CHEAT SHEET # 012 January 2025

Australian Curriculum Cheat Sheet Year 10 Mathematics

Free and always will be!

Aim: A cheat sheet for Year 10 and 10A Mathematics under the Australian Curriculum, focusing on the key areas of study with examples, formulas, and tips:

Year 10 Units Overview:

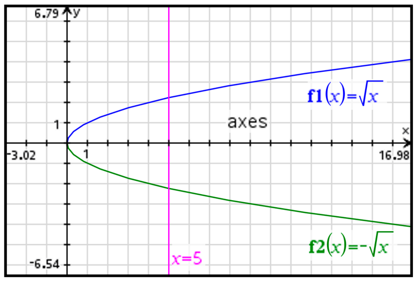
Term 1 **Unit 1** - Finance, Error, Approximation, Functions and Relations. **Unit 2** - Algebra, Equations, and Inequalities.
Term 2 **Unit 3** - Indices, and Logarithms. **Unit 4** - Trigonometry, and Pythagoras.
Term 3 **Unit 5** - Volume, Surface Area, and Networks. **Unit 6** - Algorithms, and Geometric Proofs.
Term 4 **Unit 7** - Multivariate Data Analysis, and Bias. **Unit 8** - Conditional Probability.

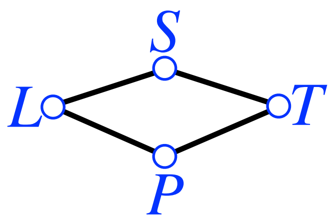
Rule for difference of cubes :

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Rule for expanding : $(a-b)^3$

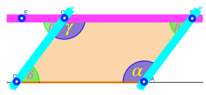
$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$



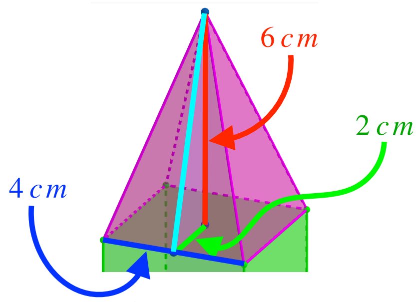


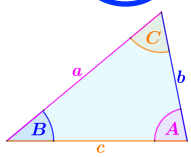
Index Form: $a^n = b$ $\leftrightarrow$ **Logarithmic Form:** $\log_a(b) = n$

Multiplication:	$\log_a(mn) = \log_a(m) + \log_a(n) = \log_a(mn)$
Division:	$\log_a\left(\frac{m}{n}\right) = \log_a(m) - \log_a(n) = \log_a\left(\frac{m}{n}\right)$
Power of a Power:	$\log_a(m)^n = n\log_a(m) = \log_a(m)^n$
Negative Power:	$\log_a\left(\frac{1}{n}\right) = \log_a\left(\frac{1}{n^1}\right) = \log_a\left(n^{-1}\right) = -1\log_a(n) = -\log_a(n)$
Zero Power:	$\log_a(1) = 0 \quad \left[a^0 = 1\right]$
Change of Base:	$\log_b(a) = \frac{\log_k(a)}{\log_k(b)} = \log_b(a)$



$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$





$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

Year 10 Mathematics Cheat Sheet

Term 1
Unit 1 - Finance, Error, Approximation, Functions and Relations

1. Number and Algebra

Finance

Simple Interest

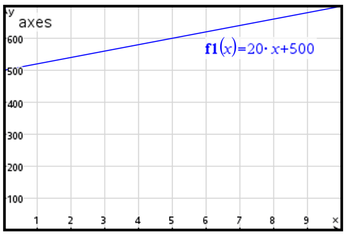
Example: A student starts a savings account with an initial deposit of $\$500$. The account earns simple interest at a rate of 4% *per annum*. Formulate a linear model to represent the balance of the account after t years.

Solution:

$$I = PRT$$

$$A = P + I$$

$$A(t) = 500 + 20t.$$



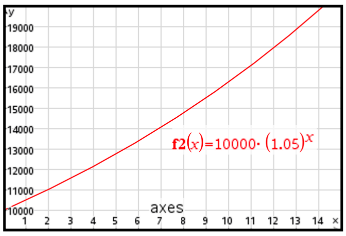
Compound Interest

Example: A small business invests $\$10{,}000$ in an account that earns compound interest at 5% *per annum, compounded annually*. Formulate an exponential model for the investment's value after t years.

Solution:

$$A = P(1+r)^t$$

$$A(t) = 10{,}000 \times 1.05^t.$$



Rounding and Approximation Errors

Finance

Example: A Year 10 student is budgeting for a school event. They estimate the cost of catering as $\$12.346$ per person, and there are 48 attendees.

To simplify calculations, they round the cost to the nearest dollar. Determine the absolute and percentage error introduced by rounding.

Solution:

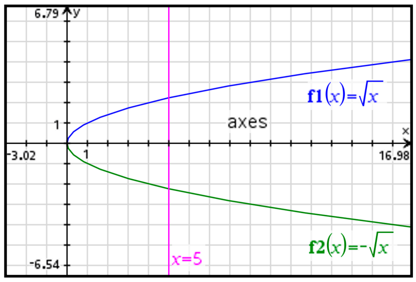
$$\text{Absolute error} = |\text{Unrounded cost} - \text{Rounded cost}|$$

$$\text{Percentage error} = \frac{\text{Absolute error}}{\text{Unrounded cost}} \times 100$$

$$\text{Percentage error} \approx 2.80\% \text{ (to 2 decimal places)}.$$

Functions and Relations

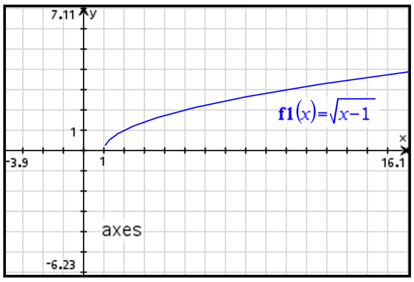
Example: Is the equation $y^2 = x$ a function?



Solution:
No, it is not a function because for any $x > 0$, there are two corresponding $y\text{-}values$. This is because $y^2 = x \rightarrow y = \pm\sqrt{x}$, (e.g., if $x = 4$, then $y = 2$ or $y = -2$). This fails the Vertical Line Test.

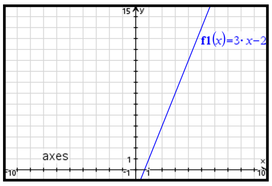
Example: Find the domain and range of the function $f(x) = \sqrt{x-1}$.

Solution:
The domain is $[1,\infty)$ or $1 < x \leq \infty$.
The range is $[0,\infty)$ or $0 < y \leq \infty$.



Example: Given $f(x) = 3x - 2$, find $f(5)$.

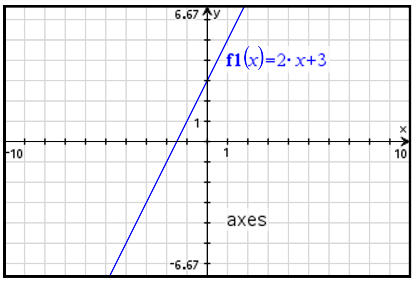
Solution:
$f(x) = 3x - 2$
$= 13$.

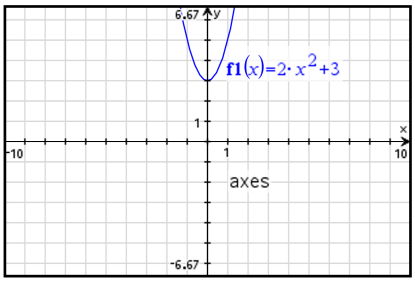


Example: If $f(x) = 2x + 3$ and $g(x) = x^2$, find $f(g(x))$.

