WORKSHEET # 024 Australian Curriculum Year 9 Mathematics WM9T4U8

ACACIA TUTORING

Statistics, and Probability of Combined Events

9

Free and always will be!

Focus: A set of questions and solutions for Year 9 students focused on 'Statistics, and Probability of Combined Events' under the "Statistics, and Probability" strand, tailored to the Australian Curriculum:

1. Understanding Combined Probabilities:

a) What does it mean to calculate the probability of combined events?

Solution:
Calculating the probability of combined events involves determining the likelihood of multiple events happening together. This can be the probability of two or more independent events occurring, or dependent events where the outcome of one event influences another.

b) Explain the difference between independent and dependent events.

Solution:
Independent Events: The occurrence of one event does not affect the probability of the other. E.g., flipping a coin twice; the result of the first flip does not change the probability of the second flip.
Dependent Events: The outcome of one event affects the probability of the other. E.g., drawing two cards without replacement from a deck; the probability of the second card depends on what was drawn first.

Events	
Independent	**Dependent**
Events where the outcome of one even ***does not affect*** the probability of the other.	Events where the outcome of one event ***affects*** the probability of the other.
Rolling a die then tossing a coin.	Probability of rain, given cloud coverage.
Drawing a card, putting in back in, then drawing another card.	Drawing a card, ***taking it out***, then drawing another card.
Pulling out a marble then, ***putting it back in***, then pulling out another marble.	Pulling out a marble, ***taking it out***, then pulling out another marble.
$P(A \ and \ B) = P(A \cap B)$ $= P(A) \times P(B)$. $=$ Probability of A $\times$ Probability of B.	$P(A \ and \ B) = P(A \cap B)$ $= P(\text{A}) \times P(B \mid A)$. $=$ Probability of A $\times$ Probability of B given A.

2. Probability of Independent Events:

a) If you roll a die twice, what is the probability of getting a 4 both times?

Solution:

$$P(\text{Event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}} = P(A) = \frac{n(A)}{n(S)}$$

$$\begin{aligned} P(A\ and\ B) &= P(A \cap B) \\ &= P(A) \times P(B)\,. \end{aligned}$$

Since each roll is independent, probability of rolling a 4 on one roll is:

$$\begin{aligned} P(4) &= \frac{n(4)}{n(S)} \\ &= \frac{1}{6}\,. \end{aligned}$$

Probability of rolling a 4 twice:

$$\begin{aligned} P(rolling\ a\ 4\ \cap\ then\ another\ 4) &= \frac{1}{6} \times \frac{1}{6} \\ &= \frac{1}{36} \approx 0.0277 \approx 2.7\,\%\,. \end{aligned}$$

b) What is the probability of flipping two coins and getting one head and one tail?

Solution:
There are four possible outcomes: $(\,HH,\ HT,\ TH,\ TT\,)$
and two of these (HT, TH) give one head and one tail:

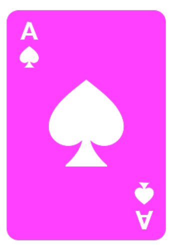
$$P(A) = \frac{n(A)}{n(S)}\,,$$

$$\begin{aligned} P(\text{One Head and One Tail}) &= \frac{2}{4} \\ &= \frac{1}{2} = 0.5 = 50\,\%\,. \end{aligned}$$

c) What is the probability of drawing an ace from a standard deck of cards?

Solution:
There are 4 Aces, out of 52 Cards:

$$P(A) = \frac{n(A)}{n(S)}\,,$$



$$\begin{aligned} P(\text{Drawing an Ace}) &= \frac{4}{52} \\ &= \frac{4 \div 4}{52 \div 4} \\ &= \frac{1}{13} \approx 0.077 \approx 7.7\,\%\,. \end{aligned}$$

3. Probability of Dependent Events:

a) A bag contains 3 red and 2 blue marbles. If you draw two marbles without replacement, what is the probability that both are red?

