UNIT TEST #021 | Australian Curriculum Mathematics | UTM9T3U5

3D Pythagoras, Spheres, Cones, and Geometric Proofs

9 Unit Test

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Instructions: Read all questions carefully to ensure you understand what is being asked. When completing your official tests / exams, your grade will be based upon your: **understanding, fluency, reasoning, and problem solving,** so ensure you show all lines of working and draw accurate, labelled diagrams where necessary. (ACiQ|9.0 Mathematics standard elaborations found on final page (general assessment marking standards)). [Practise tests are marked out of a score of 10]. For multiple choice questions, tick or fill in the circle next to the corresponding letter under the question.

Check your work if you have time. *Remember:* you don't have to start at question one, it's always best to firstly look through the test, highlight the easy looking questions and complete them first, then secondly, go back through and work on the harder questions. Good luck! And remember to breathe!

$$\sum = \frac{\quad}{10} = \quad \%$$

Part 1: Multiple Choice (2 marks)

Question 1:

a) The formula for the volume of a cone is:

A. $V = \pi r^2 h$ **B.** $V = \frac{1}{3}\pi r^2 h$ **C.** $V = \frac{4}{3}\pi r^3$ **D.** $V = 2\pi r h$

○ A ○ B ○ C ○ D

Space for question 1…

b) What is the equation for distance, d, of the space diagonal in a cube, using Pythagoras?

A. $d^2 = a^2 + b^2 + c^2$ **B.** $c^2 = a^2 + b^2$ **C.** $d = \sqrt{a^2 + b^2 + c^2}$ **D.** $d = \sqrt{b^2 + c^2}$

○ A ○ B ○ C ○ D

Space for question 1b…

Question 2:

a) What is the surface area of a sphere with radius r?

A. $4\pi r^2$ **B.** πr^2 **C.** $2\pi r$ **D.** $\frac{4}{3}\pi r^3$

○ A ○ B ○ C ○ D

Space for question 2a…

b) What is the sum of the interior angles of a triangle?

A. 90° **B.** 180° **C.** 270° **D.** 360°

○ A ○ B ○ C ○ D

Space for question 2b…

Part 2: Short Answer (4 marks)

Question 3:

a) Calculate the volume of a cone with a radius of $3\,cm$ and a height of $10\,cm$. Use $\pi \approx 3.14$.

b) Find the length of the diagonal of a rectangular prism with dimensions $5\,cm, 6\,cm,$ and $7\,cm$.

Question 4:

a) Find the surface area of a sphere with a diameter of $6\,cm$. Use $\pi \approx 3.14$.

b) Calculate the length of a wire needed to reach from one bottom corner of a rectangular room to the diagonally opposite top corner if the room's dimensions are $4\,m$, by $12\,m$, by $3\,m$ high?

Part 3: Problem Solving (4 marks)

Question 5:

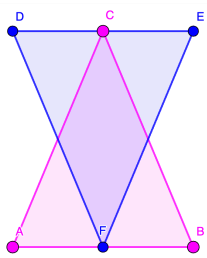
a) A cone has a base radius of $4\,cm$ and a slant height of $5\,cm$. Calculate the surface area of the cone (excluding the base). Use $\pi \approx 3.14$.

b) A civil engineer is designing a triangular support truss for a pedestrian bridge. The truss is in the shape of triangle ABC, with a second triangle DEF, proposed to reinforce the structure. The engineer provides the following measurements:

$AB = DE = 5\,m$
$AC = DF = 7\,m$
$\angle BAC = \angle EDF = 70^\circ$

The engineer needs to confirm that triangle ABC is congruent to triangle DEF, to ensure the truss design is stable and symmetrical.

Prove that triangle ABC is congruent to triangle DEF, stating the congruence criterion used.



Question 6:

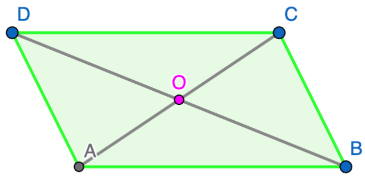
a) A hemisphere has a radius of $7\,cm$. Calculate its total surface area including the base. Calculate its volume. Use $\pi \approx 3.14$.

b) A town planner is designing a quadrilateral garden bed $ABCD$ for a local park in Queensland. To optimise irrigation, the planner wants the garden bed to be a parallelogram. The planner measures:

$AB = DC = 8\,m$
$AD = BC = 5\,m$

Diagonal AC is drawn, and the planner notes that it intersects diagonal BD at point O.
The planner needs to confirm the shape to ensure efficient irrigation piping along parallel sides.

