Statistics, and Probability of Combined Events

9 Unit Test

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Instructions: Read all questions carefully to ensure you understand what is being asked. When completing your official tests / exams, your grade will be based upon your: **understanding, fluency, reasoning, and problem solving,** so ensure you show all lines of working and draw accurate, labelled diagrams where necessary. (ACiQ|9.0 Mathematics standard elaborations found on final page (general assessment marking standards)). [Practise tests are marked out of a score of 10]. For multiple choice questions, tick or fill in the circle next to the corresponding letter under the question.

Check your work if you have time. *Remember:* you don't have to start at question one, it's always best to firstly look through the test, highlight the easy looking questions and complete them first, then secondly, go back through and work on the harder questions. Good luck! And remember to breathe!

$$\sum = \frac{\quad}{10} = \quad \%$$

Part 1: Multiple Choice (2 marks)

Question 1:

a) What does the standard deviation measure in a dataset?

A. The central tendency
B. The spread or dispersion of the data
C. The most frequent value
D. The difference between the maximum and minimum values

○ A ○ B ○ C ○ D

Space for question 1…

b) If you flip a coin twice, what is the probability of getting heads both times?

A. $\frac{1}{2}$ **B.** $\frac{1}{4}$ **C.** $\frac{3}{4}$ **D.** $\frac{1}{8}$

○ A ○ B ○ C ○ D

Space for question 1b…

Question 2:

a) If all the values in a dataset are identical, what would the standard deviation be?

A. 0 **B. 1** **C. Equal to the mean** **D. Equal to the range**

○ A ○ B ○ C ○ D

Space for question 2a…

b) A two-way table shows that out of 100 students, 60 play football, 50 play basketball, and 30 play both. What is the probability that a randomly selected student plays neither sport?

A. 0.2 **B.** 0.3 **C.** 0.4 **D.** 0.5

◯ A ◯ B ◯ C ◯ D

Space for question 2b…

Part 2: Short Answer (4 marks)

Question 3:

a) What is the probability of drawing a king from a standard deck of cards?

b) If $P(A) = 0.5, P(B) = 0.4,$ **and** $P(A \cap B) = 0.1,$ **find** $P(A \cup B)$ **.**

Question 4:

a) Construct a tree diagram for rolling a die followed by flipping a coin. What is the probability of rolling an even number and then getting heads?

b) The probability of rain is 0.25 , and the probability of snow is 0.05 on a given day. If these are mutually exclusive events, what's the probability of either rain or snow?

Part 3: Problem Solving (4 marks)

Question 5:

a) A survey results in a two-way table showing that out of 150 people, 90 like tea (T), 75 like coffee (C), and 40 like both (T and C). How many like tea but not coffee? What is the probability that a randomly selected person likes only coffee? Use a two-way table to solve the problem, or a Venn diagram.

b) Given the dataset: $\{ 5, 7, 9, 11, 13 \}$ **, calculate the standard deviation.**

Question 6:

a) Choose either I) or II):

I) A bag contains 6 red and 4 blue marbles. If you draw two marbles without replacement, what is the probability that both are red?

II) You have two bags. Bag A contains 3 red and 2 blue balls, and Bag B contains 1 red and 4 blue balls. You randomly choose a bag and then draw a ball from it. Use a tree diagram to find the probability of drawing a red ball.

b) Given: $P(A) = 0.3, P(B) = 0.25,$ **and** $P(A \cap B) = 0.1,$ **find** $P(B | A)$**.**

Solutions

1a. (0.5 marks)

B. The spread or dispersion of the data.

Standard deviation quantifies how much the numbers in a dataset deviate from the mean.

b. (0.5 marks)

B. $\frac{1}{4}$.

The probability of heads on each flip is:

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2},$$

So for two independent events, it's:

$$P(A\ and\ B) = P(A \cap B) = P(A) \times P(B).$$

$$= \frac{1}{2} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4} = 0.25 = 25\,\%.$$

2a. (0.5 marks)

A. 0 .

With all values being the same, there is no dispersion from the mean, hence the standard deviation is 0.

b. (0.5 marks)

A. 0.2 .

Using the principle of inclusion-exclusion, students playing at least one sport are:

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 60 + 50 - 30 \\ &= 80. \end{aligned}$$

Thus, the number playing neither is:

$$100 - 80 = 20,$$

So the probability is:

$$\begin{aligned} P(A) &= \frac{n(A)}{n(S)} \\ &= \frac{20}{100} \\ &= \frac{2\cancel{0}}{10\cancel{0}} \\ &= 0.2 = \frac{1}{5} = 20\,\% . \end{aligned}$$

3a. (1 mark)

There are 4 Kings, out of 52 Cards:

$$P(A) = \frac{n(A)}{n(S)},$$

$$\begin{aligned} P(\text{Drawing a King}) &= \frac{4}{52} \\ &= \frac{4 \div 4}{52 \div 4} \\ &= \frac{1}{13} \approx 0.077 \approx 7.7\,\% . \end{aligned}$$

b. (1 mark)

Use the formula for the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.5 + 0.4 - 0.1$$

$$= 0.8 = \frac{4}{5} = 80\,\% .$$

4a. (1 mark)

