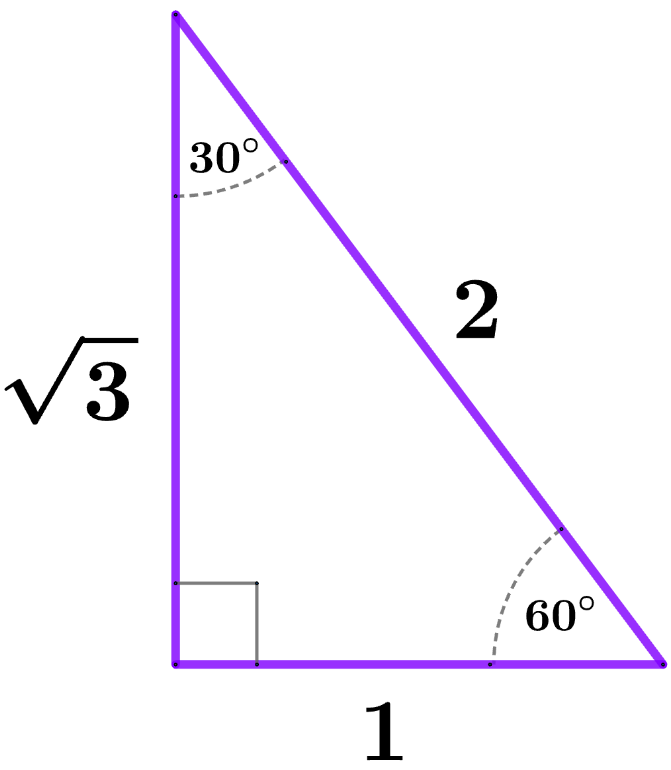


fig. 2

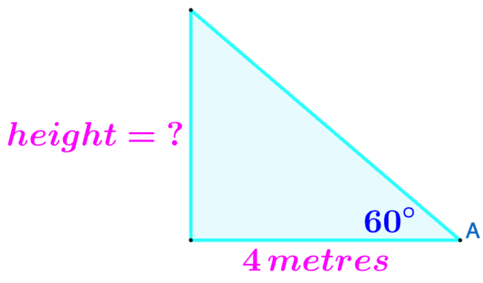
5. Real-World Applications:

a) A ladder leans against a wall at an angle of 60° with the ground. If the base of the ladder is $4\,metres$ from the wall, how high does the ladder reach?

Solution:

$$\theta = 60^\circ.$$
$$length = 4\,metres.$$
$$height = ?$$

SOH CAH TOA



Using the sine function:

$$sin(\theta) = \frac{Opposite}{Hypotenuse}$$
$$\sin(60^\circ) = \frac{height}{4\,metres}$$
$$0.866 \times 4 \approx \frac{height}{\cancel{4}} \times \cancel{4}$$
$$3.46 \approx height$$
$$height \approx 3.46$$
$$\approx 3.46\,metres.$$

OR

Using Exact Values:

$$\theta = 60^\circ.$$
$$length = 4\,metres.$$
$$height = ?$$

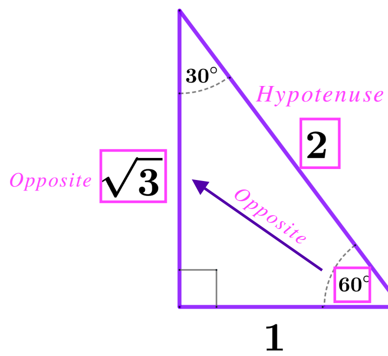
SOH CAH TOA

$$sin(\theta) = \frac{Opposite}{Hypotenuse}$$
$$\sin(60^\circ) = \frac{height}{4\,metres}$$

Using calculator, OR triangle to find exact value (see triangle far right).

$$\frac{\sqrt{3}}{2} \times 4 = \frac{height}{\cancel{4}} \times \cancel{4}$$
$$4 \times \frac{\sqrt{3}}{2} = height$$
$$\frac{4}{2} \times \sqrt{3} = height$$
$$2 \times \sqrt{3} = height$$
$$height = 2\sqrt{3}$$
$$\approx 3.46\,metres.$$

$$\sin(\theta) = \frac{Opp.}{Hyp.}$$
$$\sin(60^\circ) = \frac{Opp.}{Hyp.}$$
$$\frac{Opp.}{Hyp.} = \sin(60^\circ)$$
$$\approx 0.866 \approx \frac{\sqrt{3}}{2} = \sin(60^\circ)$$



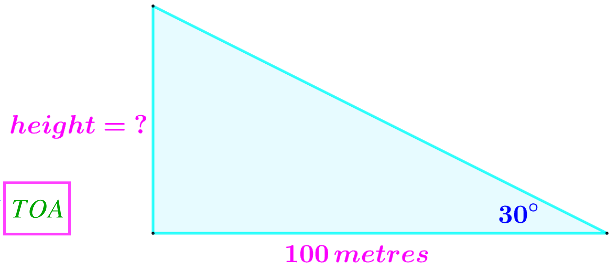
b) A ship is $100\,metres$ from the base of a lighthouse. The angle of elevation to the top of the lighthouse from the ship is 30° . How tall is the lighthouse?