I) Prove that quadrilateral $ABCD$ is a parallelogram using geometric reasoning.
II) Suppose the planner finds that $\angle ABC = 70^\circ$. Determine the measure of $\angle ADC$, and explain how the parallelogram's properties simplify the irrigation design.



Solutions

1a. (0.5 marks)

B. $V = \frac{1}{3}\pi r^2 h$.

b. (0.5 marks)

C. $d = \sqrt{a^2 + b^2 + c^2}$.

2a. (0.5 marks)

A. $4\pi r^2$.

This formula gives the total surface area of a sphere.

b. (0.5 marks)

B. 180° .

3a. (1 mark)

$$V_{cone} = \frac{1}{3}\pi r^2 h$$

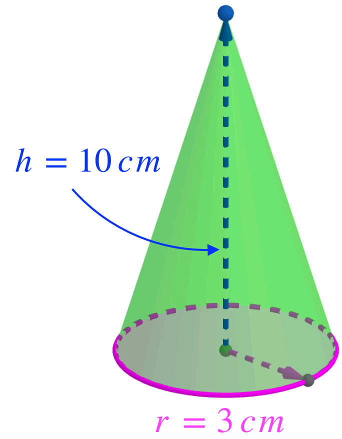
$$r = 3\,cm\,.$$

$$h = 10\,cm\,.$$

$$= \frac{1}{3} \times 3.14 \times 3^2 \times 10$$

$$= \frac{1}{3} \times 3.14 \times 9 \times 10$$

$$= 94.2\,cm^3\,.$$



b. (1 mark)

Using the 3D Pythagorean theorem :

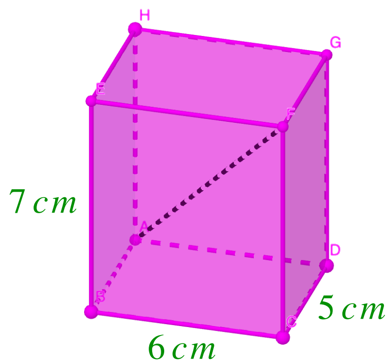
$$d = \sqrt{a^2 + b^2 + c^2}$$

$$d = \sqrt{5^2 + 6^2 + 7^2}$$

$$= \sqrt{125 + 36 + 49}$$

$$= \sqrt{110}$$

$$\approx 10.50\,cm\,.$$



4a. (1 mark)

Radius $r = \dfrac{6}{2}$

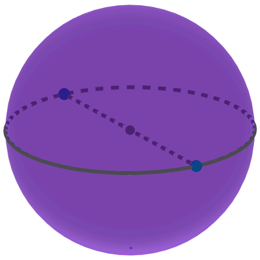
$= 3cm$,

$SA_{sphere} = 4\pi r^2$

$= 4 \times 3.14 \times 3^2$

$= 4 \times 3.14 \times 9$

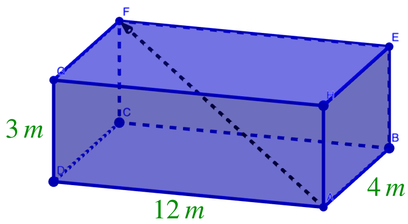
$= 113.04\, cm^2$.



b. (1 mark)

$$d = \sqrt{a^2 + b^2 + c^2}$$

This scenario requires us to use the 3D Pythagorean theorem:



$$d = \sqrt{3^2 + 4^2 + 12^2}$$

$$= \sqrt{9 + 16 + 144}$$

$$= \sqrt{169}$$

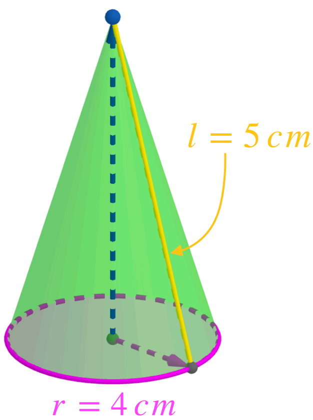
$$\approx 13\, metres\,.$$

5a. (1 mark)

$SA_{cone} = \pi r l$

$r = 4\, cm$.