Tree Diagram:

$$P(A) = \frac{n(A)}{n(S)}$$

First roll of die:

Branches $1,\ 2,\ 3,\ 4,\ 5,\ 6$ (each $P = \frac{1}{6}$)

From each die outcome, branches for H or T (each $P = \frac{1}{2}$)

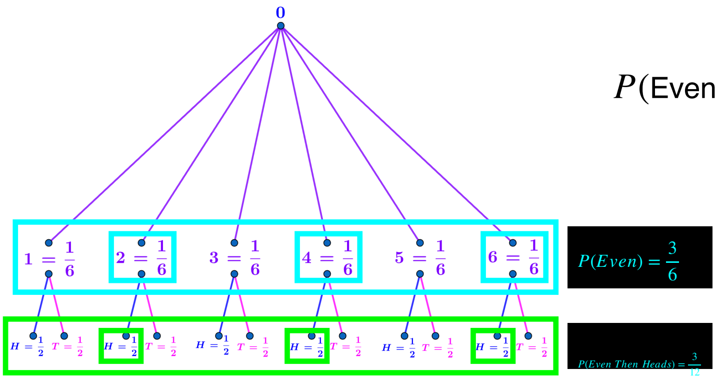
Probability of an even number $(2,\ 4,\ 6)$:

$$P(Even) = \frac{3}{6}.$$

Probability of Heads :

$$P(Heads) = \frac{1}{2}.$$

$$P(A\ and\ B) = P(A \cap B)$$
$$= P(A) \times P(B).$$



$$P(\text{Even then Heads}) = P(\text{Even Number}) \times P(\text{Heads})$$
$$= \frac{3}{6} \times \frac{1}{2}$$
$$= \frac{3 \times 1}{6 \times 2}$$
$$= \frac{3}{12}$$
$$= \frac{3 \div 3}{12 \div 3}$$
$$= \frac{1}{4}.$$

b. (1 mark)

Since they are mutually exclusive:

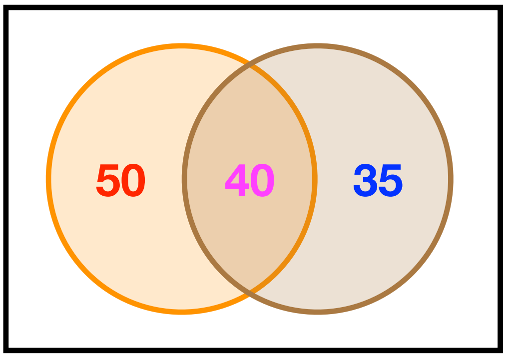
$$P(A \cup B) = P(A) + P(B)$$
$$P(rain \cup snow) = 0.25 + 0.05$$
$$= 0.3 = \frac{3}{10} = 30\,\%.$$

5a. (1 mark)

Two-way Table:

	Like Tea	Dislike Tea	**Total**
Like Coffee	40	35	75
Dislike Coffee	50	25	75
Total	90	60	150

Venn Diagram:



$$\text{Like tea but not coffee} = 90 - 40 = 50.$$

$$\text{Like only coffee} = 75 - 40 = 35.$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$\text{Probability of liking only coffee} = \frac{35}{150} = \frac{35 \div 5}{150 \div 5} = \frac{7}{30} = 0.23\dot{3} = 23.\dot{3}\,\%.$$

b. (1 mark)

$$\text{Mean} = \frac{\sum_{i=1}^{n} x_i}{n}$$
$$= \frac{\text{Sum of scores}}{\text{Number of Scores}}$$

$$\text{Mean } = \mu = \frac{5+7+9+11+13}{5}$$
$$= \frac{45}{5}$$
$$\mu = 9\,.$$

$$\text{Variance (for population)} = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{n}$$
$$= \frac{(5-9)^2 + (7-9)^2 + (9-9)^2 + (11-9)^2 + (13-9)^2}{5}$$

Deviations from the mean:

$$5-9 = -4, 7-9 = -2, 9-9 = 0, 11-9 = 2, 13-9 = 4\,.$$

Squared deviations:

$$(-4)^2 = 16, (-2)^2 = 4, 0^2 = 0, 2^2 = 4, 4^2 = 16\,.$$

$$\text{Sum of squared deviations } = 16+4+0+4+16$$
$$= 40\,.$$

$$\text{Variance (for population) } = \frac{40}{5}$$
$$= 8\,.$$

$$\text{Standard Deviation } = \sigma = \sqrt{\text{Variance}} = \sqrt{8}$$
$$\approx 2.83\,.$$