Solution:
$f(x) = 2x + 3$
$g(x) = x^2$

$= 2x^2 + 3$.

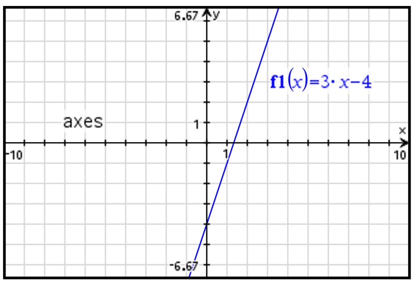


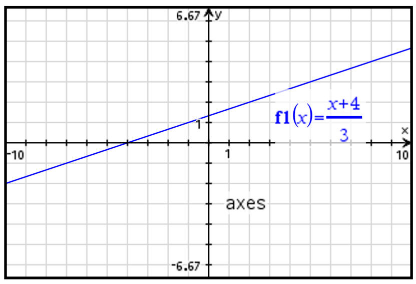


Example: Find the inverse of the function $f(x) = 3x - 4$.

Solution:
Inverse function :

$$f^{-1}(x) = \frac{x+4}{3}$$





Unit 2 - Algebra, Inequalities, and Equations:

Algebra

Algebraic Techniques:

$(+\times - = -)$ If signs are:
opposite → change to $-$
same → change to $+$

Example: Find the product of $(2x-1)$ and $(3x+4)$.

Solution:
$(2x-1)\times(3x+4) = (2x-1)(3x+4)$

Combine like terms : $= 6x^2+5x-4$.

Example: Expand $(x+6)^2$.

Rule for expanding : $(a-b)^3$
$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Solution:
Re-write, expand, then collect like terms:

$$(x+6)^2 = (x+6)(x+6)$$ *Use Crab Claw*

$$= x^2+12x+36.$$

OR
Using rule for perfect squares:

$$(a+b)^2 = a^2+2ab+b^2$$
$$(x+6)^2 = x^2 + 2\cdot x\cdot 6 + 6^2$$
$$= x^2+12x+36.$$

Rules for perfect squares:
$(a+b)^2 = a^2+2ab+b^2$
$(a-b)^2 = a^2-2ab+b^2$

Example: Factorise x^2+5x+6.

Solution:
$(x+2)(x+3)$.

Example: Factorise $3x^2+15x$.

Solution:
Factor out the greatest common factor $(3x)$:

$$3x^2+15x = 3x\cdot x + 3x\cdot 5$$
$$= 3x(x+5).$$

Example: Factorise x^3-8 using the difference of cubes.

Solution:
$x^3-8 = x^3-2^3$
$= (x-2)(x^2+2x+4)$.

Rule for difference of cubes :
$a^3-b^3 = (a-b)(a^2+ab+b^2)$

Example: Factorise $x^2 - 16$.

Solution:
This is a difference of two squares:

Rule for difference of two squares:

$a^2 - b^2 = (a+b)(a-b)$

$$\begin{aligned} x^2 - 16 &= x^2 - 4^2 \\ &= (x+4)(x-4)\,. \end{aligned}$$

Example: Complete the square for $x^2 + 6x$.

Solution:
Take half of the coefficient of $x\ = (6/2)$, square it $\ = (9)$,

Then, add and subtract it:

Remember, $(+b - b = 0)$, so by adding and subtracting $b\ (= \frac{a}{2})$,
we aren't changing the equation, just making it look different.

$$\begin{aligned} &\rightarrow x^2 + 6x \\ &= x^2 + 6x + (\frac{6}{2})^2 - (\frac{6}{2})^2 \\ &= [x^2 + 6x + 3^2] - 9\,, \end{aligned}$$

Rule for perfect squares:

$(a+b)^2 = a^2 + 2ab + b^2$

$a^2 + 2ab + b^2 = (a+b)^2$

Using rule for perfect squares:

$$\begin{aligned} [x^2 + 6x + 3^2] - 9 &= [x^2 + 2 \cdot x \cdot 3 + 3^2] - 9 \\ &= [(x+3)^2] - 9\,. \\ &= (x+3)^2 - 9\,. \end{aligned}$$

Linear Inequalities:
Example: Solve the linear inequality $-3x - 7 > 11$.

Solution:
Add 7 to both sides:

$$\begin{aligned} -3x - 7 &> 11 \\ -3x &> 18\,, \end{aligned}$$

Remember, when manipulating inequalities by multiplying or dividing by a negative number, the inequality must be reversed.

Divide by -3 ,

$$x < -6\,.$$

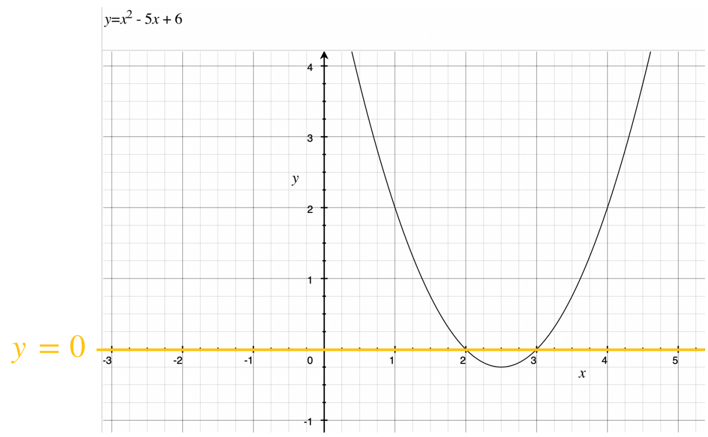
Quadratic Equations:
Example: Solve the equation $x^2 - 5x + 6 = 0$ by factorisation.

Solution:
Factorise:

$$\begin{aligned} &x^2 - 5x + 6 = 0 \\ \rightarrow\ &(x-2)(x-3) = 0 \end{aligned}$$

This gives:

$$x = 2 \ or\ x = 3\,.$$



Algebraic Fractions

Example: Simplify $\dfrac{(x-2)^3}{(x-2)(x+3)}$ by cancelling where possible.

Solution:

$$\frac{(x-2)^3}{(x-2)(x+3)} = \frac{(x-2)^1(x-2)^2}{(x-2)^1(x+3)}$$

$$= \frac{(x-2)^2}{x+3} \text{ for } x \neq 2 \text{ and } x \neq -3.$$

Introduction to Optimisation

Example: A local market stall sells homemade candles. The weekly profit P in dollars is modelled by the quadratic function $P(n) = -2n^2 + 80n - 200$, where n is the number of candles sold. Interpret the model to determine the number of candles that maximises profit.

Solution:

The quadratic function $P(n) = -2n^2 + 80n - 200$

Has a maximum at the vertex :

$$v = -\frac{b}{2a}$$

$$v = 20.$$

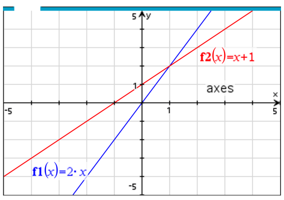
The profit is maximised when 20 candles are sold .

Simultaneous Equations:

Example: Solve $y = x + 1,\ y = 2x$.

Solution:

$x + 1 = 2x, \rightarrow x = 1,\ y = 2$.



Example: A ball is thrown, and its height h in $metres$ after t $seconds$ is given by $h = -5t^2 + 20t + 1$. At what time(s) does it reach a height of 11 $metres$?