Solution:

$$P(A) = \frac{n(A)}{n(S)}.$$
$$P(A\ and\ B) = P(A \cap B)$$
$$\text{Probability of } A \times \text{Probability of } B \text{ given } A = P(A) \times P(B|A).$$

$$\text{Probability of first red marble: } = \frac{3}{5}.$$

After drawing one red marble, there are 2 red marbles left out of 4 marbles :

$$\text{Probability of second red marble: } = \frac{2 \div 2}{4 \div 2}$$
$$= \frac{1}{2}.$$

$$\text{Combined probability } = \frac{3}{5} \times \frac{1}{2},$$
$$= \frac{3 \times 1}{5 \times 2},$$
$$= \frac{3}{10} = 0.3 = 30\,\%.$$

b) From a deck of cards, what's the probability of drawing two aces in succession without replacement?

Solution:

$$P(A) = \frac{n(A)}{n(S)}$$
$$P(A\ and\ B) = P(A \cap B)$$
$$= P(\text{A}) \times P(B|A).$$
$$= \text{Probability of A} \times \text{Probability of } B \text{ given } A.$$

$$\text{Probability of first ace } = \frac{4}{52}$$
$$= \frac{1}{13}.$$
$$\text{Probability of second ace after already drawing one } = \frac{3}{51}.$$
$$\text{Combined probability } = \frac{1}{13} \times \frac{3}{51}$$
$$= \frac{1}{221} \approx 0.0045 \approx 0.45\,\%.$$

4. Using Venn Diagrams for Combined Events:

a) If $P(A) = 0.4, P(B) = 0.3,$ **and** $P(A \cap B) = 0.1,$ **find** $P(A \cup B)$ **.**

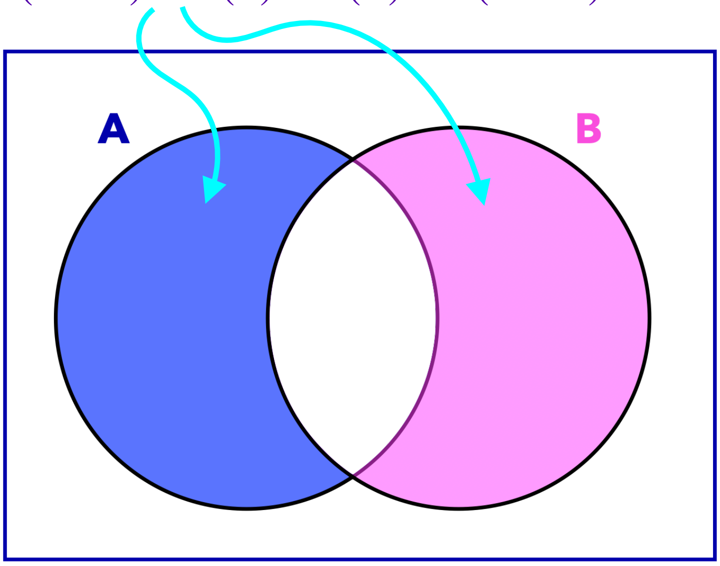
Solution:
Use the formula for the union of two events: (Principle of Inclusion / Exclusion)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cup B) = 0.4 + 0.3 - 0.1$$
$$= 0.6 = \frac{3}{5} = 60\,\%\,.$$

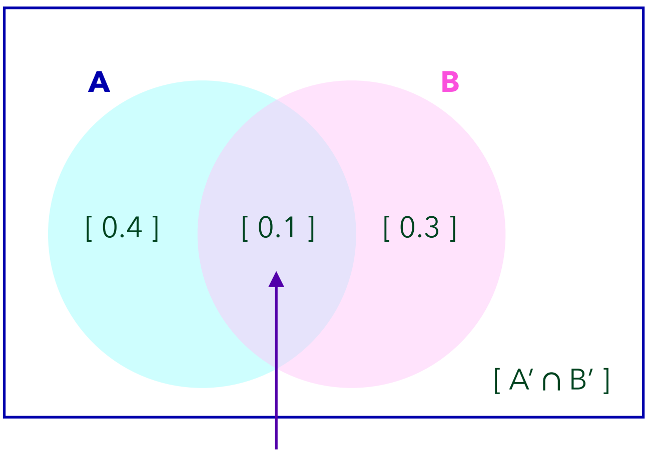
b) Draw a Venn diagram to illustrate the above probabilities.

