UNIT TEST #020 Australian Curriculum Mathematics UTM9T2U4

Simultaneous Equations, and Circles

9 Unit Test

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Instructions: Read all questions carefully to ensure you understand what is being asked. When completing your official tests / exams, your grade will be based upon your: **understanding, fluency, reasoning, and problem solving,** so ensure you show all lines of working and draw accurate, labelled diagrams where necessary. (ACiQ|9.0 Mathematics standard elaborations found on final page (general assessment marking standards)). [Practise tests are marked out of a score of 10]. For multiple choice questions, tick or fill in the circle next to the corresponding letter under the question.

Check your work if you have time. *Remember:* you don't have to start at question one, it's always best to firstly look through the test, highlight the easy looking questions and complete them first, then secondly, go back through and work on the harder questions. Good luck! And remember to breathe!

$$\sum = \frac{\quad}{10} = \quad \%$$

Part 1: Multiple Choice (2 marks)

Question 1:

a) What does it mean when a set of simultaneous equations has no solutions?

A. Their graphs are the same
B. Their graphs meet at zero
C. Their graphs never meet
D. Their graphs are the opposite of each other

○ A ○ B ○ C ○ D

Space for question 1a…

b) In a circle, an angle subtended by a diameter at the circumference is always:

A. Acute **B.** Obtuse **C.** Right **D.** Reflex

○ A ○ B ○ C ○ D

Space for question 1b…

Question 2:

a) Solve the following pair of simultaneous equations: $\begin{cases} y = 3 \\ y = 3x \end{cases}$.

A. $(1,\ 1)$ **B.** $(-1,\ 1)$ **C.** $(3,\ 1)$ **D.** $(1,\ 3)$

○ A ○ B ○ C ○ D

Space for question 2a…

b) What is true about angles at the circumference that are subtended by the same arc?

A. They are equal.
B. They add up to $180\ degrees$.
C. They are complementary.
D. They are supplementary.

◯ **A** ◯ **B** ◯ **C** ◯ **D**

Space for question 2b…

Part 2: Short Answer (4 marks)

Question 3:

a) Solve using elimination: $\begin{cases} x + y = 8 \\ x - 3y = 4 \end{cases}$.

b) Explain the difference between a chord and a diameter of a circle.

Question 4:

a) Saxon has 10 more cups of coffee than Jason. Together they have 15 cups of coffee. How many cups of coffee does each have?

b) An angle at the centre of a circle is 60°. What is the measure of the angle at the circumference subtended by the same arc?

Part 3: Problem Solving (4 marks)

Question 5:

a) Solve this system where one equation is linear and one is quadratic: $\begin{cases} y = x^2 - 9 \\ y = 3x \end{cases}$.

Extra space for question 5a…

b) In a circle, two chords AB and CD intersect at point P . If $AP = 4\,cm$, $PB = 6\,cm$, and $CP = 3\,cm$, what is the length of PD ?

Question 6:

a) A shop sells small and large stickers. Small stickers cost $\$2$ each, and large stickers cost $\$5$ each. If a total of 24 stickers were sold for $\$88$, how many of each type were sold?

b) A tangent to a circle at point T makes an angle of 40° with chord AB . What is the measure of angle ATB ?

Solutions

1a. (0.5 marks)

C. Their graphs never meet.

b. (0.5 marks)

C. Right.

An angle inscribed in a semicircle (subtended by a diameter) is always 90 degrees (a right angle).

2a. (0.5 marks)

D. $(1,\ 3)$

b. (0.5 marks)

A. They are equal.

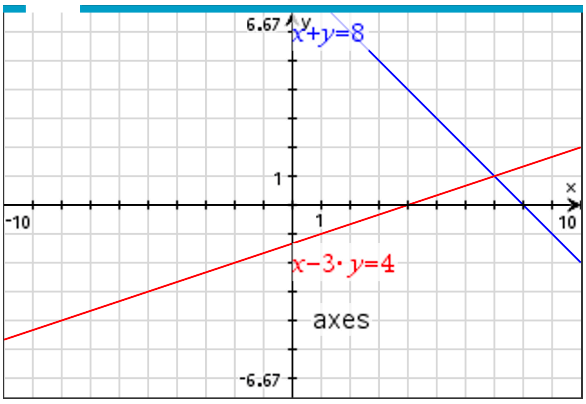
Angles subtended by the same arc at the circumference are equal.