Solution:

Set up the equation :

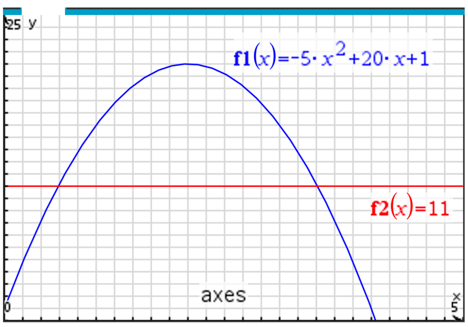
$$h = -5t^2 + 20t + 1$$

$$11 = -5t^2 + 20t + 1$$

$$t^2 - 4t + 2 = 0$$

Solve using the quadratic formula :

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Times :

$t = 2 + \sqrt{2}$ $seconds$ and $t = 2 - \sqrt{2}$ $seconds$.

$t \approx 3.41$ $seconds$ and $t \approx 0.586$ $seconds$.

Term 2
Unit 3 - Indices and Logarithms:

Index Laws:

Multiplication:	$a^m \times a^n = a^{m+n}$
Division:	$a^m \div a^n = a^{m-n}$
Power of a Power:	$(a^m)^n = a^{m\times n}$
Negative Power:	$a^{-m} = \dfrac{1}{a^m}$
Rational Power:	$a^{\frac{m}{n}} = \sqrt[n]{a^m} = a^{\frac{1}{n}\times m} = \left(\sqrt[n]{a}\right)^m = a^{m\times\frac{1}{n}} = \sqrt[n]{(a^m)}$
Zero Power:	$a^0 = 1$

Example: Simplify $16^{3/4}$.

Solution:

$$\begin{aligned} 16^{3/4} &= \left(\sqrt[4]{16}\right)^3 \\ &= 2^3 \\ &= 8\,. \end{aligned}$$

Example: Simplify $\dfrac{x^4 \cdot x^{-2}}{x^3}$.

Solution:

$$\frac{1}{x}\,.$$

Example: Solve $3^{x-1} = 27$.

Solution:
Recognise that :

$$\begin{aligned} 27 &= 3^3 \\ 3^{x-1} &= 3^3 \\ x &= 4\,. \end{aligned}$$

Example: Simplify 3^{-2}.

Solution:

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}\,.$$

Logarithmic Laws:

Index Form: $a^n = b$ $\leftrightarrow$	**Logarithmic Form:** $\log_a(b) = n$
Multiplication:	$\log_a(m\,n) = \log_a(m) + \log_a(n) = \log_a(m\,n)$
Division:	$\log_a\left(\frac{m}{n}\right) = \log_a(m) - \log_a(n) = \log_a\left(\frac{m}{n}\right)$
Power of a Power:	$\log_a(m)^n = n\,\log_a(m) = \log_a(m)^n$
Negative Power:	$\log_a\left(\frac{1}{n}\right) = \log_a\left(\frac{1}{n^1}\right) = \log_a\left(n^{-1}\right) = -1\log_a(n) = -\log_a(n)$
Zero Power:	$\log_a(1) = 0$ $\left[a^0 = 1\right]$
Change of Base:	$\log_b(a) = \frac{\log_k(a)}{\log_k(b)} = \log_b(a)$

Example: Use the laws to simplify $\log_2(8) + \log_2(4) - \log_2(2)$.

Solution:
Evaluate each term individually : $\log_2(8) + \log_2(4) - \log_2(2)$

$$\log_2(8) = 3\,.$$
$$\log_2(4) = 2\,.$$
$$\log_2(2) = 1\,.$$

Therefore, $3 + 2 - 1 = 4$.

Solving Logarithmic Equations:

Example: Solve for x in $\log_3(x) + \log_3(5x) = 2$.

Solution:

Use the product law :

$$\log_a(M) + \log_a(N) = \log_a(MN)$$
$$\log_3(5x^2) = 2\,.$$

Convert to exponential form :

$$5x^2 = 3^2$$
$$= \pm\frac{3\sqrt{5}}{5}$$

Since the log of a negative number isn't defined in real numbers, choose the positive solution :

$$x = +\frac{3\sqrt{5}}{5}\,.$$

Change of Base Formula:

Example: Use the change of base formula to find $\log_{64}(8)$.

Solution:

$$\log_{64}(8) = \frac{\log_{10}(8)}{\log_{10}(64)}$$
$$= \frac{\log_{10}(8)}{\log_{10}(8)^2}$$

So, $\log_{64}(8) = \frac{1}{2} \cdot \left(\sqrt{64} = 64^{\frac{1}{2}} = 8\right)$

Population Modelling

Example: The population of a city doubles every $10\ years$. Express the population in terms of the initial population, using indices. Then, check your equation is correct by substituting in $t = 10\ years$ and then $t = 20\ years$.

Solution:

$P = 2^{t/10}$ represents doubling every 10 years.

e.g. for $t = 10\ years$

Population = Initial Population $\times\ 2^{10/10}$

$P_{(t=10)} = P_{(t=0)} \times 2$

for $t = 20\ years$

Population = Initial Population $\times\ 2^{20/10}$

$P_{(t=20)} = P_{(t=0)} \times 4$

Logarithmic Scales

Example: The decibel (dB) scale measures sound intensity logarithmically, defined as $L = 10 \times log_{10}(I/I_0)$, where I is the sound intensity (in watts per square metre, W/m^2), and $I_0 = 10^{-12}\ W/m^2$ is the threshold of human hearing. Calculate the decibel level of a sound with an intensity of $10^{-6}\ W/m^2$.

Solution:

Calculate decibel level :

$$I/I_0 = 10^6$$

$$L = 10 \times log_{10}(I/I_0)$$
$$= 60 .$$

Decibel level $= 60\, dB$.

Unit 4 - Trigonometry, and Pythagoras:

2. Measurement and Space

Triangles

Non-Right-Angled Triangles:

Example: State the Law of Sines and the Law of Cosines.

Solution:

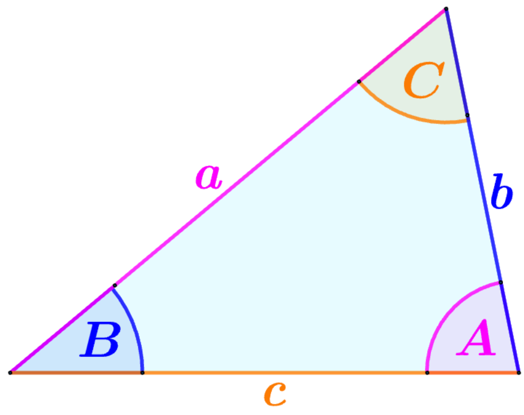
Law of Sines :

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines :

$$c^2 = a^2 + b^2 - 2ab\cos C$$

(and similar for other sides).



Example: State the Sine Area Rule:

Solution:

Area rule :

$$Area = \frac{1}{2}ab\sin(C)$$

Example: In a triangle ABC, if $a = 7$, $b = 8$, and $\angle A = 30^\circ$, find $\angle B$.