$l = 5\, cm$.



The surface area of the cone (excluding the base) involves only the lateral surface area:

[Lateral Surface Area] $= \pi r l$

$= 3.14 \times 4 \times 5$

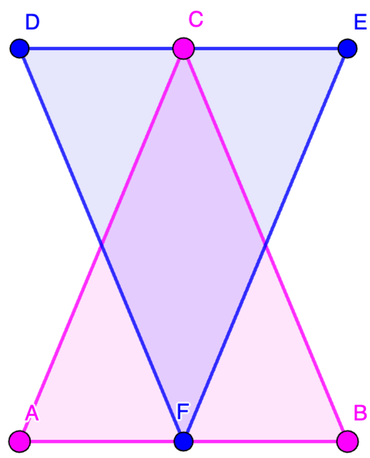
$= 62.8\, cm^2$.

b. (1 mark)

To prove triangle $ABC \cong DEF$, we examine the given information:

$AB = DE = 5\, m$ (one pair of equal sides).
$AC = DF = 7\, m$ (another pair of equal sides).
$\angle BAC = \angle EDF = 70^\circ$ (angle between the pairs of sides).

This matches the **Side-Angle-Side (SAS)** congruence criterion, where two sides and the included angle of one triangle are equal to the corresponding two sides and included angle of another triangle.



6a. (1 mark)

$$
\begin{aligned}
SA_{hemisphere} &= \frac{1}{2} \times \text{Surface Area of Sphere } + \text{ Base} \\
&= \frac{1}{2} \times 4\pi r^2 + \pi r^2 \\
&= 2\pi r^2 + \pi r^2 \\
&= 2\pi r^2 + 1\pi r^2 \\
&= 3\pi r^2
\end{aligned}
$$

$$
\begin{aligned}
V_{hemisphere} &= \frac{1}{2} \times \frac{4}{3}\pi r^3 \\
&= \frac{2}{3}\pi r^3
\end{aligned}
$$

$$r = 7\,cm\,.$$

Total Surface Area (including base):

$$
\begin{aligned}
SA_{hemisphere} &= 3\pi r^2 \\
&= 3 \times 3.14 \times 7^2 \\
&= 3 \times 3.14 \times 49 \\
&= 461.58 \text{ cm}^2\,.
\end{aligned}
$$

Volume of hemisphere:

$$
\begin{aligned}
V_{hemisphere} &= \frac{2}{3}\pi r^3 \\
&= \frac{2}{3} \times 3.14 \times 7^3 \\
&= \frac{2}{3} \times 3.14 \times 343 \\
&\approx 718\,cm^3\,.
\end{aligned}
$$

b. (1 mark)

Part I: Proving $ABCD$ is a Parallelogram

To prove $ABCD$ is a parallelogram, we use the property that a quadrilateral is a parallelogram if both pairs of opposite sides are equal.

Proof:

Given $AB = DC = 8\,m$, the opposite sides AB and DC are equal.
Given $AD = BC = 5\,m$, the opposite sides AD and BC are equal.

By the definition of a parallelogram, a quadrilateral with both pairs of opposite sides equal is a parallelogram. To reinforce this, consider the property that diagonals of a parallelogram bisect each other. In $ABCD$, diagonals AC and BD intersect at O . We can examine triangles formed by the diagonals, such as $\triangle AOB$ and $\triangle COD$:

$AB = DC = 8\,m$ (opposite sides).
$AD = BC = 5\,m$ (opposite sides, though not directly in these triangles).

Since opposite sides are equal, we hypothesise O is the midpoint of both diagonals. However, the side-length equality is sufficient for proof. Thus, $ABCD$ is a parallelogram because both pairs of opposite sides are equal.