3a. (1 mark)

$$\begin{cases} x + y = 8 \\ x - 3y = 4 \end{cases}$$

Take the second equation away from the first :

$$x - x + y - (-3y) = 8 - 4$$
$$\cancel{x} - \cancel{x} + y - (-3y) = 8 - 4$$
$$y + 3y = 4$$
$$4y = 4$$
$$\frac{\cancel{4}y}{\cancel{4}} = \frac{4}{4}$$
$$y = 1\,.$$



Substitute $y = 1$, back into the first equation :

$$x + 1 = 8$$
$$x \cancel{+1} \cancel{-1} = 8 - 1$$
$$x = 7\,.$$

Their graphs meet at : $(7,\ 1)$

b. (1 mark)

A chord is a straight line segment whose endpoints lie on the circle.

A diameter is a special chord that passes through the centre of the circle and is the longest chord, being twice the length of the radius.

4a. (1 mark)

Let S be the number of coffees Saxon has and J the number Jason has.

Equations:

$$\begin{cases} S = J + 10\,, \\ S + J = 15\,. \end{cases}$$

Substitute S from the first equation into the second:

Using $S = J + 10$:

$$\begin{aligned} \rightarrow S + J &= 15 \\ (J + 10) + J &= 15 \\ 2J + 10 &= 15 \\ 2J \cancel{+10} \cancel{-10} &= 15 - 10 \\ 2J &= 5 \\ \frac{\cancel{2}J}{\cancel{2}} &= \frac{5}{2} \\ J &= \frac{5}{2}\,(= 2.5\,)\,. \end{aligned}$$

And $S = J + 10$

$$\begin{aligned} \rightarrow S &= \frac{5}{2} + 10 \\ S &= \frac{5}{2} + \frac{20}{2} \\ S &= \frac{25}{2}\,(= 12.5\,)\,. \end{aligned}$$

Saxon has 2.5 cups of coffee, and Jason has 12.5 cups.

b. (1 mark)

The angle at the circumference subtended by an arc is half the angle at the centre subtended by the same arc:

$$\text{Angle at circumference} = \frac{60^\circ}{2} = 30^\circ.$$

5a. (1 mark)

Set the equations equal to each other:

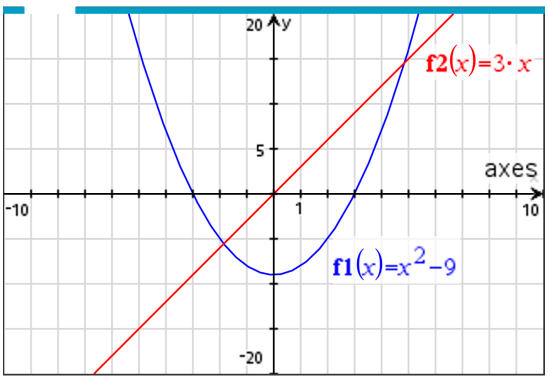
$$3x = x^2 - 9$$
$$\cancel{3x} \cancel{-3x} = x^2 - 9 - 3x$$
$$0 = x^2 - 9 - 3x$$
$$x^2 - 3x - 9 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

Solve the quadratic equation:

Use the quadratic formula with $a = 1,\ b = -3,\ c = -9$:

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times -9}}{2 \times 1}$$



$$x = \frac{+3 \pm \sqrt{9 + 36}}{2}$$
$$= \frac{3 \pm \sqrt{45}}{2}$$
$$= \frac{3 \pm \sqrt{9 \times 5}}{2}$$
$$x = \frac{3}{2} \pm \frac{3\sqrt{5}}{2}$$

Find corresponding y values:

For $x = \frac{3}{2} + \frac{3\sqrt{5}}{2},\ y = 3(\frac{3}{2} + \frac{3\sqrt{5}}{2})$

For $x = \frac{3}{2} - \frac{3\sqrt{5}}{2},\ y = 3(\frac{3}{2} - \frac{3\sqrt{5}}{2})$

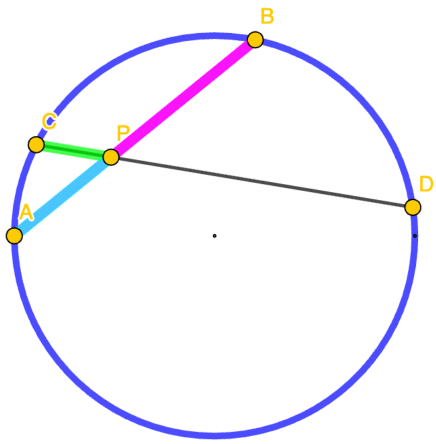
Solutions: $\left(\frac{3 + 3\sqrt{5}}{2}, \frac{9 + 9\sqrt{5}}{2}\right)$ and $\left(\frac{3 - 3\sqrt{5}}{2}, \frac{9 - 9\sqrt{5}}{2}\right)$.