Solution:

Using the Law of Sines :

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{7}{\sin 30^\circ} = \frac{8}{\sin B}$$

$$\sin 30^\circ = \frac{1}{2} = 0.5 \text{, so :}$$

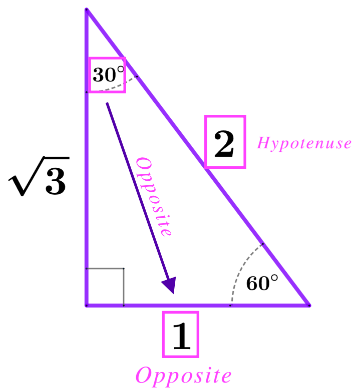
$$\angle B \approx 34.85^\circ.$$

SOH CAH TOA

$$\sin(\theta) = \frac{Opp.}{Hyp.}$$

$$\sin(30^\circ) = \frac{1}{2}$$

$$\sin(30^\circ) = 0.5$$

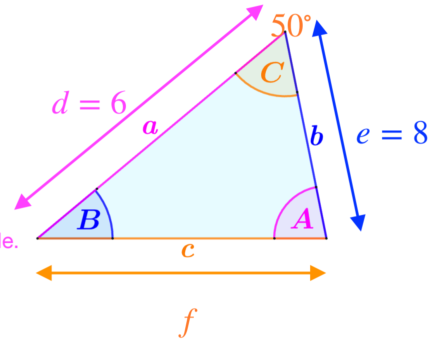


(Note: there could be another solution due to the sine function's periodicity, but given the standard triangle configuration, we only consider this one.)

Example: In triangle DEF, $d = 6$, $e = 8$, and $\angle F = 50^\circ$, find the length of f.

Solution:
Using the Law of Cosines :

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$
$$f \approx \sqrt{38.2912}$$
$$\approx 6.19\,.$$



Note: Triangle **NOT** drawn to scale.

Trigonometry (Right-Angled):
Formulas: $SOHCAHTOA$.

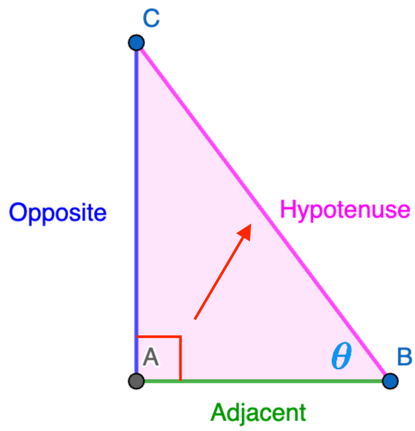
$$SOH \rightarrow \sin(\theta) = \frac{opposite}{hypotenuse} = \frac{O}{H}$$

$$CAH \rightarrow \cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{A}{H}$$

$$TOA \rightarrow \tan(\theta) = \frac{opposite}{adjacent} = \frac{O}{A}$$

$SOH\ CAH\ TOA$

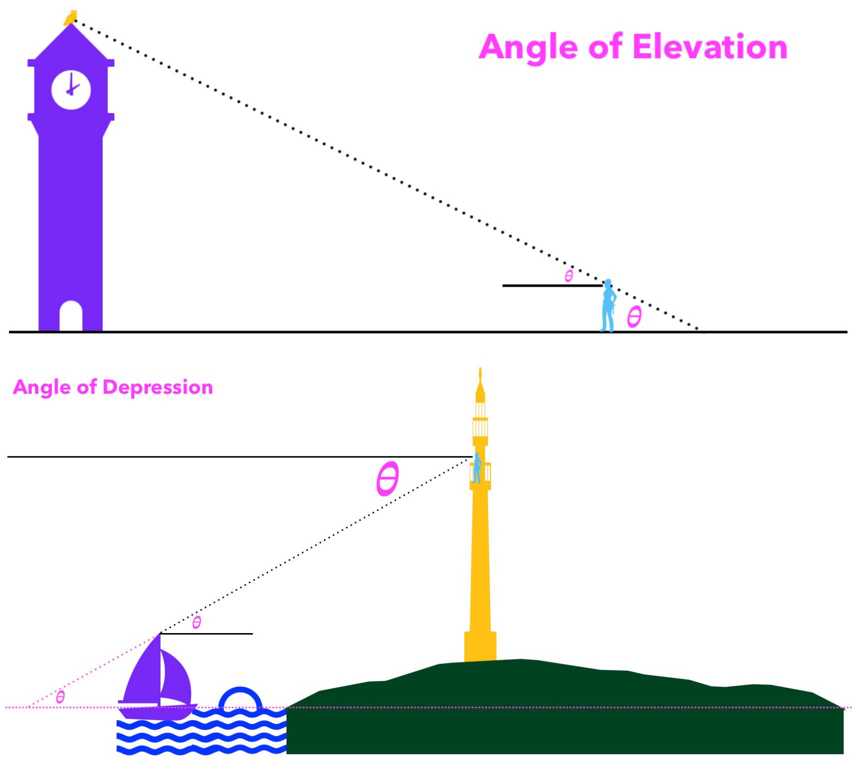
Some Old Hags, SOH
Can't Always Hide, CAH
Their Old Age. TOA



Example: $\angle A = 30^\circ,\ hypotenuse = 10\,cm$.

Solution:
$Opposite = 5\,cm$.

Angles of Elevation and Depression



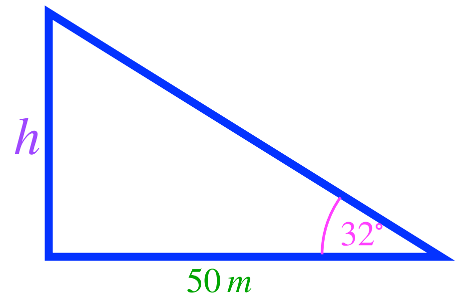
Example: From a point $50\,m$ away from the base of a tree, the angle of elevation to the top of the tree is 32° . Find the height of the tree, correct to one decimal place.

Solution:
Draw a diagram, identify the trigonometric ratio:
Use the tangent ratio :

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan(32^\circ) = \frac{h}{50}$$

$$h \approx 31.24345\,.$$



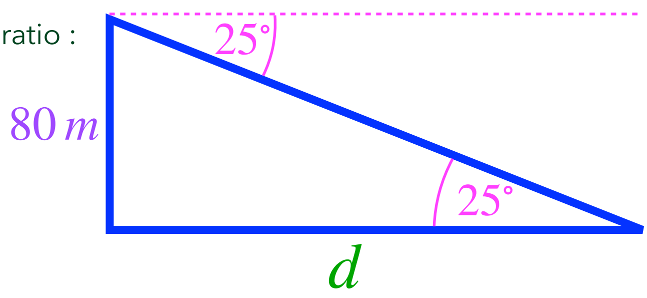
Example: A hiker stands on a cliff $80\,m$ above sea level and observes a boat on the water. The angle of depression to the boat is 25° , and the boat is on a bearing of 140° from the hiker. Calculate the horizontal distance from the hiker to the boat, correct to the nearest metre.

Solution:
Draw a diagram, identify the trigonometric ratio :

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

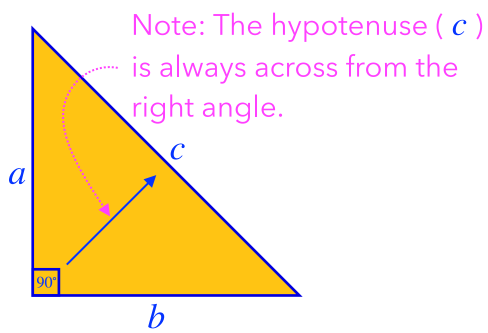
$$\tan(25^\circ) = \frac{80}{d}$$

$$d \approx 171.586\,.$$



Pythagorean Theorem:
Example: Legs $3\,cm$, $4\,cm$.

Solution:
$c = 5\,cm$.



Example: A $3\,m$ ladder is leaning against a vertical wall. The base of the ladder is $2\,m$ from the wall, how far up the wall does the ladder reach, correct to two decimal places?

