UNIT TEST #017 | Australian Curriculum Mathematics | UTM9T1U1

Real Numbers, Indices, and Surds

9 Unit Test

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Instructions: Read all questions carefully to ensure you understand what is being asked. When completing your official tests / exams, your grade will be based upon your: **understanding, fluency, reasoning, and problem solving,** so ensure you show all lines of working and draw accurate, labelled diagrams where necessary. (ACiQ|9.0 Mathematics standard elaborations found on final page (general assessment marking standards)). [Practise tests are marked out of a score of 10]. For multiple choice questions, tick or fill in the circle next to the corresponding letter under the question.

Check your work if you have time. *Remember:* you don't have to start at question one, it's always best to firstly look through the test, highlight the easy looking questions and complete them first, then secondly, go back through and work on the harder questions. Good luck! And remember to breathe!

$$\sum = \frac{}{10} = \quad \%$$

Laws of Indices

Multiplication: $a^m \times a^n = a^{m+n}$

Division: $a^m \div a^n = a^{m-n}$

Power of a Power: $(a^m)^n = a^{m \times n}$

Negative Power: $a^{-m} = \frac{1}{a^m}$

Rational Power: $a^{\frac{m}{n}} = \sqrt[n]{a^m} = a^{\frac{1}{n} \times m} = \left(\sqrt[n]{a}\right)^m = a^{m \times \frac{1}{n}} = \sqrt{(a^m)}$

Zero Power: $a^0 = 1$

Part 1: Multiple Choice (2 marks)

Question 1:

a) Which of the following is an irrational number?

A. $\frac{1}{2}$ **B.** 0.75 **C.** $\sqrt{9}$ **D.** $\sqrt{2}$

◯ A ◯ B ◯ C ◯ D

b) Simplify $\sqrt{72}$.

A. $6\sqrt{2}$ **B.** $3\sqrt{8}$ **C.** $8\sqrt{9}$ **D.** $9\sqrt{8}$

◯ **A** ◯ **B** ◯ **C** ◯ **D**

Space for question 2a…

Question 2:

a) Simplify $2^3 \times 2^5$.

A. 2^{15} **B.** 2^{-2} **C.** 128 **D.** 256

◯ **A** ◯ **B** ◯ **C** ◯ **D**

Space for question 1…

b) Simplify $\frac{5^{10}}{5^6}$.

A. 125 **B.** 425 **C.** 625 **D.** 825

◯ **A** ◯ **B** ◯ **C** ◯ **D**

Part 2: Short Answer (4 marks)

Question 3:

a) Explain the difference between rational and irrational numbers.

b) What is the result of $\sqrt{50} + \sqrt{32}$ in its simplest surd form?.

Question 4:

a) What are indices, and how do they affect numbers? Define what a surd is. Give examples of both.

b) Simplify $5^{2/3}$.

Part 3: Problem Solving (4 marks)

Question 5:

a) Estimate the value of $\sqrt{10}$ between two consecutive whole numbers without using a calculator.

b) The length of a rectangle is $\sqrt{20}\,cm$, and the width is $\sqrt{5}\,cm$. Find the perimeter of the rectangle in simplest surd form.

Question 6:

a) Rationalise the denominator of $\frac{10}{\sqrt{3}}$.

b) Solve for x in the equation $2^{5x} = 256$.

Solutions

1a. (0.5 marks)

$D.\ \sqrt{2}$

$\sqrt{2}$ cannot be expressed as a simple fraction and does not terminate or repeat as a decimal, making it irrational.

b. (0.5 marks)

$A.\ 6\sqrt{2}$

- Simplify by factorising 72 :

$$\begin{aligned} 72 &= 8 \times 9 \\ &= 2^3 \times 3^2 \\ &= 2^2 \times 2^1 \times 3^2 \\ 72 &= 2^2 \times 2 \times 3^2 \text{ , so} \end{aligned}$$

$$\begin{aligned} \rightarrow \sqrt{72} &= \sqrt{2^2 \times 2 \times 3^2} \\ &= \sqrt{2^2 \times 3^2 \times 2} \\ &= \sqrt{2^2 \times 3^2} \times \sqrt{2} \\ &= \sqrt{(2 \times 3)^2} \times \sqrt{2} \\ &= \cancel{\sqrt{(2 \times 3)^2}} \times \sqrt{2} \\ &= (2 \times 3) \times \sqrt{2} \\ \sqrt{72} &= 6\sqrt{2}\,. \end{aligned}$$

2a. (0.5 marks)

D. 256.

Using the multiplication law:

$$\begin{aligned} 2^3 \times 2^5 &= 2^{3+5} \\ &= 2^8 \\ &= 256\,. \end{aligned}$$

b. (0.5 marks)

C. 625.

Using the division law:

$$\begin{aligned} \frac{5^{10}}{5^6} &= 5^{10-6} \\ &= 5^4 \\ &= 5^2 \times 5^2 \\ &= 25 \times 25 \\ &= 625\,. \end{aligned}$$

3a. (0.5 marks)

Rational numbers can be expressed as a fraction, where both the numerator and the denominator are integers, and the denominator is not zero. They either terminate or repeat when written as decimals. Irrational numbers cannot be expressed as a simple fraction; their decimal representations neither terminate nor repeat. Examples include π , $\sqrt{2}$, and e .

b. (0.5 marks)

First, simplify each term:

$$\sqrt{50} = \sqrt{25 \times 2}$$
$$\sqrt{50} = \sqrt{25} \times \sqrt{2}$$
$$= 5\sqrt{2},$$

$$\sqrt{32} = \sqrt{16 \times 2}$$
$$\sqrt{32} = \sqrt{16} \times \sqrt{2}.$$
$$= 4\sqrt{2}$$

Then add:

$$\sqrt{50} + \sqrt{32} = 5\sqrt{2} + 4\sqrt{2}$$
$$= (5+4)\sqrt{2}.$$
$$= 9\sqrt{2}.$$

4a. (1 mark)

Indices (or exponents) indicate how many times a number (the base) is multiplied by itself. For example, 3^4 means 3 is multiplied by itself 4 times: $3 \times 3 \times 3 \times 3 = 81$.
They can significantly increase or decrease the magnitude of numbers.