$(\approx 4.85,\ \approx 14.56)$ and $(\approx -1.85,\ \approx -5.56)$

b. (1 mark)

Using the Intersecting Chords Theorem:

$$AP \times PB = CP \times PD$$
$$4 \times 6 = 3 \times PD$$
$$\frac{24}{3} = \frac{\cancel{3} \times PD}{\cancel{3}}$$
$$\frac{24}{3} = PD$$
$$PD = \frac{24}{3}$$
$$= 8\,cm.$$



6a. (1 mark)

Let s be the number of small stickers and L be the number of large stickers.

Equations:

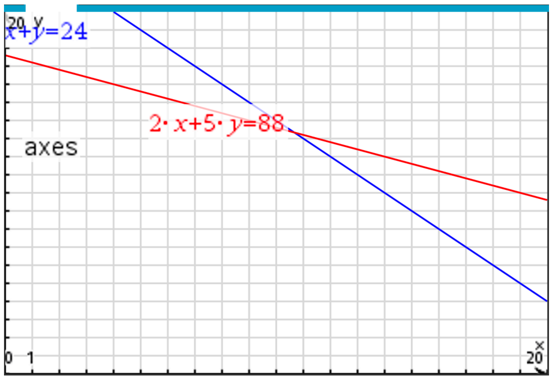
$s + L = 24$ stickers ,

$s + L = 24$.

Re-arrange to get $s = \ldots$,

$s + \cancel{L} \cancel{-L} = 24 \; -L$

$\rightarrow s = 24 - L$.



$\$2s + \$5L = \$88$,

$2s + 5L = 88$.

Solve using substitution (or elimination) :

Using $s = 24 - L$,

substitute into the second equation :

$\rightarrow 2s + 5L = 88$

$$2(24 - L) + 5L = 88$$
$$48 - 2L + 5L = 88$$
$$48 + 3L = 88$$
$$\cancel{48} + 3L \cancel{-48} = 88 \; -48$$
$$3L = 40$$
$$\frac{\cancel{3}L}{\cancel{3}} = \frac{40}{3}$$
$$L = \frac{40}{3}$$

$L \approx 13.33$ (we can't have a fraction of a sticker,

so we round down to 13) :

$\rightarrow L \approx 13$.

$$s = 24 - L$$
$$s \approx 24 - 13$$

$\rightarrow s \approx 11$.

Solution:

13 large stickers and 11 small stickers were sold .

b. (1 mark)

The angle between a tangent and a chord through the point of tangency is equal to the angle in the alternate segment:

$\angle ATB = 40^\circ$.

$$\sum = \frac{\quad}{10} = \quad \%$$

General Assessment Marking Standards

Remember: When your official tests are marked, they won't be a score out of 10, they will be a grade (A,B,C,D,E) based on the following standards:

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Year 9 Mathematics standard elaborations

		A	B	C	D	E
		The folio of student work contains evidence of the following:				
Mathematical proficiencies	Understanding	accurate and consistent identification, representation, description and connection of mathematical concepts and relationships in complex unfamiliar, complex familiar, and simple familiar situations	accurate identification, representation, description and connection of mathematical concepts and relationships in complex familiar and simple familiar situations	identification, representation, description and connection of mathematical concepts and relationships in simple familiar situations	partial identification, representation and description of mathematical concepts and relationships in some simple familiar situations	fragmented identification, representation and description of mathematical concepts and relationships in isolated and obvious situations
	Fluency	choice, use and application of comprehensive facts, definitions, and procedures to find solutions in complex unfamiliar, complex familiar, and simple familiar situations	choice, use and application of effective facts, definitions, and procedures to find solutions in complex familiar and simple familiar situations	choice, use and application of facts, definitions, and procedures to find solutions in simple familiar situations	choice and use of partial facts, definitions, and procedures to find solutions in some simple familiar situations	choice and use of fragmented facts, definitions and procedures to find solutions in isolated and obvious situations
	Reasoning	comprehensive explanation of mathematical thinking, strategies used, and conclusions reached in complex unfamiliar, complex familiar, and simple familiar situations	detailed explanation of mathematical thinking, strategies used, and conclusions reached in complex familiar and simple familiar situations	explanation of mathematical thinking, strategies used, and conclusions reached in simple familiar situations	partial explanation of mathematical thinking, strategies used, and conclusions reached in some simple familiar situations	fragmented explanation of mathematical thinking, strategies used, and conclusions reached in isolated and obvious situations
	Problem-solving	purposeful use of problem-solving approaches to find solutions to problems.	effective use of problem-solving approaches to find solutions to problems.	use of problem-solving approaches to find solutions to problems.	partial use of problem-solving approaches to make progress towards finding solutions to problems.	fragmented use of problem-solving approaches to make progress towards finding solutions to problems.

Key shading emphasises the qualities that discriminate between the A–E descriptors

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