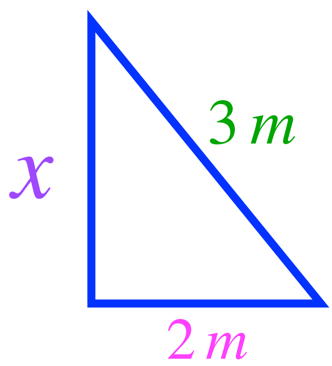
Solution:
Draw a diagram:

Apply Pythagoras' theorem :

For a right-angled triangle , $a^2 + b^2 = c^2$, where c is the hypotenuse.

$$2^2 + x^2 = 3^2$$

$$x \approx 2.24\,m\,.$$



Term 3
Unit 5 - Volume, Surface Area, and Networks:

Space

Volume and Surface Area:

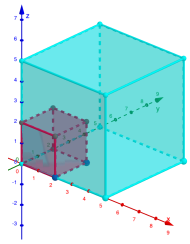
Example: Calculate the volume of a solid made by removing a smaller cube of side length $2\,cm$ from one corner of a larger cube with a side length of $5\,cm$.

Solution:

$$V_{cube} = s^3$$

Volume of composite solid :

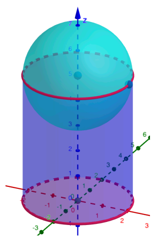
$$\text{Volume} = V_{large\ cube} - V_{small\ cube}$$
$$= 117\,cm^3.$$



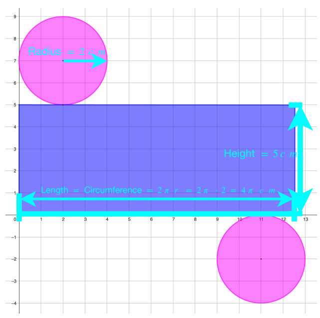
Example: Calculate the surface area of a cylinder with a radius of $2\,cm$ and height of $5\,cm$ if a hemisphere of radius $2\,cm$ is attached to one of its circular bases.

Solution:

Surface area of the cylinder without bases (rectangle) :



$$A_{rectangle} = \text{Length} \times \text{Width} = \text{Circumference of Circle} \times \text{Height}$$
$$= 2\pi r \times h$$



Area of one circular base of the cylinder :

$$A_{circle} = \pi r^2$$

Surface area of the hemisphere (excluding base) :

$$A_{whole\ sphere} = 4\pi r^2$$

$$A_{hemisphere} = \frac{1}{2} \times 4\pi r^2$$

Total surface area :

$$20\pi + 4\pi + 8\pi = 32\pi cm^2.$$

Example: A pyramid with a square base of side $4\,cm$ and height $6\,cm$ sits atop a cube with a side length of $4\,cm$. Find the volume and total surface area.

Solution:

Cube's volume :

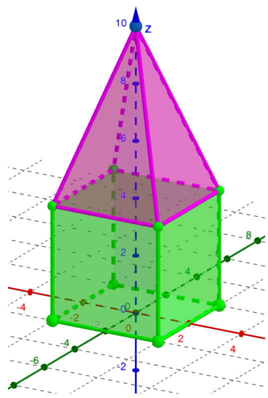
$$V_{cube} = s^3$$

Pyramid's volume :

$$V_{pyramid} = \frac{1}{3} A_{base} h$$

Total volume :

$$64 + 32 = 96\,cm^3.$$



Calculate height using Pythagoras :

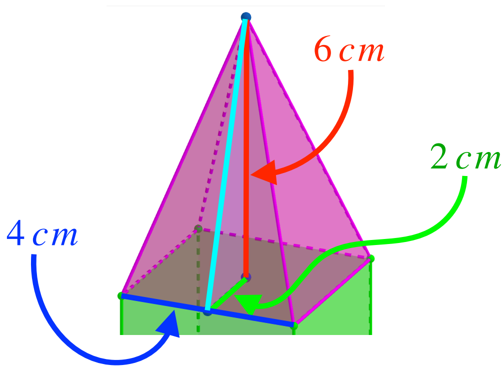
$$c^2 = a^2 + b^2$$

We now have the triangle's perpendicular height

$$A_{exposed\ pyramid} = 4 \times \frac{1}{2}bh$$

Total Surface Area :

$$80 + 50.6 = 130.6\, cm^2.$$



Networks

Example: A small town has four locations: a Library L, a Post Office P, a Supermarket S, and a Train Station T. The connections between them are represented in a network diagram as follows :

L is connected to P and S.
P is connected to L and T.
S is connected to L and T.
T is connected to P and S.

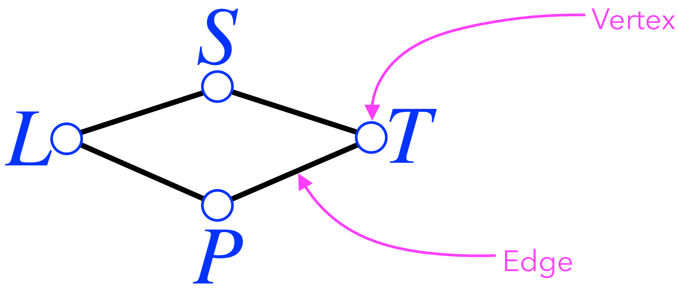
Draw the network diagram based on the description, then describe the connectedness of the network, and list the degree of each vertex.

Solution:

Vertices: L (Library), P (Post Office), S (Supermarket), T (Train Station).

Edges: $L - P,\ L - S,\ P - T,\ S - T$.

The network can be visualised as a quadrilateral with vertices $L,\ P,\ S,\ T,$ where:



A network is *connected* if there is a path between every pair of vertices.

The network is *connected* because there is at least one path (direct or indirect) between every pair of vertices.

Degree of vertices :

L : Degree 2 (connected to P, S).
P : Degree 2 (connected to L, T).
S : Degree 2 (connected to L, T).
T : Degree 2 (connected to P, S).

A *connected graph*, is one where there exists at least one path (direct or indirect) between each pair of distinct vertices.

The network forms a simple cycle ($L - P - T - S - L$), indicating strong connectedness.

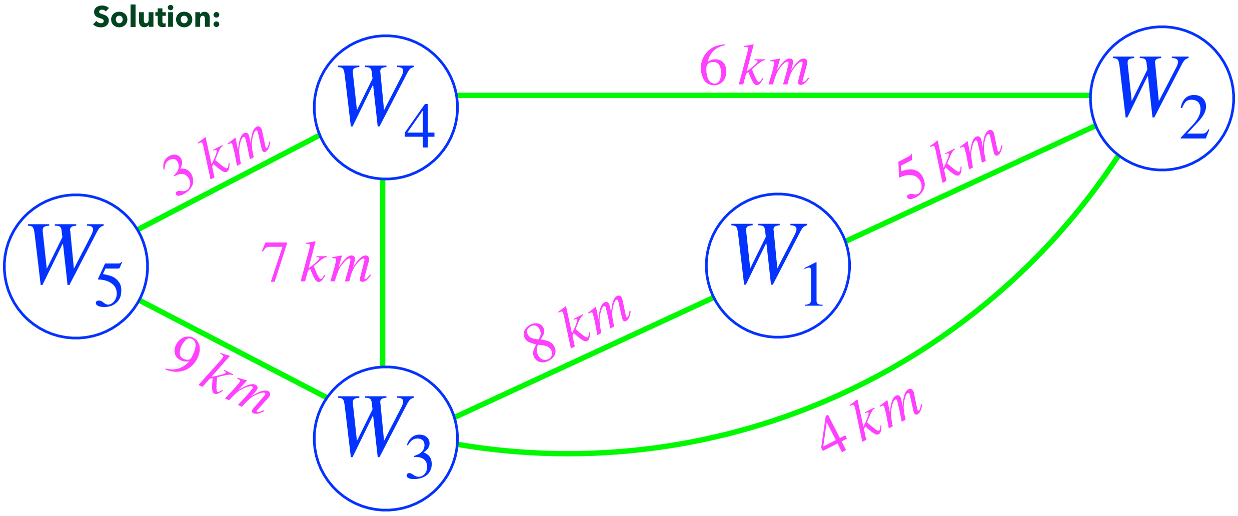
Example: A delivery company operates between five warehouses : W_1, W_2, W_3, W_4, and W_5 .

