



Expanding and Factorising

8

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Focus: A set of questions and solutions for Year 8 students focused on 'Expanding and Factoring' under the "Number and Algebra" strand, tailored to the Australian Curriculum:

1. Understanding Expansion:

a) What does it mean to expand an algebraic expression?

Solution:

Expanding an algebraic expression involves multiplying out the terms to remove brackets, using the *distributive property*.

b) Expand $2(x + 3)$.

Solution:

Using the distributive property:

$$\begin{aligned}
 & 2(x + 3) \\
 &= 2 \times x + 2 \times 3. \\
 &= 2x + 6
 \end{aligned}$$

$$a(b + c) = ab + ac$$

2. Expanding Binomials:

a) Expand $(x + 4)(x - 2)$. (Use Crab Claw)

Solution:

Use the FOIL (First, Outer, Inner, Last) method (also known as **Crab Claw**):

$$\begin{aligned}
 & \rightarrow (x + 4)(x - 2) \\
 &= x \cdot x + x \cdot (-2) + 4 \cdot x + 4 \cdot (-2) \\
 &= x^2 - 2x + 4x - 8 \\
 &\text{Combine like terms: } = x^2 + 2x - 8.
 \end{aligned}$$

(+ × - = -) If signs are:

opposite → change to -

same → change to +

Remember, mathematicians sometimes use the symbol (\cdot) instead of (\times)

so we don't get (\times) confused with (x)

E.g. $2 \times x \equiv 2 \cdot x$



b) Expand $(3y - 1)(2y + 5)$. *(Use Crab Claw)*

Solution:

$$\begin{aligned} &\rightarrow 3y \cdot 2y + 3y \cdot 5 - 1 \cdot 2y - 1 \cdot 5 \\ &= 6y^2 + 15y - 2y - 5 \end{aligned}$$

Combine like terms:

$$= 6y^2 + 13y - 5.$$

$(- \times + = -)$ If signs are:

opposite \rightarrow change to $-$

same \rightarrow change to $+$

3. Factorising Quadratics:

a) What is factorising in algebra, and why is it useful?

Solution:

Factorising involves writing an expression as a product of its factors. It's useful for solving equations, simplifying expressions, and understanding the structure of polynomials.

b) Factorise $x^2 + 5x + 6$.

Solution:

Look for two numbers that multiply to 6 (the constant term) and add to 5 (the coefficient of x):
Numbers are 2 and 3, so:

$$\begin{aligned} &x^2 + 5x + 6 \\ &_ \times _ = 6 \text{ and } _ + _ = 5 \quad \text{OR} \quad _ \times _ = +6 \text{ and } _ + _ = +5 \\ &\rightarrow 3 \times 2 = 6 \text{ and } 3 + 2 = 5 \quad \rightarrow 3 \times 2 = +6 \text{ and } 3 + 2 = +5 \\ &= (x + 3)(x + 2). \quad = (x + 3)(x + 2). \end{aligned}$$

c) Factorise $x^2 - 7x + 12$.

Solution:

Find pairs of numbers that multiply to 12 and add to -7 :
Numbers are -3 and -4 , so:

$$\begin{aligned} &x^2 - 7x + 12 \\ &_ \times _ = 12 \text{ and } _ + _ = -7 \quad \text{OR} \quad _ \times _ = +12 \text{ and } _ + _ = -7 \\ &\rightarrow -3 \times -4 = 12 \text{ and } -3 + -4 = -7 \quad \rightarrow -3 \times -4 = +12 \text{ and } -3 + -4 = -7 \\ &= (x - 3)(x - 4). \quad = (x - 3)(x - 4). \end{aligned}$$



d) Factorise $2x^2 + 5x + 2$.