Solution:

$$\theta = 30^\circ.$$
$$distance = 100\,m.$$

Using the tangent function:

$$\tan(\theta) = \frac{Opposite}{Adjacent} \quad SOH\ CAH\ TOA$$
$$\tan(30^\circ) = \frac{height}{100}$$
$$0.57735 \approx \frac{height}{100}$$
$$0.57735 \times 100 \approx \frac{height}{100} \times 100$$
$$0.57735 \times 100 \approx height$$
$$\rightarrow height \approx 0.57735 \times 100$$
$$\rightarrow height \approx 57.735\,metres.$$



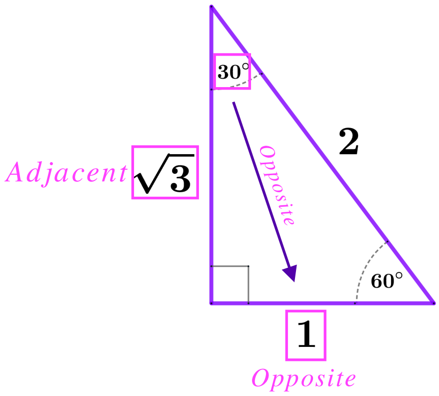
OR

Using Exact Values:

$$\theta = 30^\circ.$$
$$distance = 100\,m.$$
$$height = ?$$

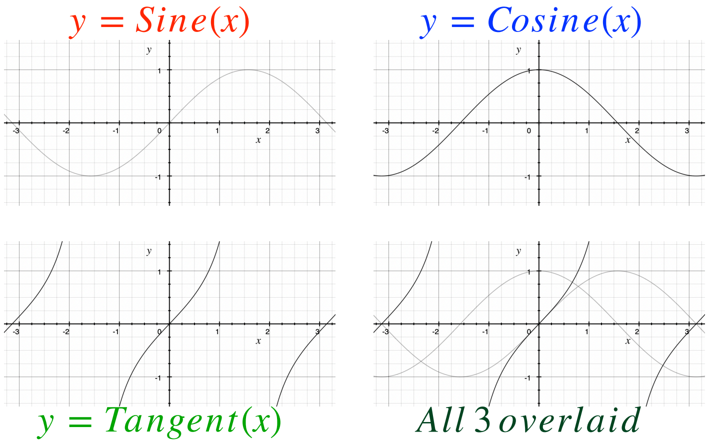
$$\tan(\theta) = \frac{Opposite}{Adjacent}$$
$$\tan(30^\circ) = \frac{height}{100}$$
$$\frac{1}{\sqrt{3}} \times 100 = \frac{height}{100} \times 100$$
$$\frac{1}{\sqrt{3}} \times 100 = height$$
$$\rightarrow height = \frac{1}{\sqrt{3}} \times 100$$
$$\rightarrow height = \frac{100}{\sqrt{3}}$$
$$\approx 57.7\,metres.$$

$$\tan(\theta) = \frac{Opp}{Adj.}$$
$$\tan(30^\circ) = \frac{Opp.}{Adj.}$$
$$\frac{Opp.}{Adj.} = \tan(30^\circ)$$
$$\approx 1.732 \approx \frac{1}{\sqrt{3}} = \tan(30^\circ)$$



6. Trigonometric Identities:

Prove the identity $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$ **, using the definitions of sine and cosine.**



Solution:
By definition,

$$\tan\theta = \frac{Opposite}{Adjacent},$$

$SOH\ CAH\ TOA$ $$\sin\theta = \frac{Opposite}{Hypotenuse},$$

and $$\cos\theta = \frac{Adjacent}{Hypotenuse}.$$

Therefore,

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{Opposite}{Hypotenuse}}{\frac{Adjacent}{Hypotenuse}}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{Opposite}{\cancel{Hypotenuse}} \times \frac{\cancel{Hypotenuse}}{Adjacent}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{Opposite}{Adjacent}$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}.$$

When dividing by a fraction, change the times to divide, and flip the fraction on the right (the one being divided by).