Possible routes with distances (in $kilometres$) :

W_1 to W_2: $5\,km$
W_1 to W_3: $8\,km$
W_2 to W_3: $4\,km$
W_2 to W_4: $6\,km$
W_3 to W_4: $7\,km$
W_4 to W_5: $3\,km$
W_3 to W_5: $9\,km$

Draw and interpret the network to identify all possible paths from W_1 to W_5 , then determine the shortest path from W_1 to W_5 and its total distance.

Solution:



Calculate the total distance for each path:
Path 1: $17\,km$, Path 2: $18\,km$, Path 3: $14\,km$, Path 4: $18\,km$, Path 5: $19\,km$.

The shortest path is $W_1 - W_2 - W_4 - W_5$ with a total distance of $14\,km$.

Unit 6 - Algorithms, and Geometric Proofs

Introduction to Algorithms:

Example: A robot is programmed to move on a 5×5 grid, starting at the origin $(0,0)$ and needing to reach the point $(4,4)$. The robot can only move right R or up U, and each move increments its $x\text{-}coordinate$ or $y\text{-}coordinate$ by 1. Design an algorithm to find one possible path for the robot,

Solution:

Step 1:

Understand the problem

The robot starts at $(0,0)$ and must reach $(4,4)$. To move from $(0,0)$ to $(4,4)$, the robot needs to increase its $x\text{-}coordinate$ by $+4$ (4 right moves) and $y\text{-}coordinate$ by $+4$ (4 up moves). Total moves required: 4 right + 4 up = 8 moves. The path is a sequence of 8 moves (e.g., $RRRRUUUU$ or $URURURUR$), and we need to find one valid sequence.

Step 2:

Design an Algorithm

Pseudocode:

```
START at position (0,0)
SET x = 0, y = 0
WHILE x < 4
    MOVE right
    INCREMENT x by 1
ENDWHILE
WHILE y < 4
    MOVE up
    INCREMENT y by 1
ENDWHILE
STOP at position (4,4)
```

Example: A treasure is hidden in a 4×4 grid (with cells labeled from $(1,1)$ to $(4,4)$), and a robot must locate it by checking cells one at a time. The robot can move to any adjacent cell (up, down, left, or right) in one step and starts at $(1,1)$. Design an algorithm to ensure the robot checks every cell in the grid (covering all 16 cells), represent the solution using a digital tool (e.g., pseudocode or a flowchart), test the algorithm, and justify why it guarantees finding the treasure.

Solution:

Step 1:

Understand the Problem

The 4×4 grid has cells at coordinates (x, y) where $x, y \in 1,2,3,4$.

The robot's moves are: up $(y + 1)$, down $(y - 1)$, left $(x - 1)$, right $(x + 1)$.

The goal is to visit all 16 cells, ensuring no cell is missed.

This is a path-planning problem, similar to a Hamiltonian path or a grid traversal (e.g., snake-like pattern or spiral).

Step 2:

Design an Algorithm

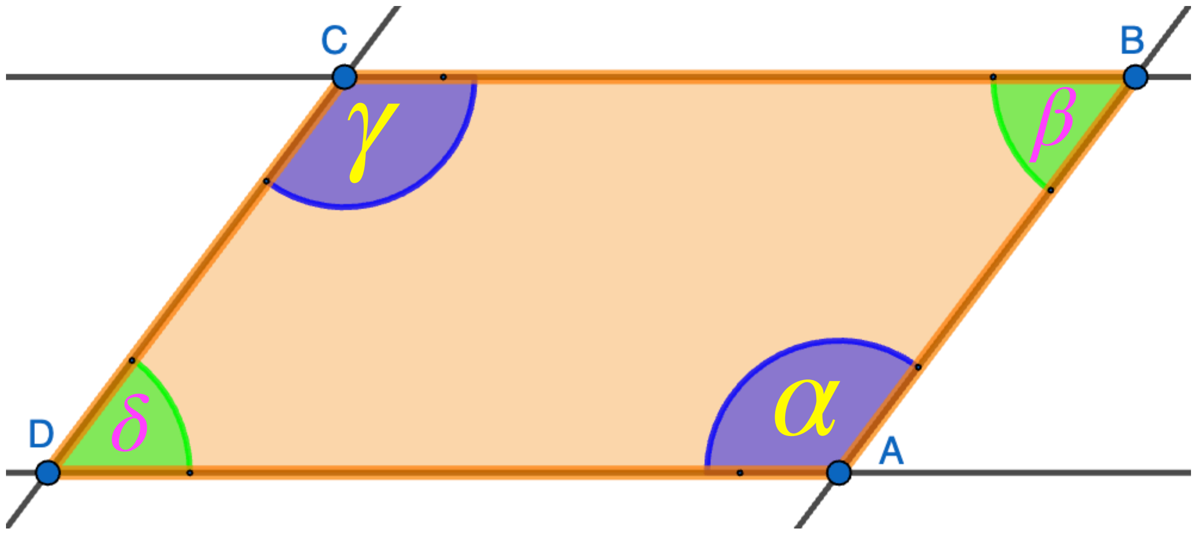
A simple and systematic approach is to move in a zig-zag (snake-like) pattern: traverse each row from left to right, then move down to the next row and traverse right to left, alternating directions. This ensures all cells are visited efficiently.

Pseudocode:

```
START at position (1,1)
SET visited_cells = [(1,1)]
SET current_x = 1, current_y = 1
WHILE visited_cells count < 16
    IF current_y is odd
        WHILE current_x < 4 AND (current_x + 1, current_y) not visited
            MOVE right
            INCREMENT current_x by 1
            ADD (current_x, current_y) to visited_cells
        ENDWHILE
        IF current_y < 4
            MOVE down
            INCREMENT current_y by 1
            ADD (current_x, current_y) to visited_cells
        ENDIF
    ELSE
        WHILE current_x > 1 AND (current_x - 1, current_y) not visited
            MOVE left
            DECREMENT current_x by 1
            ADD (current_x, current_y) to visited_cells
        ENDWHILE
        IF current_y < 4
            MOVE down
            INCREMENT current_y by 1
            ADD (current_x, current_y) to visited_cells
        ENDIF
    ENDIF
ENDWHILE
STOP
```

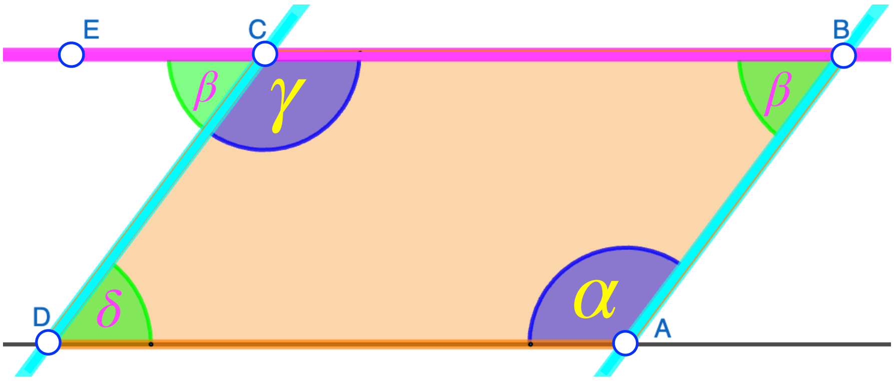
Geometric Proofs:

Example: Prove that the opposite angles of a parallelogram are equal using deductive reasoning.