Solution:

$$2x^2 + 5x + 2$$

Here, consider pairs for 2 times 2, (4), that add to +5:

Numbers are 1 and 4, but we need to account for the leading coefficient:

$$\begin{aligned} &\rightarrow 2x^2 + 5x + 2 \\ &= (2x + 1)(x + 2). \end{aligned}$$

OR

$$2x^2 + 5x + 2$$

We need numbers that multiply to give $(2 \times 2) = 4$ and add to +5 :

These numbers are 1 and 4,

so we break $5x$ into $x + 4x$,

$$\begin{aligned} &\rightarrow 2x^2 + 5x + 2 \\ &= 2x^2 + x + 4x + 2 \\ &= x \times 2x + x \times 1 + 2 \times 2x + 2 \times 1 \\ &= x \times 2x + x \times 1 + 2 \times 2x + 2 \times 1 \\ &= x(2x + 1) + 2(2x + 1) \\ &= (2x + 1)(x + 2). \end{aligned}$$

4. Factorising Common Factors:

a) Factorise $3x + 12$.

Solution:

The common factor is 3 :

$$\begin{aligned} &\rightarrow 3x + 12 \\ &= 3 \cdot x + 3 \cdot 4 \\ &= 3 \cdot (x + 4) \\ &= 3(x + 4). \end{aligned}$$

b) Factorise $6a^2 + 9a$.

Solution:

The common factor is $3a$:

$$\begin{aligned} &\rightarrow 6a^2 + 9a \\ &= 3a \cdot 2a + 3a \cdot 3 \\ &= 3a \cdot (2a + 3) \\ &= 3a(2a + 3). \end{aligned}$$



5. Practical Application:

If the area of a rectangle is given by $x^2 + 5x + 6$ square units, what could be the dimensions of the rectangle?

Solution:

$$\text{Area} = x^2 + 5x + 6$$

$$\begin{aligned} _ \times _ &= 6 \text{ and } _ + _ = 5 \\ \rightarrow 3 \times 2 &= 6 \text{ and } 3 + 2 = 5 \\ &= (x + 3)(x + 2). \end{aligned}$$

So factorising gives:

$$\begin{aligned} \text{Area} &= (x + 3) \times (x + 2) \\ &= \text{Length} \times \text{Width}, \text{ which means the dimensions could be:} \\ &\rightarrow (x + 3) \text{ units by } (x + 2) \text{ units.} \end{aligned}$$

6. Combining Expansion and Factorisation:

Expand $(x - 1)(x - 1)$ and then factorise the result back to check your work.

Solution:

Expand: $(x - 1)(x - 1)$ (Use Crab Claw)

$$\begin{aligned} &= x^2 - x - x + 1 \\ &= x^2 - 2x + 1. \end{aligned}$$

Factorise:

$$x^2 - 2x + 1 = (x - 1)(x - 1).$$

This equals the original expression, confirming the expansion.

7. Factorising Special Cases:

Difference of two squares:

$$\begin{aligned} &x^2 - a^2 \\ &= (x - a)(x + a). \end{aligned}$$

a) Factorise $x^2 - 9$.

Solution:

This is a difference of two squares:

$$\begin{aligned} x^2 - 9 &= x^2 - 3^2 \\ &= (x - 3)(x + 3). \end{aligned}$$

b) Factorise $x^2 - 25$.

Solution:

This is a difference of two squares:

$$\begin{aligned} x^2 - 25 &= x^2 - 5^2 \\ &= (x - 5)(x + 5). \end{aligned}$$



Additional Notes for Teachers:

Learning Outcomes: Students should be able to expand and factorise algebraic expressions, understand the relationship between these processes, and apply them in solving problems.

Teaching Strategies: Use visual aids like algebra tiles to demonstrate expansion and factorisation. Encourage students to check their factorisation by expanding back to the original expression. Provide varied examples, including real-world applications like area calculations or optimisation problems.

Assessment: Assess through exercises that require both expansion and factorisation, emphasising understanding of the process rather than just memorisation.

Resources: Algebra tiles, online algebra tools, or worksheets with a mix of problems to practice these skills.

This question set aligns with the Australian Curriculum for Year 8, focusing on the proficiencies of understanding, fluency, problem-solving, and reasoning in number and algebra, specifically in expanding and factorising expressions.

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