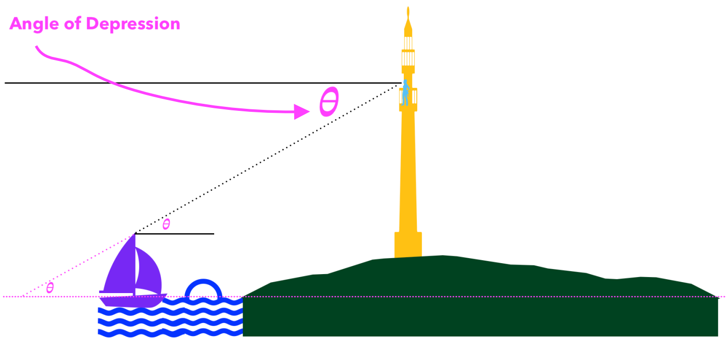
$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d}$$

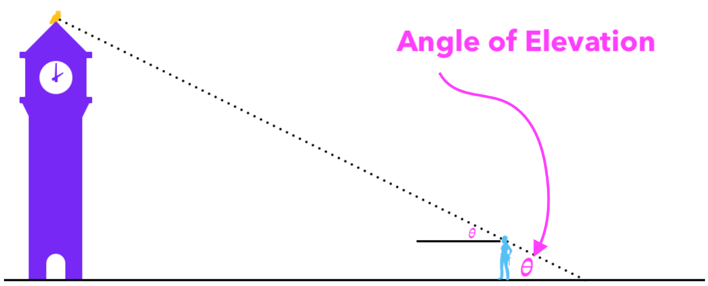
$$= \frac{a}{b} \times \frac{d}{c}$$

7. Angles of Elevation and Depression:

a) Explain the difference between angle of elevation and angle of depression.

Solution:
Angle of Elevation:
The angle above the horizontal line of sight, looking up at an object.
Angle of Depression:
The angle below the horizontal line of sight, looking down from an elevated point.





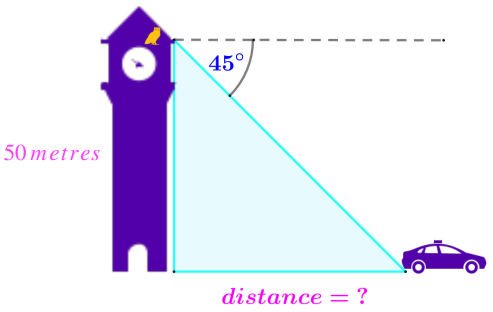
b) If the angle of depression from the top of a building to a car on the ground is 45°, and the building is $50\,metres$ tall, how far is the car from the base of the building?

Solution:

$$\theta = 45^{\circ}.$$
$$height = 50\,m.$$
$$distance = ?$$

The angle of depression is equal to the angle of elevation from the car, to the top of the building (alternate interior angles).

Therefore:



$$\tan(\theta) = \frac{Opp.}{Adj.} \quad SOH\ CAH\ TOA$$
$$\tan(45^{\circ}) = \frac{50}{distance}$$
$$1 = \frac{50}{distance}$$
$$1 \times distance = \frac{50}{\cancel{distance}} \times \cancel{distance}$$
$$distance = 50$$
$$= 50\,metres.$$

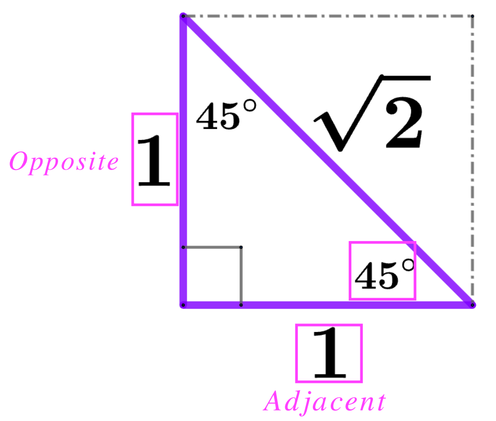
Or

Using Exact Values:

$$\theta = 45^{\circ}.$$
$$height = 50\,metres.$$
$$distance = ?$$

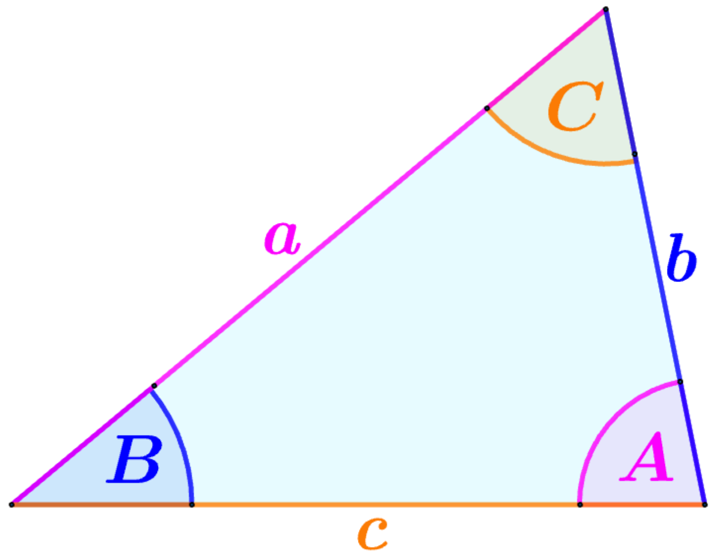
$$tan(\theta) = \frac{Opp.}{Adj.}$$
$$\tan(45^{\circ}) = \frac{50}{distance}$$
$$\frac{1}{1} = \frac{50}{distance}$$
$$1 \times distance = \frac{50}{\cancel{distance}} \times \cancel{distance}$$
$$distance = 50$$
$$= 50\,metres.$$

$$\tan(\theta) = \frac{Opp.}{Adj.}$$
$$\tan(45^{\circ}) = \frac{Opp.}{Adj.}$$
$$\frac{Opp.}{Adj.} = \tan(45^{\circ})$$
$$\frac{1}{1} = \tan(45^{\circ})$$
$$\tan(45^{\circ}) = 1$$