6a. (1.5 marks)
I)

$$P(A) = \frac{n(A)}{n(S)}.$$

$$P(A\ and\ B) = P(A \cap B)$$

$$\text{Probability of } A \times \text{Probability of } B \text{ given } A = P(A) \times P(B|A).$$

$$\text{Probability of first red marble: } = \frac{6}{10}.$$

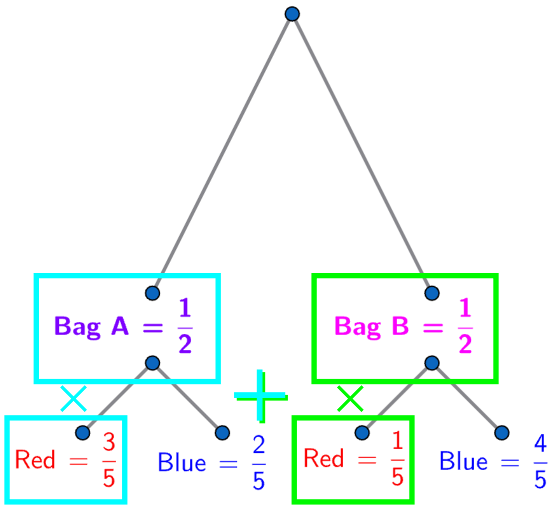
After drawing one red marble, there are 5 red marbles left out of 9 marbles :

$$\text{Probability of second red marble: } = \frac{5}{9}$$

$$\begin{aligned}
\text{Combined probability } &= \frac{6}{10} \times \frac{5}{9}, \\
&= \frac{6 \times 5}{10 \times 9}, \\
&= \frac{30}{90}, \\
&= \frac{3\cancel{0}}{9\cancel{0}}, \\
&= \frac{3 \div 3}{9 \div 3}, \\
&= \frac{1}{3} = 0.3\dot{3} \approx 33\,\%.
\end{aligned}$$

II. (1.5 marks)

Tree Diagram:



$$P(A) = \frac{n(A)}{n(S)}$$

$$\begin{aligned} P(A \ and \ B) &= P(A \cap B) \\ &= P(A) \times P(B)\,. \end{aligned}$$

Choose Bag A: $\frac{1}{2}$, then Red : $\frac{3}{5}$

Choose Bag B: $\frac{1}{2}$, then Red : $\frac{1}{5}$

Probability of drawing a red ball:

$$P(\text{Red}) = \left(P(A) \times P(Red)\right) + \left(P(B) \times P(Red)\right)$$

$$\begin{aligned} P(\text{Red}) &= \left(\frac{1}{2} \times \frac{3}{5}\right) + \left(\frac{1}{2} \times \frac{1}{5}\right) \\ &= \frac{3}{10} + \frac{1}{10} \\ &= \frac{3+1}{10} \\ &= \frac{4 \div 2}{10 \div 2} \\ &= \frac{2}{5} = 0.4 = 40\,\%\,. \end{aligned}$$

b. (0.5 marks)
Use the conditional probability formula:

$$
\begin{aligned}
P(B|A) &= \frac{P(A \cap B)}{P(A)} \\
&= \frac{0.1}{0.3} \\
&= \frac{0.1 \times 10}{0.3 \times 10} \\
&= \frac{1}{3} = 0.3\dot{3} \approx 33\,\%.
\end{aligned}
$$

$$\sum = \frac{}{10} = \quad \%$$

General Assessment Marking Standards

Remember: When your official tests are marked, they won't be a score out of 10, they will be a grade (A,B,C,D,E) based on the following standards:

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Year 9 Mathematics standard elaborations

Mathematical proficiencies	A	B	C	D	E
	The folio of student work contains evidence of the following:				
Understanding	accurate and consistent identification, representation, description and connection of mathematical concepts and relationships in complex unfamiliar, complex familiar, and simple familiar situations	accurate identification, representation, description and connection of mathematical concepts and relationships in complex familiar and simple familiar situations	identification, representation, description and connection of mathematical concepts and relationships in simple familiar situations	partial identification, representation and description of mathematical concepts and relationships in some simple familiar situations	fragmented identification, representation and description of mathematical concepts and relationships in isolated and obvious situations
Fluency	choice, use and application of comprehensive facts, definitions, and procedures to find solutions in complex unfamiliar, complex familiar, and simple familiar situations	choice, use and application of effective facts, definitions, and procedures to find solutions in complex familiar and simple familiar situations	choice, use and application of facts, definitions, and procedures to find solutions in simple familiar situations	choice and use of partial facts, definitions, and procedures to find solutions in some simple familiar situations	choice and use of fragmented facts, definitions and procedures to find solutions in isolated and obvious situations
Reasoning	comprehensive explanation of mathematical thinking, strategies used, and conclusions reached in complex unfamiliar, complex familiar, and simple familiar situations	detailed explanation of mathematical thinking, strategies used, and conclusions reached in complex familiar and simple familiar situations	explanation of mathematical thinking, strategies used, and conclusions reached in simple familiar situations	partial explanation of mathematical thinking, strategies used, and conclusions reached in some simple familiar situations	fragmented explanation of mathematical thinking, strategies used, and conclusions reached in isolated and obvious situations
Problem-solving	purposeful use of problem-solving approaches to find solutions to problems.	effective use of problem-solving approaches to find solutions to problems.	use of problem-solving approaches to find solutions to problems.	partial use of problem-solving approaches to make progress towards finding solutions to problems.	fragmented use of problem-solving approaches to make progress towards finding solutions to problems.

Key shading emphasises the qualities that discriminate between the A–E descriptors

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