Solution:

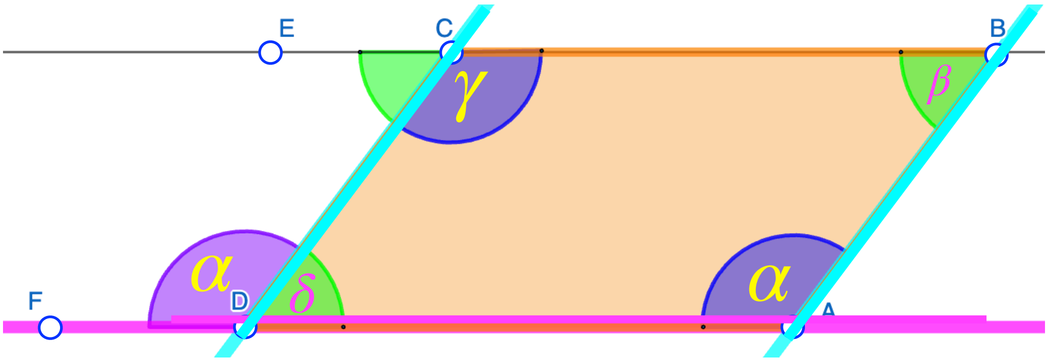
Prove that the opposite angles of a parallelogram are equal



Since $AB||DC$, the alternate interior angles formed by transversal BC are equal (by the Alternate Interior Angles Theorem).

Therefore, $\angle ABC = \angle DCE$. (Let's call this angle $pair\ 1$.)

Now consider the transversal AD intersecting AB at A and DC at D.



Since AB || DC, the alternate interior angles formed by transversal AD are equal.

Therefore, $\angle BAD = \angle CDF$. (Let's call this angle $pair\ 2$.)

Thus, using $pair\ 1$ and $\angle ABC + \angle BCD = 180^\circ$, i.e $\beta + \gamma = 180^\circ$.

And $\angle ADC + \angle BCD = 180^\circ$, i.e $\delta + \gamma = 180^\circ$.

$$\angle ABC + \angle BCD = 180^\circ = \angle ADC + \angle BCD$$
$$\rightarrow \angle ABC = \angle ADC$$
$$\text{i.e. } \beta = \delta$$

(both are supplements of the same angle $\angle BCD\ (\gamma)$).

Similarly, using $pair\ 2$ and $\angle BAD + \angle ABC = 180^\circ$, i.e $\alpha + \beta = 180^\circ$.

$$\rightarrow \angle BAD = \angle BCD$$
$$\text{i.e. } \alpha = \gamma$$

(both are supplements of the same angle $\angle ADC\ (\delta)$).

∴ The opposite angles of a parallelogram are equal.

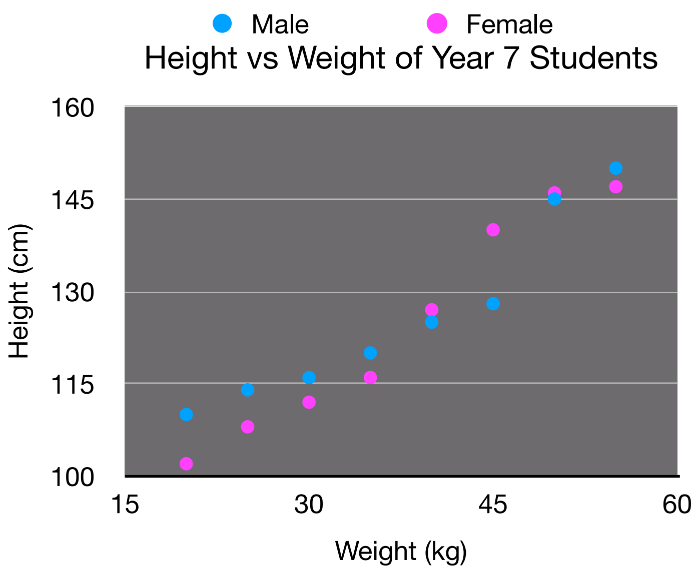
Term 4
Unit 7 - Multivariate Data Analysis, and Bias

3. Statistics and Probability

Multivariate Data:

Scatterplots

Example: Interpret the following scatter plot where height (cm) is plotted against weight (kg):



Solution:

The scatter plot suggests a positive correlation between height and weight. As height increases, weight generally increases, indicating that taller individuals tend to weigh more.

Line of Best Fit

Example: Given data points $(1, 2)$, $(2, 4)$, $(3, 5)$, and $(4, 7)$, find the equation of the line of best fit using the first and last points.

Solution:

Slope , $(m) \rightarrow m = \dfrac{(y_2 - y_1)}{(x_2 - x_1)}$

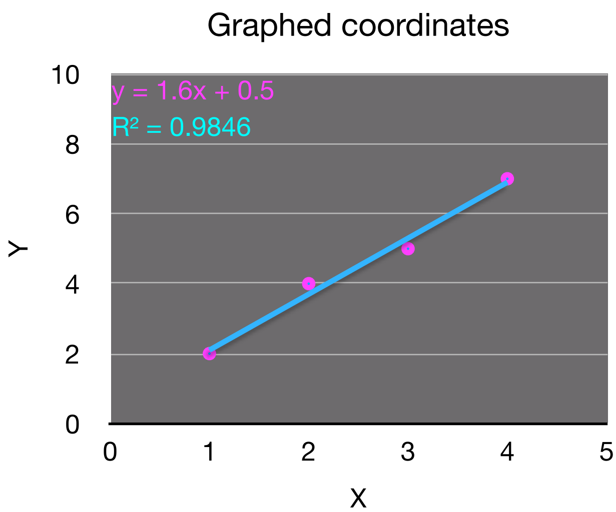
$m = 1.6\dot{6}$.

Y-intercept (c) (using $(x_1, y_1) = (1, 2)$ and $m \approx 1.7$)

$\rightarrow y = mx + c$

$c = 0.3$.

Equation : $y = 1.7x + 0.3$.



Two-way tables

Example: A class of 30 Year 10 students was surveyed to determine their preferred after-school activity (Sports or Arts) and whether they own a pet (Yes or No). The results are summarised as follows:

12 students prefer Sports and own a pet.
8 students prefer Sports and do not own a pet.
6 students prefer Arts and own a pet.
4 students prefer Arts and do not own a pet.

Calculate the marginal totals and the grand total.

Solution:

	Sports	**Arts**	Total
Pet: Yes	12	6	18
Pet: No	8	4	12
Total	20	10	30

Road Statistics

Example: According to research from the Queensland University of Technology : mobile phone distraction was estimated to contribute to 18 % of fatal crashes and 5 % of injury crashes. Despite the dangers and illegality, approximately half of drivers in Queensland admitted to using their mobile phone for browsing or texting while driving. Naturalistic studies show hand-held phone use increases crash risk by a factor of 3.63 . In 2025 there were a total of 1,196 fatalities, determine what number of these are estimated to be from mobile phone distraction.

Solution:
18 % of fatalities due to moblie phone distraction,
1,196 crashes :
≈ 215 fatalities due to mobile phone distraction.