8. Sine and Cosine Rule:

For non-right-angled triangles:

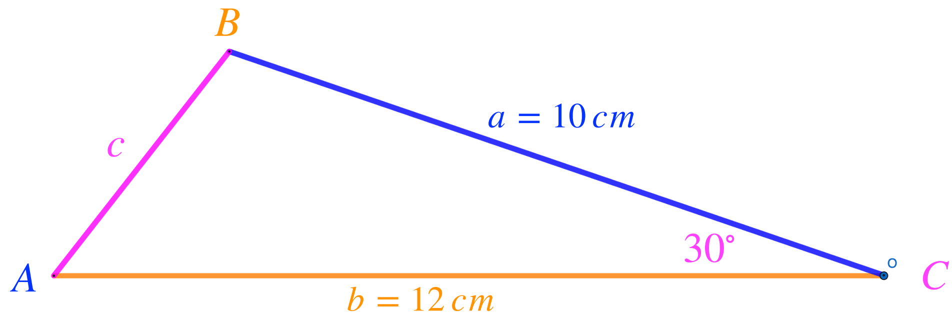


Cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Sine Rule : $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

a) In triangle ABC, $AC = 12\,cm$, $BC = 10\,cm$, and $\angle BCA = 30^\circ$. Find the length of AB using the cosine rule.

Solution:



Here, $c = c$,

$b = 12$,

$a = 10$,

and $\cos 30^\circ = \dfrac{\sqrt{3}}{2}$

Cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

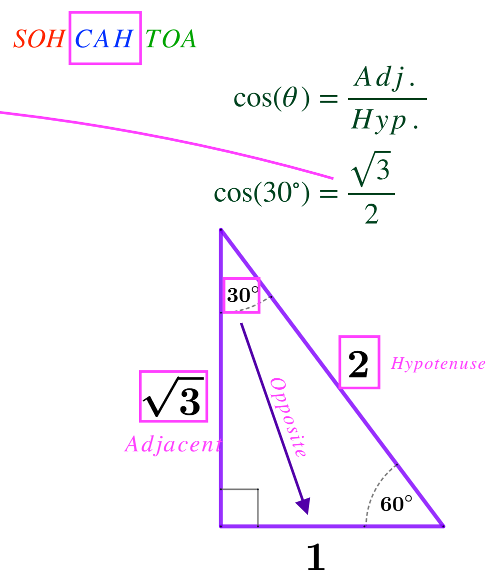
$$c^2 = 10^2 + 12^2 - 2 \times 10 \times 12 \times \frac{\sqrt{3}}{2}$$

$$c^2 = 100 + 144 - 120\sqrt{3}$$

$$c^2 = 244 - 120\sqrt{3}$$

$$c \approx \sqrt{244 - 120\sqrt{3}}$$

$$\approx 6.013\,cm\,.$$

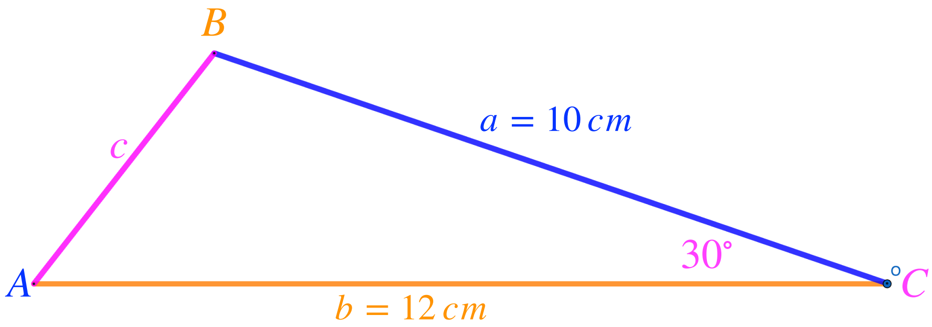


b) Find $\angle B$ in the same triangle ABC using the sine rule.