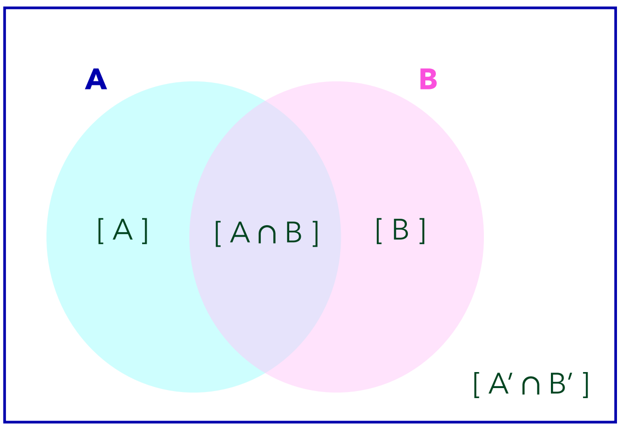
Solution:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6\,.$$



$\rightarrow$ Blue Section + Pink Section
= Blue Circle + Pink Circle − White Intersection
$= P(A) + P(B) - P(A \cap B)$





(∩) = Intersection

5. Practical Application:

In a game, you need to roll a die and flip a coin. What's the probability of rolling a 6 and flipping heads?

Solution:

$$P(A) = \frac{n(A)}{n(S)}$$

These are independent events:

$$\text{Probability of rolling a } 6 = \frac{1}{6}.$$

$$\text{Probability of flipping heads} = \frac{1}{2}.$$

$$\begin{aligned} P(A\ and\ B) &= P(A \cap B) \\ &= P(A) \times P(B). \end{aligned}$$

$$\begin{aligned} \text{Combined probability} = P(A\ and\ B) &= \frac{1}{6} \times \frac{1}{2} \\ &= \frac{1 \times 1}{6 \times 2} \\ &= \frac{1}{12} \approx 0.083 \approx 8.3 \quad . \end{aligned}$$

6. Conditional Probability:

Given: $P(A) = 0.6, P(B) = 0.5,$ **and** $P(A \cap B) = 0.3,$ **find** $P(B|A)$**.**

Solution:
Use the conditional probability formula:

$$\begin{aligned} P(B|A) &= \frac{P(A \cap B)}{P(A)} \\ &= \frac{0.3}{0.6} \\ &= \frac{0.3 \times 10}{0.6 \times 10} \\ &= \frac{3}{6} \\ &= \frac{3 \div 3}{6 \div 3} \\ &= \frac{1}{2} = 0.5 = 50\,\%. \end{aligned}$$

7. Mutually Exclusive Events:

a) If two events are mutually exclusive, how do you calculate the probability of either occurring?

Solution:
For mutually exclusive events A and B,

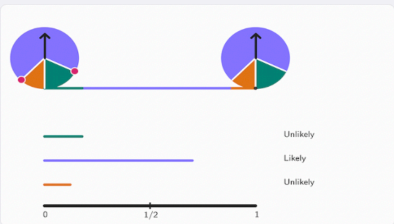
$$P(A \cup B) = P(A) + P(B).$$

b) The probability of rain is 0.2, and the probability of snow is 0.1 on a given day. If these are mutually exclusive events, what's the probability of either rain or snow?

Solution:
Since they are mutually exclusive:

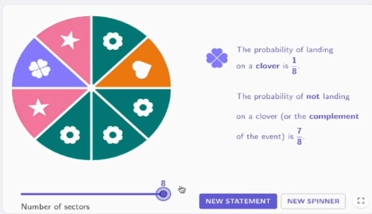
$$\begin{aligned} P(A \cup B) &= P(A) + P(B) \\ P(rain \cup snow) &= 0.2 + 0.1 \\ &= 0.3 = \frac{3}{10} = 30\,\%. \end{aligned}$$

GeoGebra Interactive, accessed 15 March 2025, from: [https://www.geogebra.org/ - Probability](https://www.geogebra.org/)