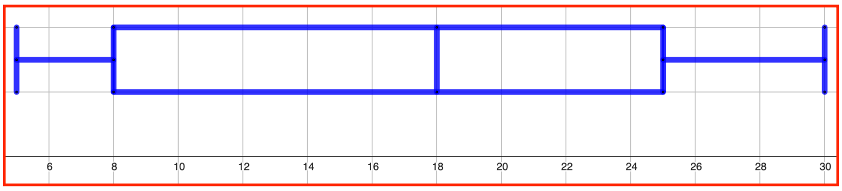
Univariate Data:

Five-Number Summaries and Box-Plots

Example: Given the following dataset: $\{ 5, 7, 8, 12, 15, 18, 20, 22, 25, 28, 30 \}$. List the five-number summary, then construct a box plot.

Solution:

Minimum:	5
Q_1 :	Median of the lower half (5, 7, 8, 12, 15) is 8
Median (Q_2):	18 (middle number)
Q_3 :	Median of the upper half (20, 22, 25, 28, 30) is 25
Maximum:	30



Example: What would you do if you had to check for outliers before drawing the box plot?

Solution:

Use the Interquartile Range (IQR) method:

Any data point,

Below : ($Q_1 - 1.5 \times IQR$) or,

Above : ($Q_3 + 1.5 \times IQR$) is considered an outlier.

Lower bound: $8 - 1.5 \times 17$

$= -17.5$. (no data points / outliers here)

→ There are no data points below -17.5 in the dataset.

Upper bound: $25 + 1.5 \times 17$

$= 50.5$. (no data points / outliers here either)

→ There are no data points above 50.5 in the dataset.

Bias in the Media

Example: A news article claims: "$80\,\%$ *of teenagers prefer online learning over traditional classroom learning, based on a survey of* 100 *students from a single private school."* Identify a potential source of bias in this claim.

Solution:

The survey is biased due to sampling bias. The sample of 100 students is drawn from a single private school, which is not representative of all teenagers. Private school students may have different access to technology or educational preferences compared to students in public schools or rural areas, skewing the results.

Example: A social media post claims: *"Crime rates have doubled in the past year, based on police reports from one suburb."* Critique the inference made in this claim.

Solution:

The inference that "crime rates have doubled" is misleading because it generalises data from one suburb to an entire region or population, committing a hasty generalisation fallacy. The claim lacks context about whether the suburb is representative, the sample size of the police reports, or whether the increase is statistically significant. Other factors, such as increased police reporting or a small baseline crime rate, could exaggerate the percentage increase.

Unit 8 - Conditional Probability

Conditional Probability:

Example: A card is drawn from a standard deck. What is the probability that it's a heart given that it's red?

Solution:

There are 26 red cards, and 13 of these are hearts.

$$P(Heart|Red) = \frac{P(Heart \cap Red)}{P(Red)}$$

$$= \frac{\frac{13}{52}}{\frac{26}{52}}$$

$$= \frac{13}{52} \div \frac{26}{52}$$

$$= \frac{13}{52} \times \frac{52}{26}$$

$$= \frac{1}{2} = 0.5 = 50\,\%\,.$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

Example: If $40\,\%$ of students play soccer, $30\,\%$ play basketball, and $15\,\%$ play both, what is the probability that a student plays soccer given they play basketball?

Solution:

$$P(Soccer|Basketball) = \frac{P(Soccer \cap Basketball)}{P(Basketball)}$$

$$= \frac{1}{2} = 0.5 = 50\,\%\,.$$

Independent vs. Dependent Events

Example: Two dice are rolled. Is knowing that one die shows a 4 independent of the other die showing a 2 ?, and what is the probability that the second die shows a two?

Solution:

Yes, these events are independent because the outcome of one die does not affect the outcome of the other. Therefore,

$$P(\text{ Second die shows } 2 \,|\, \text{ First die shows } 4\ ) = P(\text{ Second die shows } 2\ )$$

$$= \frac{1}{6} = 0.16\dot{6} = 16.6\dot{6}\,\%\,.$$

Example: An allergy test can determine if you are allergic to peanuts. Sometimes though, the results of the test are incorrect. There are two conditions which arise:

1. For people that ARE allergic, only $60\,\%$ of tests correctly read positive, i.e. the result is positive for having an allergy only 60% of the time even though all of these people are allergic. This gives $40\,\%$ $(100\% - 60\%)$ of results as "false negatives", i.e. $40\,\%$ of people that are allergic will receive a negative result for having an allergy.

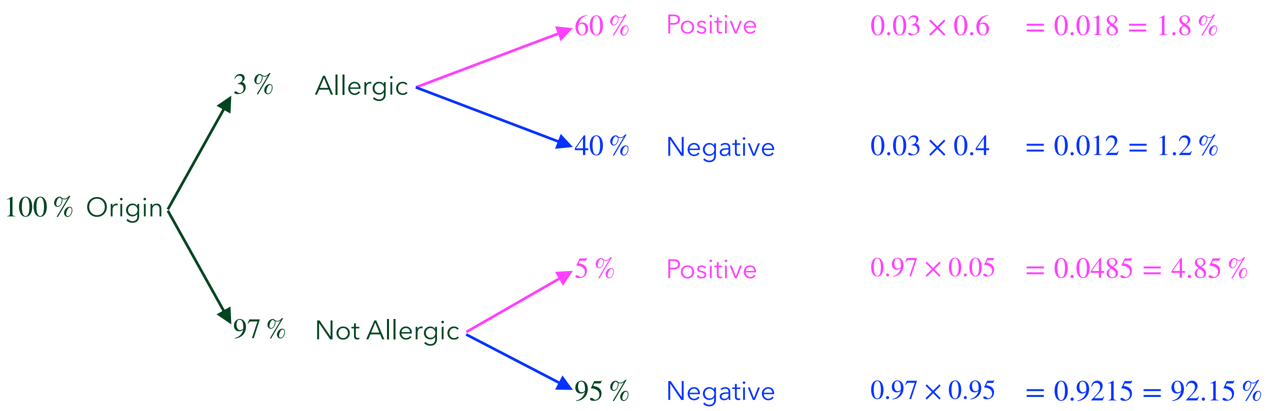
2. For people that ARE NOT allergic, the test can still be positive $5\,\%$ of the time, i.e. the result is positive for having an allergy $5\,\%$ of the time, even though these people are not allergic. This is known as a "false positive". This gives $95\,\%$ $(100\% - 5\%)$ of tests correctly reading negative.

If $3\,\%$ of the population have the allergy, and a patient claims to be allergic, what are the chances that they really have the allergy? i.e. calculate : $P(Allergic\,|\,Positive)$.

Solution:

Peanut Allergy Test Results Table:

	Positive	Negative
Allergic	60%	40% (False Negative)
Not Allergic	5% (False Positive)	95%



We want to know the chance of being allergic when the test gives a "positive" result. The positive results add to: $0.018 + 0.0485 = 0.0665$, and $3\,\%$ of the population are allergic, but only $60\,\%$ of results are correct (True) :

$P(Allergic\,|\,Positive) =$ ($3\,\%$ percent of population $\times 60\,\%$ correct results) out of ($6.65\,\%$ positive results)

$$= \frac{3\,\%\ percent\ of\ population \times 60\,\%\ correct\ results}{6.65\,\%\ positive\ results}$$

$$= \frac{0.03 \times 0.6}{0.0665}$$

$P(A\,|\,+) \approx 0.27 = 27\,\%$.

$$P(A\,|\,B) = \frac{P(A) \times P(B\,|\,A)}{P(A) \times P(B\,|\,A) + P(\neg A) \times P(B\,|\,\neg A)}$$

These cheat sheets are designed to give students a quick reference for key concepts and methods. However, for both Year 10 and 10A, deep understanding through practice and exploration of applications is crucial. Use textbooks, online resources, and practice papers tailored to the Australian Curriculum for comprehensive learning.