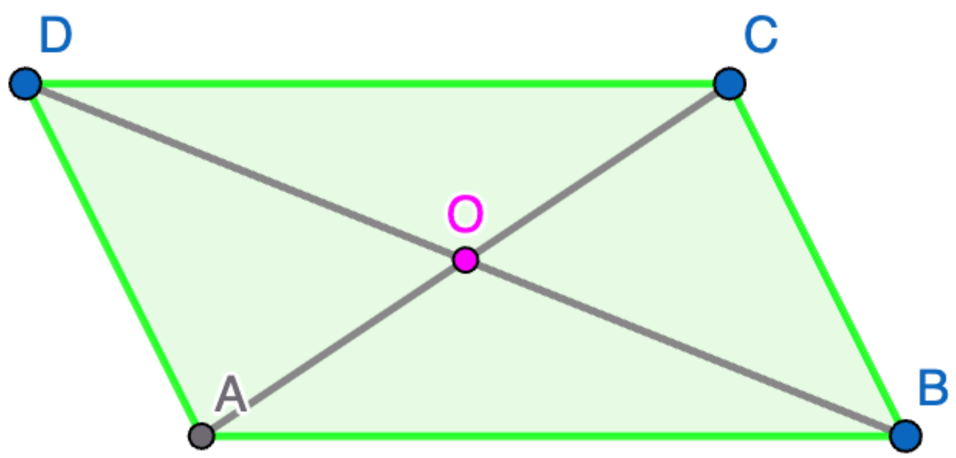
Part II: Determining Angle and Irrigation Benefit

In a parallelogram, opposite angles are equal, and adjacent angles are supplementary (sum to 180°).
Given $\angle ABC = 70^\circ$:

Since $\angle ABC$ and $\angle ADC$ are opposite angles in parallelogram $ABCD$, they are equal.
Therefore, $\angle ADC = \angle ABC = 70^\circ$.

Irrigation Design Explanation:

The parallelogram shape simplifies irrigation because opposite sides are parallel and equal in length, allowing for straightforward installation of piping along these sides. Parallel sides mean irrigation lines can be laid out symmetrically, ensuring even water distribution across the garden bed. The equal opposite angles (e.g., $\angle ABC = \angle ADC = 70^\circ$) confirm the shape's consistency, enabling the planner to use uniform fittings and reduce material waste, as the piping can follow the parallel boundaries efficiently.



$$\sum = \frac{\quad}{10} = \quad \%$$

General Assessment Marking Standards

Remember: When your official tests are marked, they won't be a score out of 10, they will be a grade (A,B,C,D,E) based on the following standards:

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Year 9 Mathematics standard elaborations

		A	B	C	D	E
		The folio of student work contains evidence of the following:				
Mathematical proficiencies	Understanding	accurate and consistent identification, representation, description and connection of mathematical concepts and relationships in complex unfamiliar, complex familiar, and simple familiar situations	accurate identification, representation, description and connection of mathematical concepts and relationships in complex familiar and simple familiar situations	identification, representation, description and connection of mathematical concepts and relationships in simple familiar situations	partial identification, representation and description of mathematical concepts and relationships in some simple familiar situations	fragmented identification, representation and description of mathematical concepts and relationships in isolated and obvious situations
	Fluency	choice, use and application of comprehensive facts, definitions, and procedures to find solutions in complex unfamiliar, complex familiar, and simple familiar situations	choice, use and application of effective facts, definitions, and procedures to find solutions in complex familiar and simple familiar situations	choice, use and application of facts, definitions, and procedures to find solutions in simple familiar situations	choice and use of partial facts, definitions, and procedures to find solutions in some simple familiar situations	choice and use of fragmented facts, definitions and procedures to find solutions in isolated and obvious situations
	Reasoning	comprehensive explanation of mathematical thinking, strategies used, and conclusions reached in complex unfamiliar, complex familiar, and simple familiar situations	detailed explanation of mathematical thinking, strategies used, and conclusions reached in complex familiar and simple familiar situations	explanation of mathematical thinking, strategies used, and conclusions reached in simple familiar situations	partial explanation of mathematical thinking, strategies used, and conclusions reached in some simple familiar situations	fragmented explanation of mathematical thinking, strategies used, and conclusions reached in isolated and obvious situations
	Problem-solving	purposeful use of problem-solving approaches to find solutions to problems.	effective use of problem-solving approaches to find solutions to problems.	use of problem-solving approaches to find solutions to problems.	partial use of problem-solving approaches to make progress towards finding solutions to problems.	fragmented use of problem-solving approaches to make progress towards finding solutions to problems.

Key shading emphasises the qualities that discriminate between the A–E descriptors

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