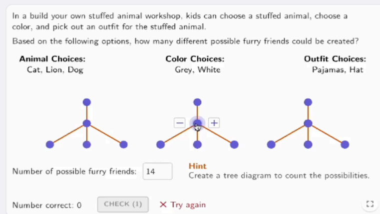
EXPLORATION GRADES 6-8

Spinning Likelihood



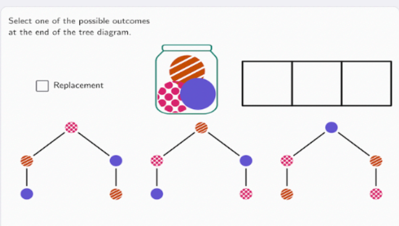
EXPLORATION GRADES 6-8

Determining the Complement of a Spinner Event



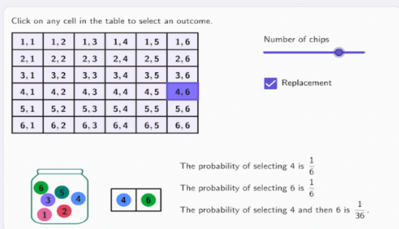
PRACTICE GRADES 6-8

Determining the Sample Space of an Event by Building Furry Friends



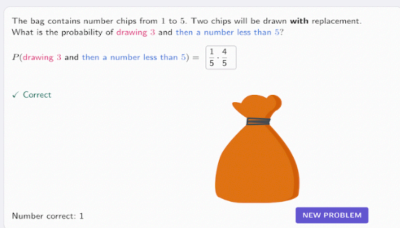
EXPLORATION GRADES 6-8

Possible Probability Outcomes With and Without Replacement



EXPLORATION GRADES 6-8

Probabilities With and Without Replacement



PRACTICE GRADES 6-8

Calculating Compound Probabilities

8. Using Probability Trees:

a) Construct a probability tree for the scenario of flipping two coins and determine the probability of getting at least one tail.

Solution:

$$P(A) = \frac{n(A)}{n(S)}$$

For each coin flip, there are two outcomes:

$Heads\ (H)$ or $Tails\ (T)$.

First flip:

$P(H) = \frac{1}{2}$ or $P(T) = \frac{1}{2}$,

Second flip:

$P(H) = \frac{1}{2}$ or $P(T) = \frac{1}{2}$ for each outcome of the first flip.

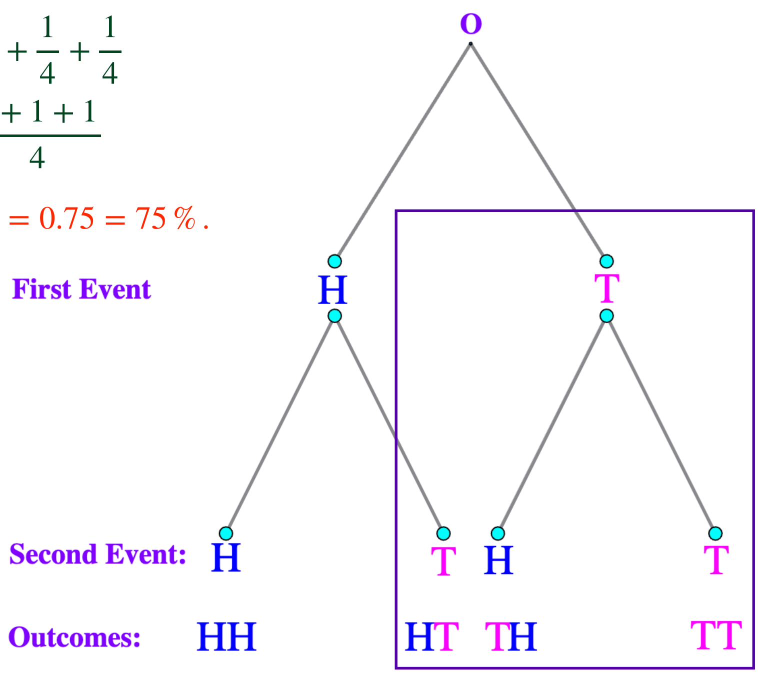
Here's the tree:

```
H ----> H (1/4)
|       |
|       T (1/4)
|
T ----> H (1/4)
        |
        T (1/4)
```

The scenarios for at least one tail are:

$\cancel{HH}, HT, TH, TT$

$$\begin{aligned}\text{Total probability } &= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \\ &= \frac{1+1+1}{4} \\ &= \frac{3}{4} = 0.75 = 75\,\%\,.\end{aligned}$$

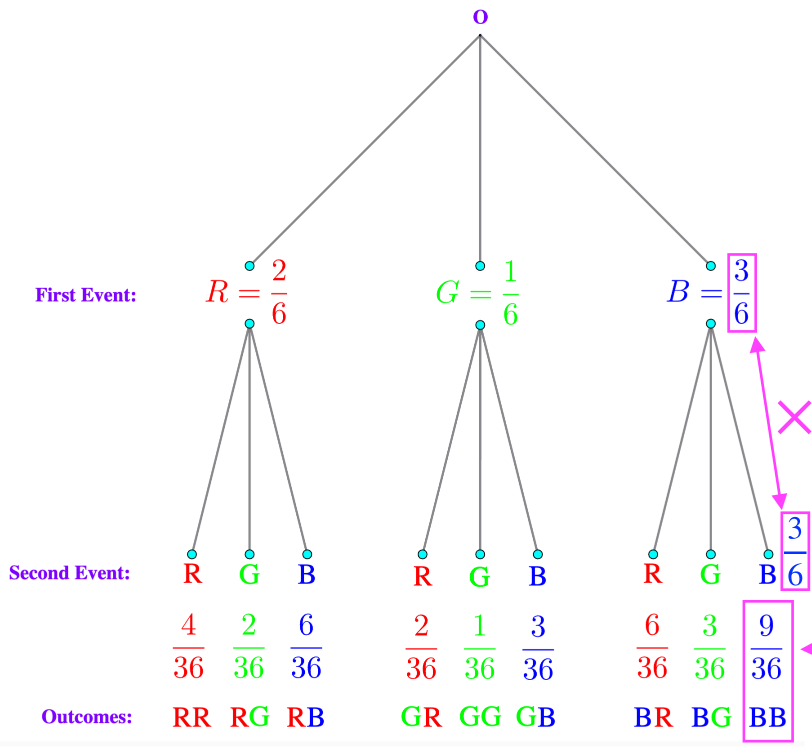


b) Use a probability tree to find the probability of getting two blue balls when drawing two balls from a bag containing 2 red, 3 blue, and 1 green ball, with replacement.

Solution:

For each draw, there are 6 possible outcomes (since we replace the ball after each draw):

```
R ----> R (4/36)
|       |
|       B (6/36)
|       |
|       G (2/36)
|
B ----> R (6/36)
|       |
|       B (9/36)
|       |
|       G (3/36)
|
G ----> R (2/36)
        |
        B (3/36)
        |
        G (1/36)
```



$P(BB) = \frac{3}{6} \times \frac{3}{6} = \frac{9}{36}$

$$P(\text{Event}) = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Outcomes}} = P(A) = \frac{n(A)}{n(S)}$$

Combined probability (since the events are independent):

$$\text{Probability of } A \times \text{Probability of } B = P(A) \times P(B).$$

Probability of drawing two blue balls with replacement:

$$\begin{aligned} P(BB) = P(B) \times P(B) &= \frac{3}{6} \times \frac{3}{6} \\ &= \frac{3 \times 3}{6 \times 6} \\ &= \frac{9}{36} \\ &= \frac{9 \div 9}{36 \div 9} \\ &= \frac{1}{4} = 0.25 = 25\,\%. \end{aligned}$$

9. Two-Way Tables

A survey results in a two-way table showing that out of 300 people, 180 like tea (T), 150 like coffee (C), and 80 like both (T and C). How many like tea but not coffee? What is the probability that a randomly selected person likes only coffee? Use a two-way table to solve the problem, or a Venn diagram.

Solution:
Two-way Table:

	Like Tea	Dislike Tea	**Total**
Like Coffee	80	70	150
Dislike Coffee	100	50	150
Total	180	120	300

$$\text{Like tea but not coffee} = 180 - 80 = 100.$$

$$\text{Like only coffee} = 150 - 80 = 70.$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$\text{Probability of liking only coffee} = \frac{70}{300} = \frac{70 \div 10}{300 \div 10} = \frac{7}{30} = 0.23\dot{3} = 23.\dot{3}\,\%.$$

10. Standard Deviation

a) What does standard deviation tell you about a dataset?