A surd is an irrational number expressed as a root (usually square or cube root) that cannot be simplified to a rational number (fraction). Examples include $\sqrt{2}$ or $\sqrt[3]{5}$.

b. (1 mark)

$$125^{2/3} = 125^{\frac{2}{3}} = 125^{2\times\frac{1}{3}}$$
$$= 125^{\frac{1}{3}\times 2}$$
$$= (125^{1/3})^2$$
$$= (\sqrt[3]{125})^2$$
$$= (5)^2$$
$$= 25.$$

5a. (1 mark)

Since $3^2 = 9$

and $4^2 = 16$,

OR

$\sqrt{9} = 3$

$\sqrt{16} = 4$

$\sqrt{10}$ must lie between 3 and 4.

A more precise estimation could place it closer to

3 than 4 because 10 is closer to 9 than to 16. ($3.16^2 \approx 10$)

b. (1 mark)

$$Perimeter = 2(length + width)$$

$$Length = \sqrt{20}$$
$$= \sqrt{4 \times 5}$$
$$= \sqrt{4} \times \sqrt{5}$$
$$= 2\sqrt{5}$$

$$Width = \sqrt{5}$$
$$= 1\sqrt{5}$$

$$Perimeter = 2(2\sqrt{5} + 1\sqrt{5})$$
$$= 2 \times 3\sqrt{5}$$
$$= 6\sqrt{5}\ cm.$$

6a. (1 mark)

Multiply numerator and denominator by $\sqrt{3}$:

$$\frac{10}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{10 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$= \frac{10 \times \sqrt{3}}{3^{\frac{1}{2}} \times 3^{\frac{1}{2}}}$$

$$= \frac{10\sqrt{3}}{3^{\frac{1}{2}+\frac{1}{2}}}$$

$$= \frac{10\sqrt{3}}{3^{1}}$$

$$= \frac{10\sqrt{3}}{3}.$$

Remember, $\frac{\sqrt{3}}{\sqrt{3}} = 1$, so we are multiplying by 1, which doesn't change the original expression , it only makes it look different .

Alternatively, $\sqrt{3} \times \sqrt{3} = 3$, because:

$$\sqrt{3} \times \sqrt{3} = 3^{\frac{1}{2}} \times 3^{\frac{1}{2}}$$
$$= (3 \times 3)^{\frac{1}{2}}$$
$$= (9)^{\frac{1}{2}}$$
$$= \sqrt{9}$$
$$= 3.$$

b. (1 mark)

$$2^{5x} = 256$$

Recognise that:

$256 = 2^8$ so,

$$2^{5x} = 256$$
$$\rightarrow 2^{5x} = 2^8,$$

Therefore:

$$5x = 8$$
$$\frac{5x}{5} = \frac{8}{5}$$
$$x = \frac{8}{5} = 1.6.$$

$$\sum = \frac{\quad}{10} = \quad \%$$

General Assessment Marking Standards

Remember: When your official tests are marked, they won't be a score out of 10, they will be a grade (A,B,C,D,E) based on the following standards:

ACiQ | v9.0

Year 9 Mathematics standard elaborations

		A	B	C	D	E
		The folio of student work contains evidence of the following:				
Mathematical proficiencies	**Understanding**	accurate and consistent identification, representation, description and connection of mathematical concepts and relationships in complex unfamiliar, complex familiar, and simple familiar situations	accurate identification, representation, description and connection of mathematical concepts and relationships in complex familiar and simple familiar situations	identification, representation, description and connection of mathematical concepts and relationships in simple familiar situations	partial identification, representation and description of mathematical concepts and relationships in some simple familiar situations	fragmented identification, representation and description of mathematical concepts and relationships in isolated and obvious situations
	Fluency	choice, use and application of comprehensive facts, definitions, and procedures to find solutions in complex unfamiliar, complex familiar, and simple familiar situations	choice, use and application of effective facts, definitions, and procedures to find solutions in complex familiar and simple familiar situations	choice, use and application of facts, definitions, and procedures to find solutions in simple familiar situations	choice and use of partial facts, definitions, and procedures to find solutions in some simple familiar situations	choice and use of fragmented facts, definitions and procedures to find solutions in isolated and obvious situations
	Reasoning	comprehensive explanation of mathematical thinking, strategies used, and conclusions reached in complex unfamiliar, complex familiar, and simple familiar situations	detailed explanation of mathematical thinking, strategies used, and conclusions reached in complex familiar and simple familiar situations	explanation of mathematical thinking, strategies used, and conclusions reached in simple familiar situations	partial explanation of mathematical thinking, strategies used, and conclusions reached in some simple familiar situations	fragmented explanation of mathematical thinking, strategies used, and conclusions reached in isolated and obvious situations
	Problem-solving	purposeful use of problem-solving approaches to find solutions to problems.	effective use of problem-solving approaches to find solutions to problems.	use of problem-solving approaches to find solutions to problems.	partial use of problem-solving approaches to make progress towards finding solutions to problems.	fragmented use of problem-solving approaches to make progress towards finding solutions to problems.

Key shading emphasises the qualities that discriminate between the A–E descriptors

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