Solution:

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

We know $b = 12,\ c = BA \approx 6.013,\ \angle C = 30^\circ$, so:



$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{12}{\sin B} = \frac{6.013}{\sin 30}$$
$$\frac{12}{\sin B} = \frac{6.013}{\sin 30}$$
$$\frac{12}{\sin B} = \frac{6.013}{0.5}$$
$$\frac{12 \times 0.5}{\sin B} = \frac{6.013}{0.5} \times 0.5$$
$$\frac{6}{\sin B} = 6.013$$
$$\frac{6}{\sin B} \times \sin B = 6.013 \times \sin B$$
$$6 = 6.013 \times \sin B$$
$$\frac{6}{6.013} = \frac{6.013 \times \sin B}{6.013}$$
$$\frac{6}{6.013} = \sin B$$
$$\sin B = \frac{6}{6.013}$$
$$\sin^{-1}(\sin B) = \sin^{-1}\left(\frac{6}{6.013}\right)$$
$$B = \sin^{-1}\left(\frac{6}{6.013}\right)$$
$$\approx 86.23^\circ.$$

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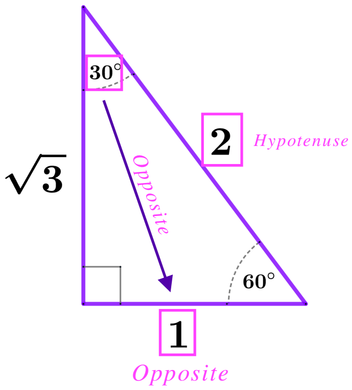
$$\sin(\theta) = \frac{Opp.}{Hyp.}$$
$$\sin(30^\circ) = \frac{1}{2}$$
$$\sin(30^\circ) = 0.5$$

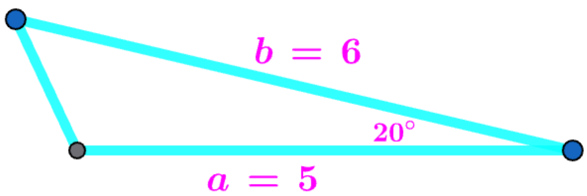


9. Area Rule:

Area Rule: $Area = \dfrac{1}{2}ab\sin(C)$

Find the area of a triangle with side lengths $a = 5$, $b = 6$, and the angle between a and b is 20°

$$Area = \frac{1}{2}ab\sin(C)$$
$$= \frac{1}{2} \cdot 5 \cdot 6 \cdot \sin(20)$$
$$= 15\sin(20)$$
$$\approx 5.13\ units.$$



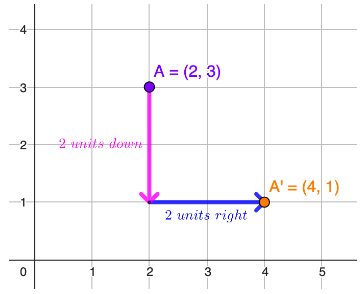
10. Transformations:

a) Describe the transformation that maps point $A(2,3)$ to point $A'(4,1)$

Solution:

To determine the transformation, calculate the change in coordinates from : $A(2,\ 3)$ to $A'(4,\ 1)$:

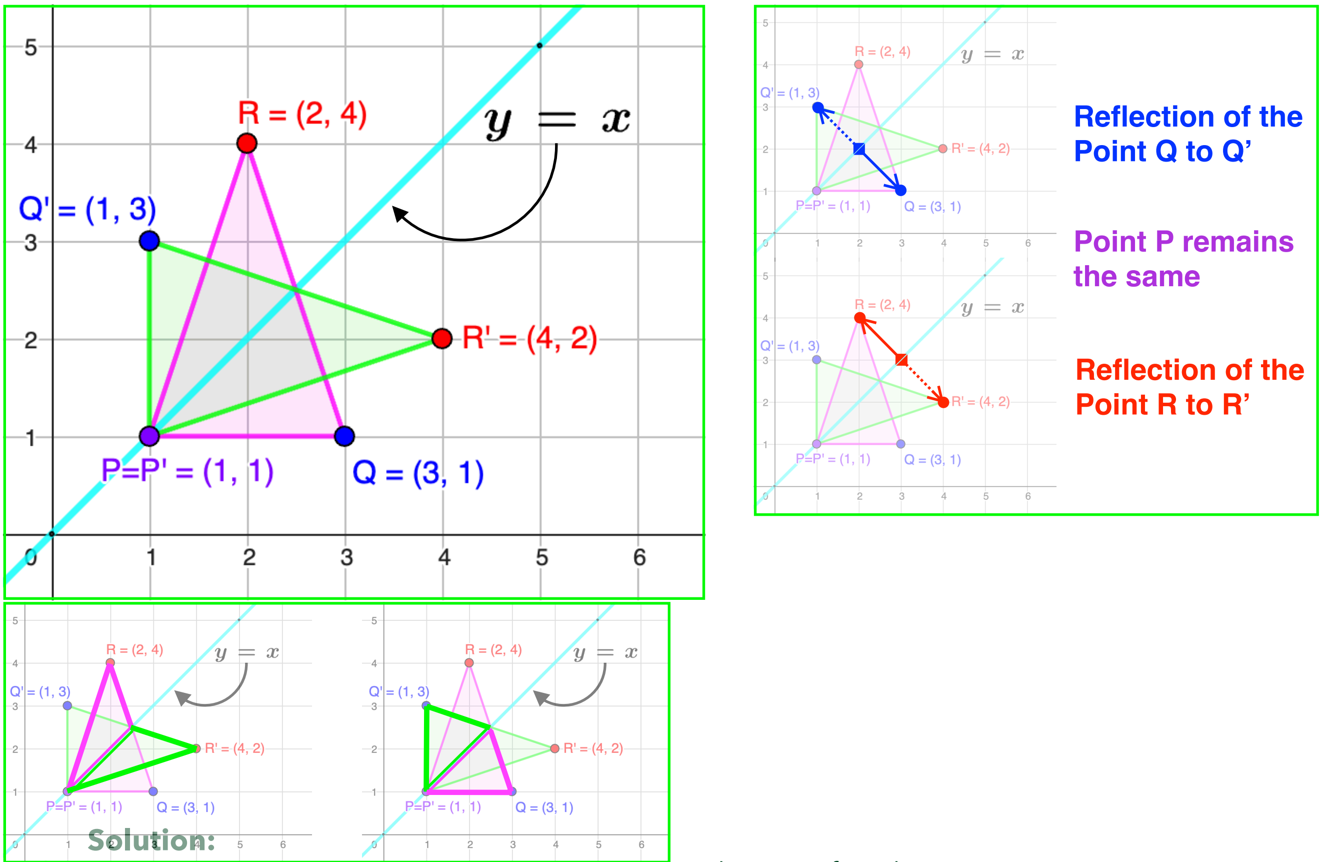
Change in x : $4-2=+2$
Change in y : $1-3=-2$