Solution:
A standard deviation indicates the spread of the data points, suggesting variability or dispersion from the mean. It can show whether the data is consistent or predictable.

A large standard deviation indicates that the data points are spread out over a larger range of values, suggesting greater variability or dispersion from the mean. This means the data is less consistent or predictable.

A small standard deviation indicates that the data points are condensed around a small range of values, suggesting low variability or dispersion from the mean. This means the data is highly consistent or predictable.

b) Given the dataset: $\{\ 10,\ 14,\ 18,\ 22,\ 26\ \}$ **, calculate the standard deviation.**

Solution:

$$\{\ 10,\ 14,\ 18,\ 22,\ 26\ \}$$

$$\text{Mean} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$= \frac{\text{Sum of scores}}{\text{Number of Scores}}$$

$$\text{Mean} = \mu = \frac{10+14+18+22+26}{5}$$

$$= \frac{90}{5}$$

$$\mu = 18\,.$$

$$\text{Variance (for population)} = \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{n}$$

$$= \frac{(10-18)^2+(14-18)^2+(18-18)^2+(22-18)^2+(26-18)^2}{5}$$

Deviations from the mean:

$$10-18 = -8, 14-18 = -4, 18-18 = 0, 22-18 = 4, 26-18 = 8\,.$$

Squared deviations:

$$(-8)^2 = 64, (-4)^2 = 16, 0^2 = 0, 4^2 = 16, 8^2 = 64\,.$$

$$\text{Sum of squared deviations} = 64+16+0+16+64$$

$$= 160\,.$$

$$\text{Variance (for population)} = \frac{160}{5}$$

$$= 32\,.$$

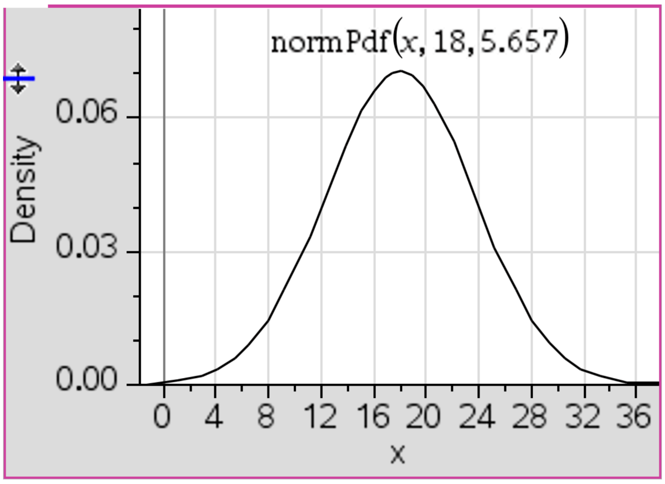
$$\text{Standard Deviation} = \sigma = \sqrt{\text{Variance}} = \sqrt{32}$$

$$= \sqrt{16\times 2}$$

$$= \sqrt{16}\times\sqrt{2}$$

$$= 4\sqrt{2}$$

$$\approx 5.66\,.$$



Standard deviation leads into Bell Curves, pictured above, lucky for you, not in year 9!

Additional Notes for Teachers:

Learning Outcomes: Students should understand how to calculate the probability of combined events, differentiate between independent and dependent events, and apply these concepts in practical situations.
Teaching Strategies:
Use dice, coins, or cards for hands-on activities to demonstrate probability concepts.
Incorporate Venn diagrams for visual learning of combined probabilities.
Discuss real-life scenarios where combined probabilities are relevant, like weather forecasts or game strategies.
Assessment: Evaluate through problems requiring calculation of probabilities for multiple events, interpretation of Venn diagrams, and solving conditional probability problems.
Resources: Physical probability tools (dice, coins), Venn diagram software or templates, or probability simulators for interactive learning.

This question set aligns with the Australian Curriculum for Year 9, focusing on the proficiencies of understanding, fluency, problem-solving, and reasoning in statistics and probability, specifically in the context of measures of central tendency, and the probability of combined events.