This indicates a translation 2 units to the right (positive x-direction), and 2 units down (negative y-direction).

Thus, the transformation is a translation by the vector $\overrightarrow{T}=(2,\ -2)$.

b) A triangle with vertices $P(1,\ 1)$, $Q(3,\ 1)$, and $R(2,\ 4)$ is reflected over the line $y=x$. What are the coordinates of the image points P', Q', and R'?



Solution:

Reflection over the line $y=x$ swaps the x and y coordinates of each point.

$P(1,\ 1)$ becomes $P'(1,\ 1)$ (since $x=y$, it remains unchanged).
$Q(3,\ 1)$ becomes $Q'(1,\ 3)$.
$R(2,\ 4)$ becomes $R'(4,\ 2)$.

The coordinates of the image points are $P'(1,\ 1)$, $Q'(1,\ 3)$, and $R'(4,\ 2)$.

c) A square with vertices $A(0,0)$, $B(2,0)$, $C(2,2)$, and $D(0,2)$ is rotated 90° counterclockwise about the origin and then translated $3\ units$ to the right. Determine the coordinates of the final image of vertex C.

Solution:

Step 1: Rotation 90° counterclockwise about the origin:

For a 90° counterclockwise rotation about the origin, the rule is $(x, y) \to (-y, x)$:

$A(0,0) \to A'(0,0)$

$B(2,0) \to B'(0,2)$

$C(2,2) \to C'(-2,2)$

$D(0,2) \to D'(-2,0)$

The image of C after rotation is $C'(-2,2)$.

Rotaion Coordinate Rules (around the origin)

90° counterclockwise or 270° clockwise
$(x, y) \to (-y, x)$

180° clockwise or 180° counterclockwise
$(x, y) \to (-x, -y)$

90° clockwise or 270° counterclockwise
$(x, y) \to (y, -x)$

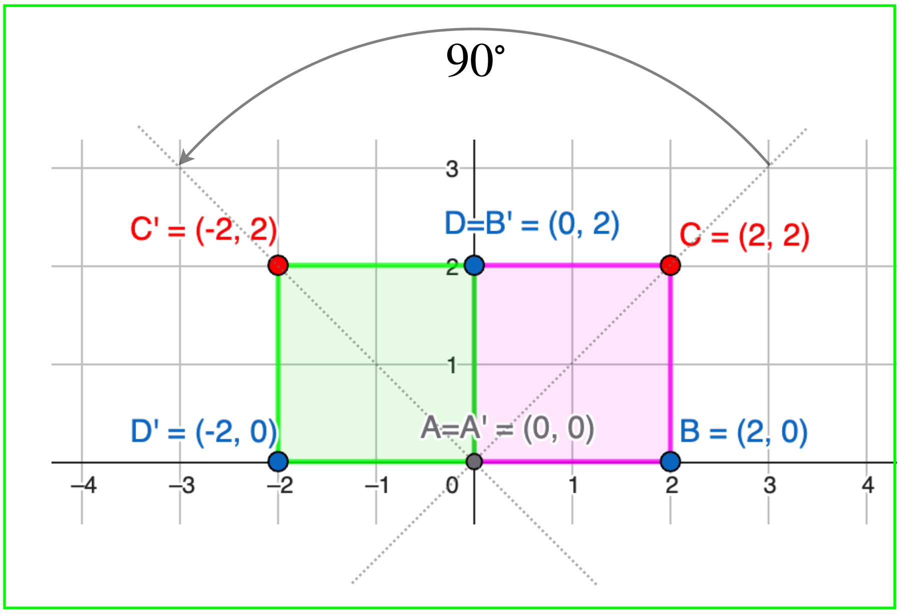
Step 2: Translation 3 units to the right:

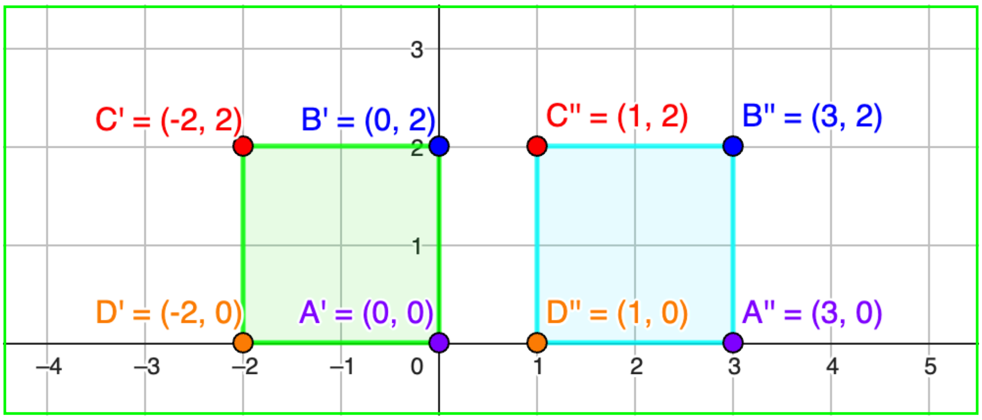
Translation 3 units right adds 3 to the x-coordinate:

$(x, y) \to (x+3, y)$:

$C'(-2,2) \to C''(-2+3, 2) = (1,2)$

The final coordinates of C are $(1,2)$.





d) A logo design is created by dilating a triangle with vertices $T(1, 1)$, $U(3, 1)$, and $V(2, 3)$ by a scale factor of 2 from the origin, followed by a reflection over the $x - axis$. The designer wants the final image to fit within a rectangular frame with a height of $6\ units$. Verify if the final image meets this requirement by calculating the height of the transformed triangle.

Solution:

Step 1: Dilation by a scale factor of 2 from the origin:

Dilation from the origin with scale factor 2 multiplies each coordinate by 2 :

$(x, y) \to (2x, 2y)$:

$T(1, 1) \to T'(2, 2)$

$U(3, 1) \to U'(6, 2)$

$V(2, 3) \to V'(4, 6)$

Step 2: Reflection over the $x - axis$:

Reflection over the $x - axis$ changes the sign of the y-coordinate :

$(x, y) \to (x, -y)$:

$T'(2,2) \to T''(2, -2)$

$U'(6, 2) \to U''(6, -2)$

$V'(4,6) \to V''(4, -6)$

Step 3: Calculate the height of the final triangle :

The height is the vertical distance between the highest and lowest $y - coordinates$ of $T''(2, -2)$, $U''(6, -2)$, and $V''(4, -6)$:

Highest $y - coordinate$: -2

Lowest $y - coordinate$: -6

Height $= -2 - (-6)$

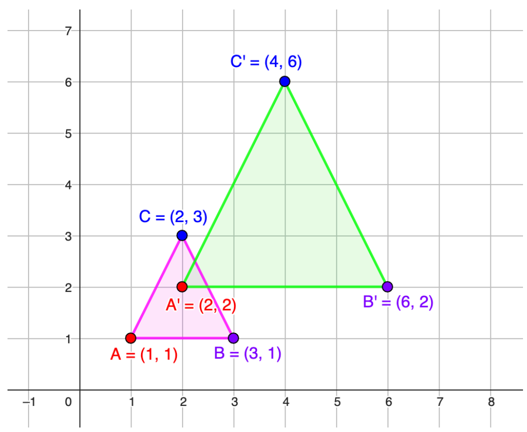
$= -2 + 6$

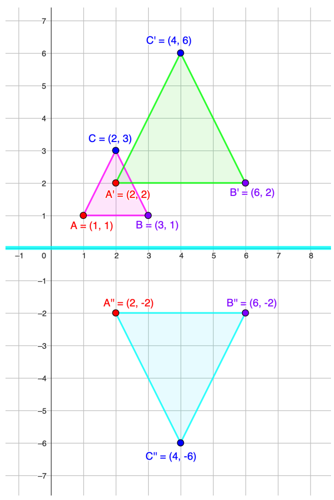
$= 4\ units$

Step 4: Verify against the frame requirement :

The frame has a height of 6 units, and the triangle's height is 4 units.

Since $4 < 6$, the final image fits within the frame.





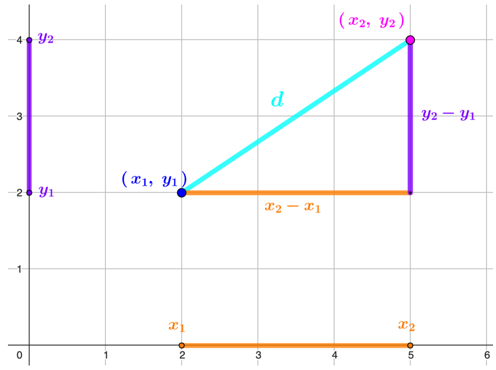
11. Distance between two points:

To find the distance between two points, given point one is (x_1, y_1) and the second is (x_2, y_2) use the formula derived from the Pythagorean Theorem :

$$c^2 = a^2 + b^2$$

$$\rightarrow \sqrt{d^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



a) Determine the distance between the two points : $A(2, 1)$ **and** $B(5, 3)$ **.**

Solution:

$(x_1, y_1) = (2, 1)$
$(x_2, y_2) = (5, 3)$

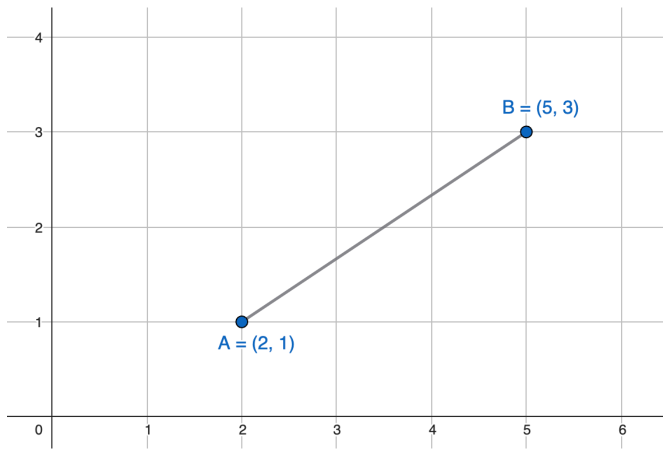
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(5-2)^2 + (3-1)^2}$$

$$= \sqrt{(3)^2 + (2)^2}$$

$$= \sqrt{9 + 4}$$

$$d = \sqrt{13} \ (\approx 3.61) .$$



b) Determine the distance between the two points : $A(2, 2)$ **and** $B(5, 1)$ **.**

Solution:

$(x_1, y_1) = (2, 2)$
$(x_2, y_2) = (5, 1)$

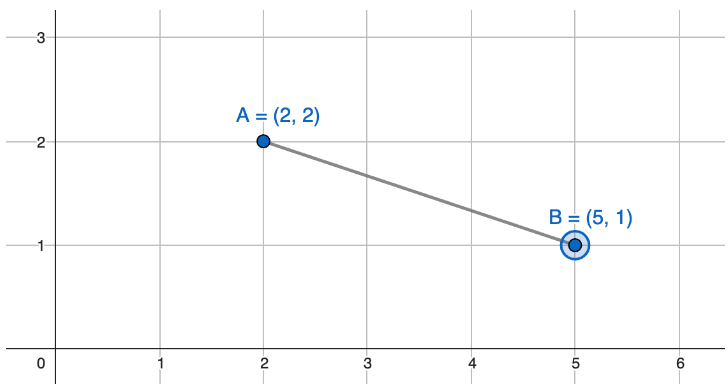
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(5-2)^2 + (1-2)^2}$$

$$= \sqrt{(3)^2 + (-1)^2}$$

$$= \sqrt{9 + 1}$$

$$d = \sqrt{10} \ (\approx 3.16) .$$



Additional Notes for Teachers:

Learning Outcomes: Students should understand the basic trigonometric ratios, how to use them in solving for sides, angles, and real-world problems, and be familiar with basic trigonometric identities.
Teaching Strategies:
Use right-angled triangles drawn on grid paper or digital tools to visualise and calculate.
Incorporate real-life scenarios like navigation, architecture, or sports to make trigonometry tangible.
Teach students to use calculators for trigonometric functions but also to understand the underlying concepts.
Assessment: Assess through problems involving calculation of unknown sides or angles, application in practical contexts, and understanding of trigonometric identities.
Resources: Trigonometry tables or calculators for function values, interactive geometry software, or physical models of right triangles for hands-on learning.

This question set aligns with the Australian Curriculum for Year 9, focusing on the proficiencies of understanding, fluency, problem-solving, and reasoning in measurement and geometry, specifically in the context of trigonometry.